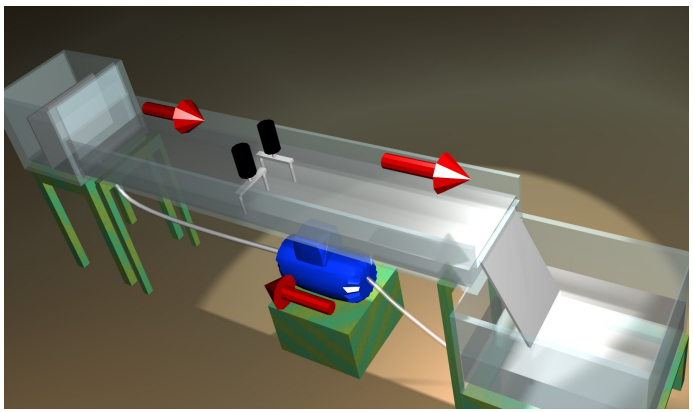
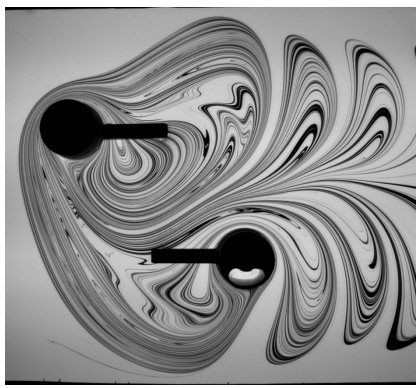
Stirring and Mixing of Viscous Fluids

- Viscous flows $\Rightarrow$ no turbulence! (laminar)
- Open and closed systems
- Active (rods) and passive





A Simple Example: Planetary Mixers

In food processing, rods are often used for stirring.

[movie 1] ©BLT Inc.

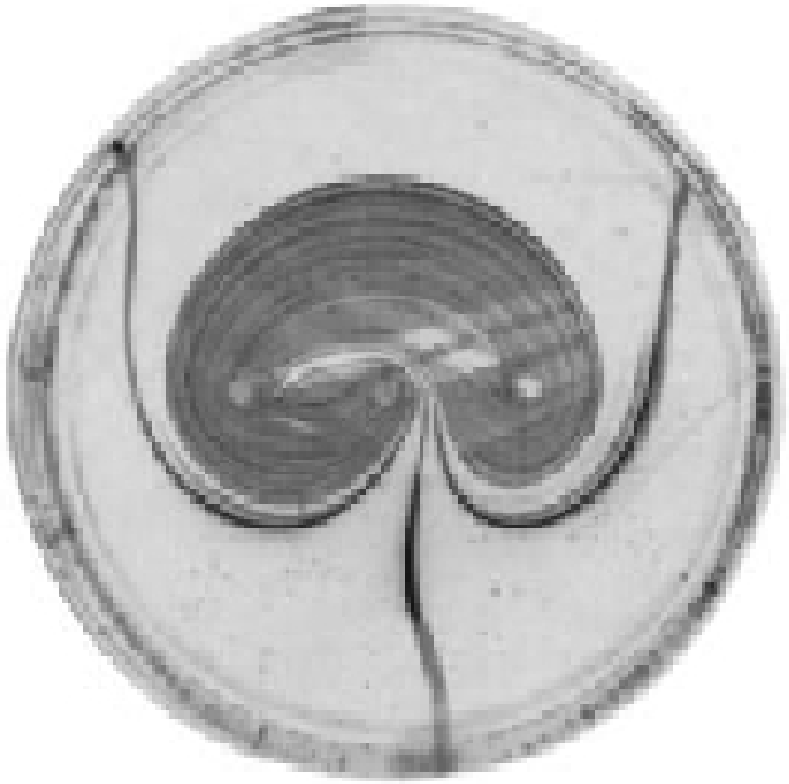
The Taffy Puller

This may not look like it has much to do with stirring, but notice how the taffy is stretched and folded exponentially.

Often the hydrodynamics are less important than the precise nature of the rod motion!

