Stirring with Braids

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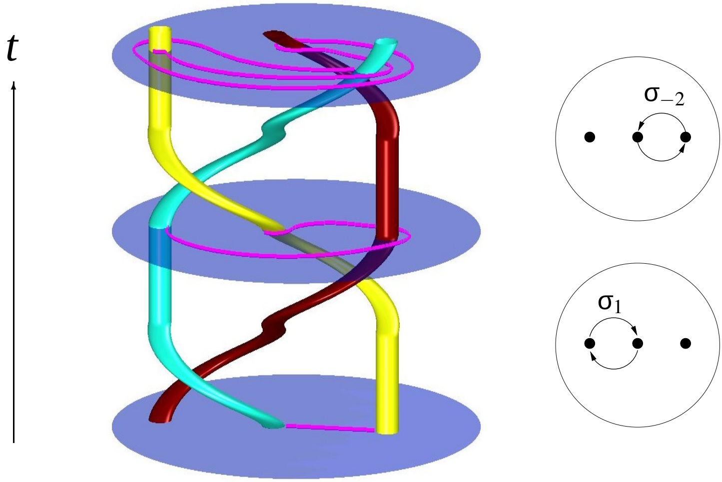
Talk for Prospective Students, 23 November 2005

The Taffy Puller

[movie 1]

The Four-pronged Taffy Puller

The Connection with Braids



What are Stirring and Mixing?

- Taffy machines are nice, but not such an important application!
- There are many applications where stirring is required.
- The coffee (or tea) cup is the classic example, but not so challenging because of turbulence.
- Much more difficult problems exist where the fluid is very viscous, and difficult to stir (example: molten glass).
- For those cases, want to stir using an optimal motion of rods.
- The theory of braids helps us choose such motions.

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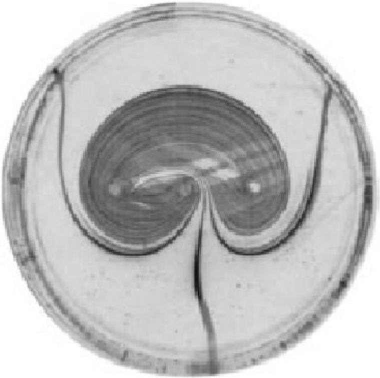
Experiment of Boyland, Aref, & Stremler

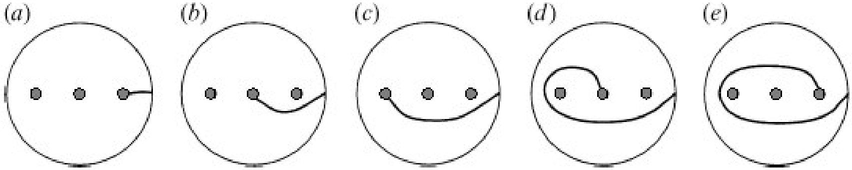


[movie 2] [movie 3]

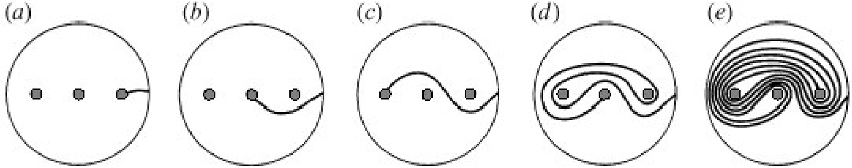
[P. L. Boyland, H. Aref, and M. A. Stremler, *J. Fluid Mech.* **403**, 277 (2000)]

The Two Stirring Methods

Bad. . .



Good!



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How Fast Does the Elastic Grow?

We can show that for the good protocol, the length of the elastic band is multiplied by a number ϕ,

$$\phi = \frac{1+\sqrt{5}}{2} = 1.6180\ldots$$

for each interchange of the rods.

Along with π, ϕ is probably the best known number in mathematics: it is the Golden Ratio! It seems to pop up everywhere...

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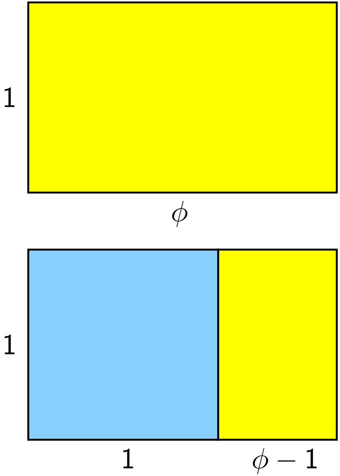
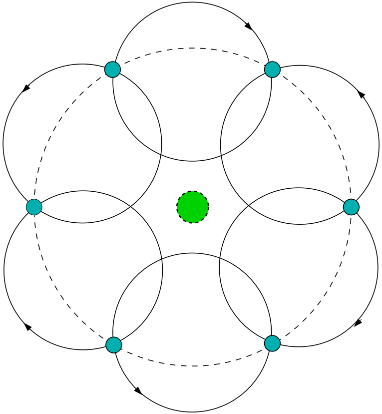
The Golden Ratio, ϕ



A rectangle has the proportions of the Golden Ratio if, after taking out a square, the remaining rectangle has the same proportions as the original:

$$\frac{\phi}{1} = \frac{1}{\phi - 1}$$

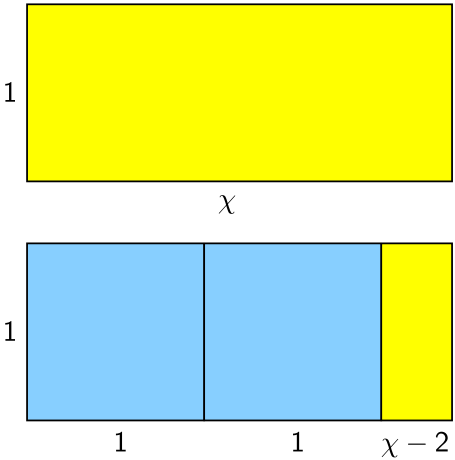
A Circular Stirring Device



For this stirring device, the elastic band grows by a factor of $1+\sqrt{2} = 2.4142\ldots$ at each cycle. This is the lesser-known Silver Ratio.

The Silver Ratio, χ

A rectangle has the proportions of the Silver Ratio if, after taking out two squares, the remaining rectangle has the same proportions as the original.



$$\frac{\chi}{1} = \frac{1}{\chi - 2}$$

Conclusions

- The motion of stirring rods can be described using braids.
- The properties of these braids tell us something about the efficiency of the stirring device for mixing real fluids.
- Together with my RA Matthew Finn and my PhD student Emmanuelle Gouillart, we have been looking for ways of improving existing devices using these principles.
- An important facet is whether the stirring device can be built! This is where mathematics meets engineering.
- This project thus covers the entire spectrum of applied mathematics: from the pure maths aspects (braid theory) to practical implementation in real applications.