[movie 2]

Experiment of Boyland, Aref, & Stremler

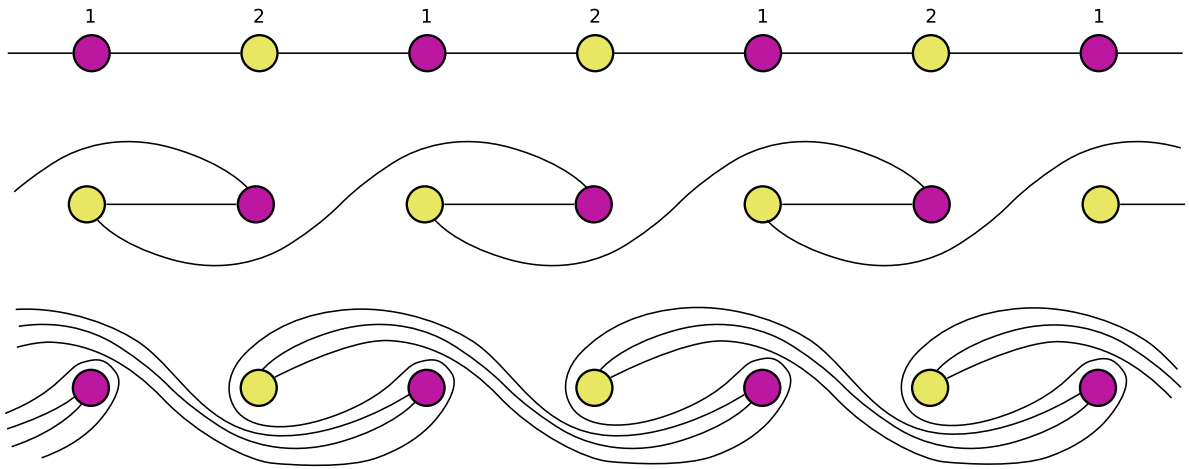


[movie 3] [movie 4]

[P. L. Boyland, H. Aref, and M. A. Stremler, *J. Fluid Mech.* **403**, 277 (2000)]

Periodic Array of Rods

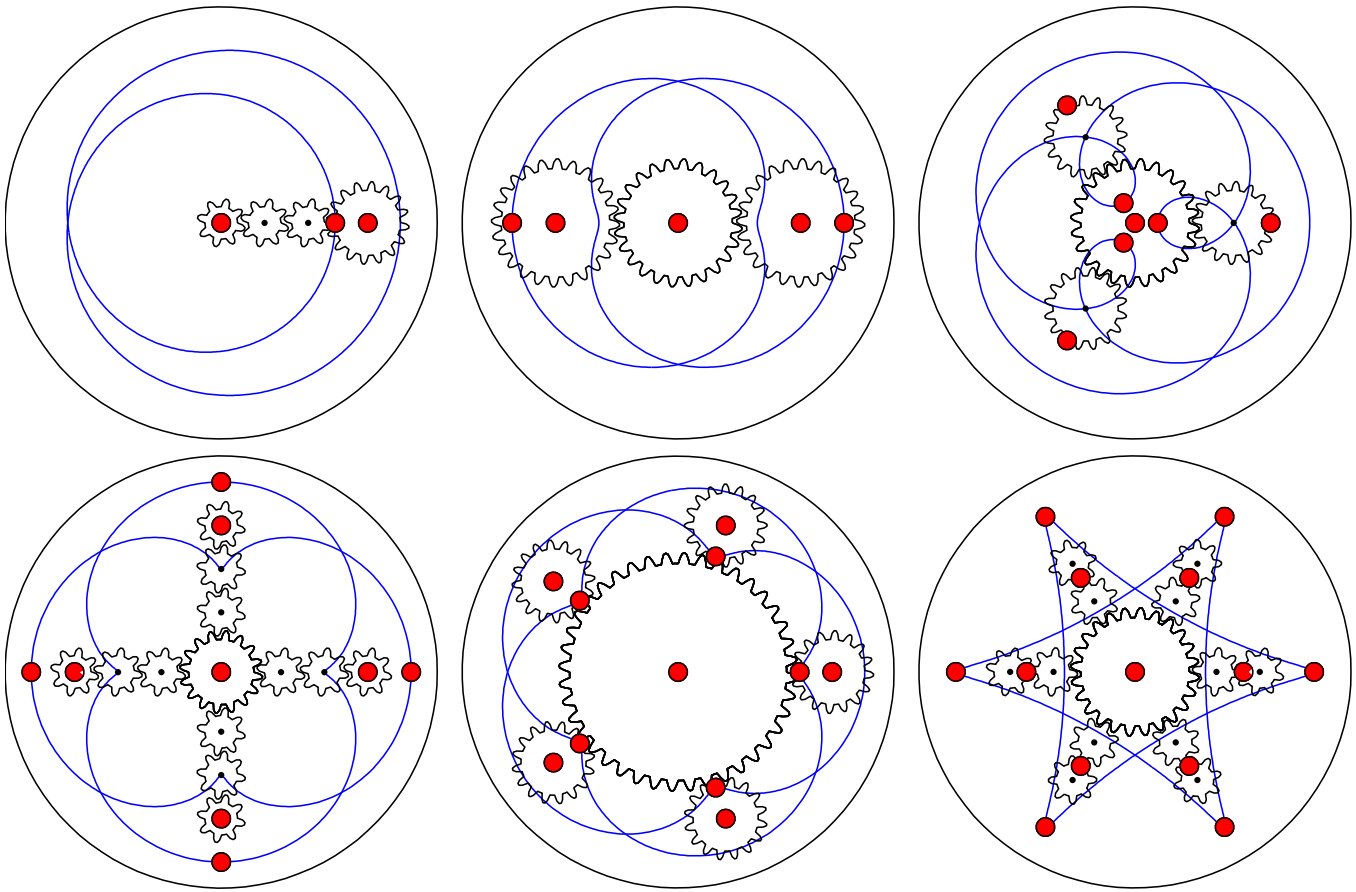
- Consider periodic lattice of rods.
- Move all the rods periodically as in Boyland et al. (2000):



- The stretching per 'switch' is $\log \chi$, where $\chi = 1 + \sqrt{2}$ is the Silver Ratio!
- This is optimal for a periodic lattice of two rods (Follows from D'Alessandro et al. (1999)).
- Work with M. D. Finn (Adelaide).

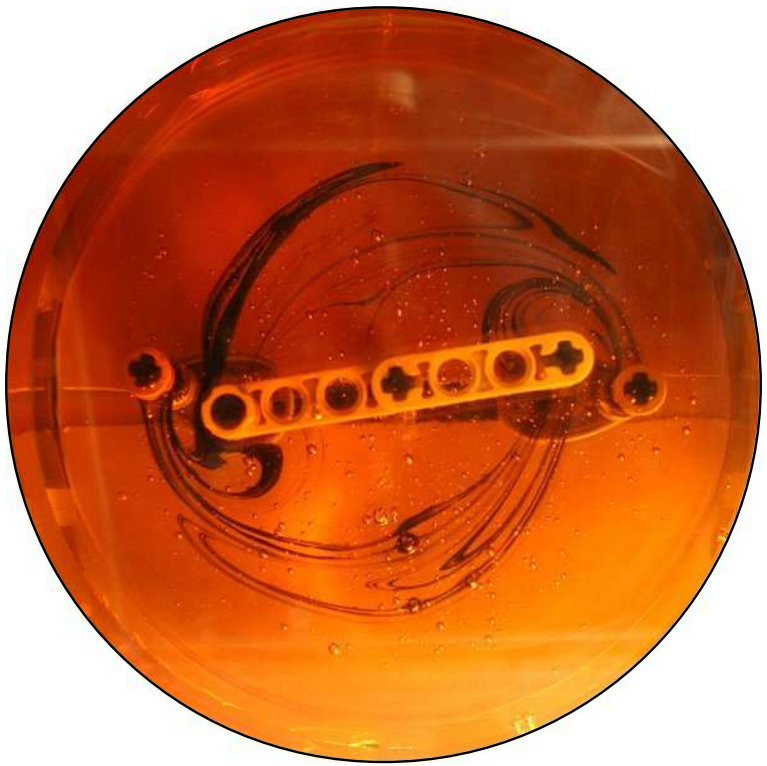
Silver Mixers!

- The designs with entropy given by the silver ratio can be realised with simple gears.
- All the rods move at once: very efficient.



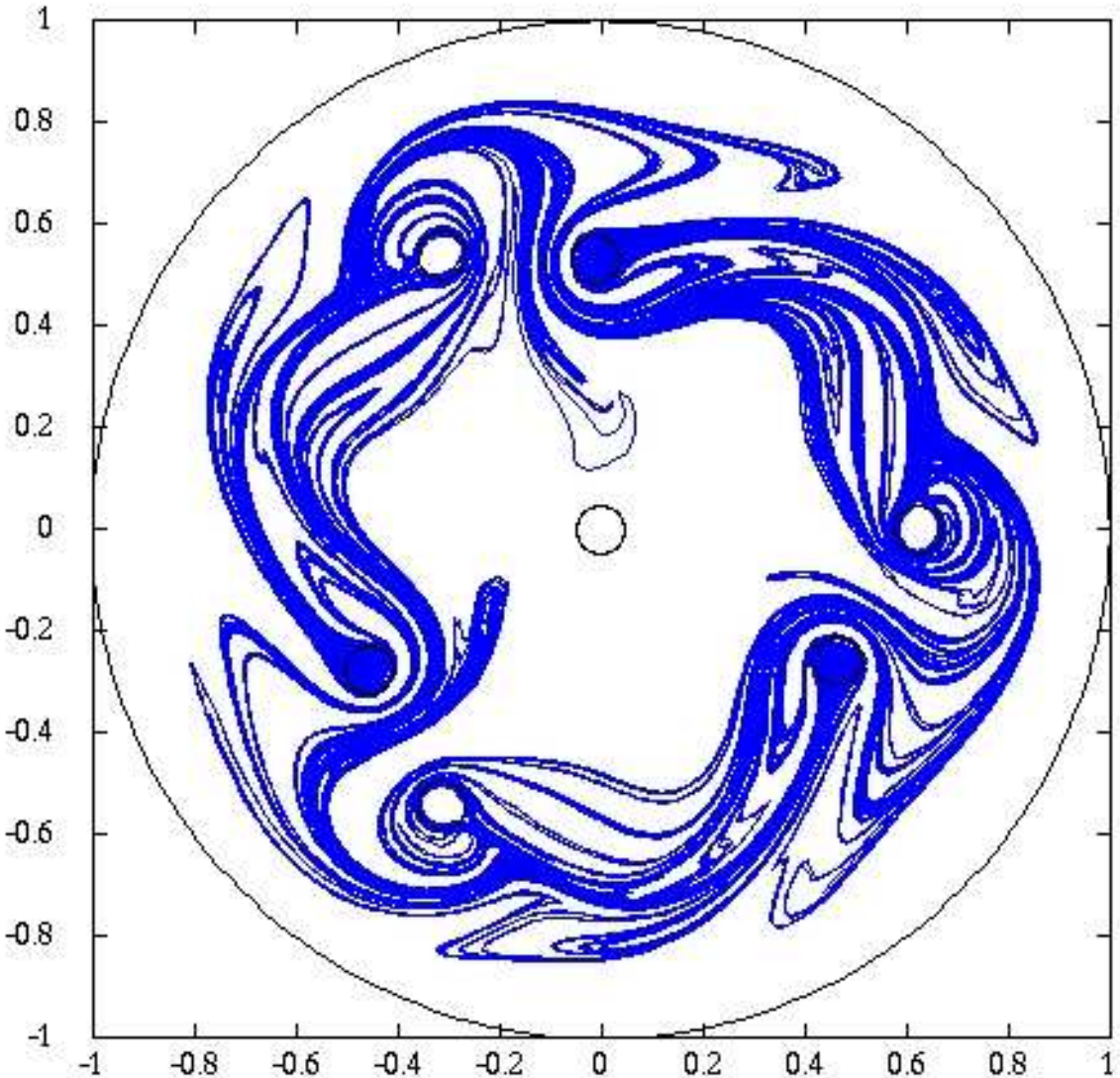
[movie 5]

Four Rods



[movie 6] [movie 7]

Six Rods



[movie 8]

Conclusions

- Having rods undergo 'braiding' motion guarantees a minimal amound of entropy (stretching of material lines).
- We have an optimal design, the silver mixers.
- Need to also optimise other mixing measures, such as variance decay rate.

Other topological aspects:

- Topology also predicts injection into the mixing region, important for open flows.
- Classify all rod motions and periodic orbits according to their topological properties.
- Train track automata allow exploration of possible pseudo-Anosovs, and can be used for rigorous proofs.

References

Aref, H. 1984 Stirring by Chaotic Advection. *J. Fluid Mech.* **143**, 1–21.

Bestvina, M. & Handel, M. 1992 Train Tracks for ad Automorphisms of Free Groups. *Ann. Math.* **134**, 1–51.

Binder, B. J. & Cox, S. M. 2007 A Mixer Design for the Pigtail Braid. *Fluid Dyn. Res.* In press.

Boyland, P. L., Aref, H. & Stremler, M. A. 2000 Topological fluid mechanics of stirring. *J. Fluid Mech.* **403**, 277–304.

Boyland, P. L., Stremler, M. A. & Aref, H. 2003 Topological fluid mechanics of point vortex motions. *Physica D* **175**, 69–95.

Chertkov, M. & Lebedev, V. 2003 Boundary Effects on Chaotic Advection-Diffusion Chemical Reactions. *Phys. Rev. Lett.* **90**, 134501.

D'Alessandro, D., Dahleh, M. & Mezić, I. 1999 Control of mixing in fluid flow: A maximum entropy approach. *IEEE Transactions on Automatic Control* **44**, 1852–1863.

Gouillart, E., Finn, M. D. & Thiffeault, J.-L. 2006 Topological Mixing with Ghost Rods. *Phys. Rev. E* **73**, 036311.

Gouillart, E., Kuncio, N., Dauchot, O., Dubrulle, B., Roux, S. & Thiffeault, J.-L. 2007 Walls Inhibit Chaotic Mixing. *Phys. Rev. Lett.* **99**, 114501.

Kobayashi, T. & Umeda, S. 2006 Realizing pseudo-Anosov egg beaters with simple mecanisms Preprint.

Moussafir, J.-O. 2006 On the Entropy of Braids. *Func. Anal. and Other Math.* **1**, 43–54.

Pierrehumbert, R. T. 1994 Tracer microstructure in the large-eddy dominated regime. *Chaos Solitons Fractals* **4**, 1091–1110.

Rothstein, D., Henry, E. & Gollub, J. P. 1999 Persistent patterns in transient chaotic fluid mixing. *Nature* **401**, 770–772.

Thiffeault, J.-L. 2005 Measuring Topological Chaos. *Phys. Rev. Lett.* **94**, 084502.

Thiffeault, J.-L. & Finn, M. D. 2006 Topology, Braids, and Mixing in Fluids. *Phil. Trans. R. Soc. Lond. A* **364**, 3251–3266.

Thurston, W. P. 1988 On the geometry and dynamics of diffeomorphisms of surfaces. *Bull. Am. Math. Soc.* **19**, 417–431.

Voth, G. A., Saint, T. C., Dobler, G. & Gollub, J. P. 2003 Mixing rates and symmetry breaking in two-dimensional chaotic flow. *Phys. Fluids* **15**, 2560–2566.