
Braiding and Mixing

Periodic Boundary Conditions and Periodic Orbits

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Matthew Finn

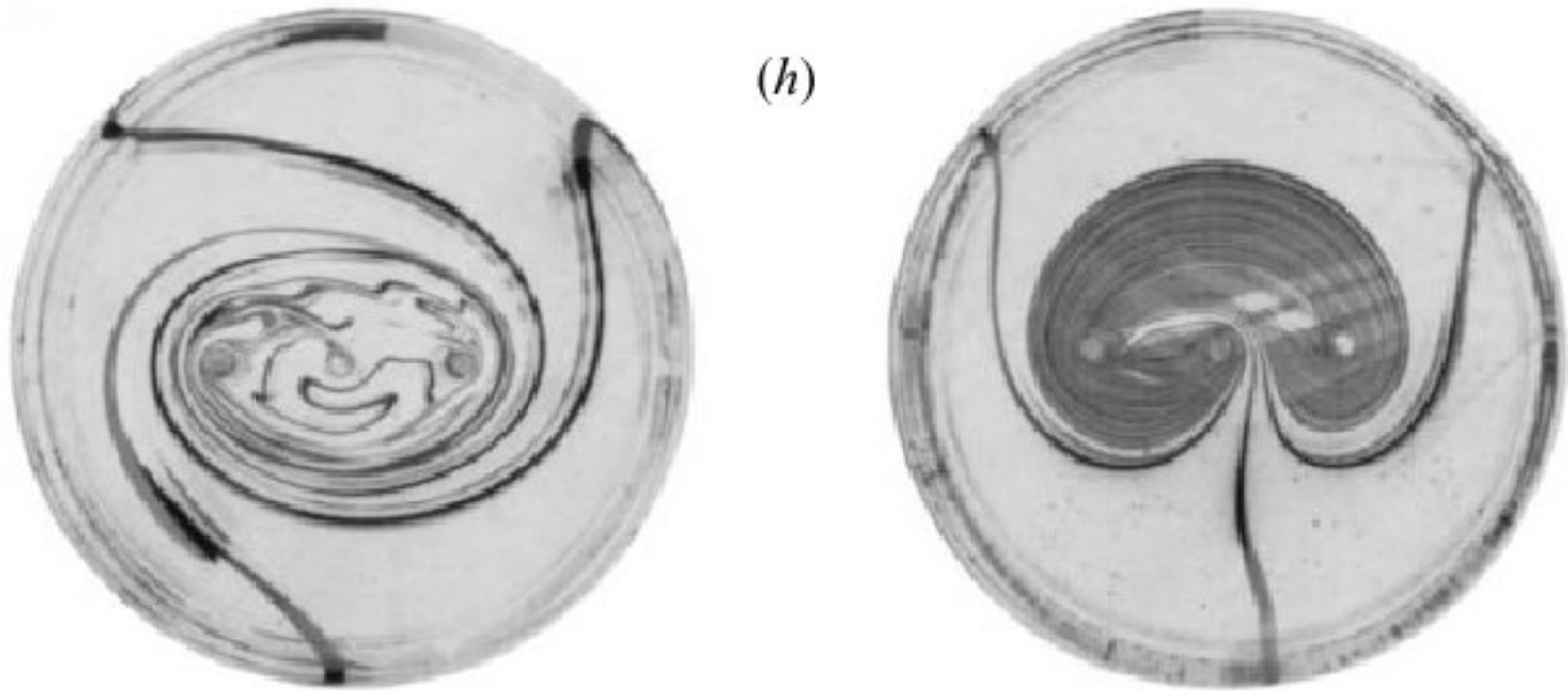
Emmanuelle Guillart

<http://www.ma.imperial.ac.uk/~jeanluc>

Department of Mathematics

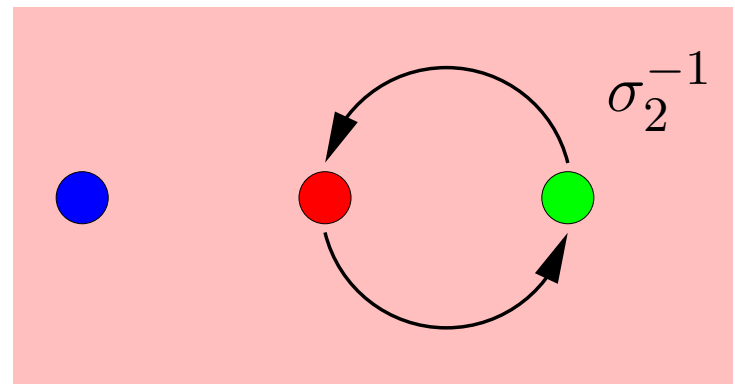
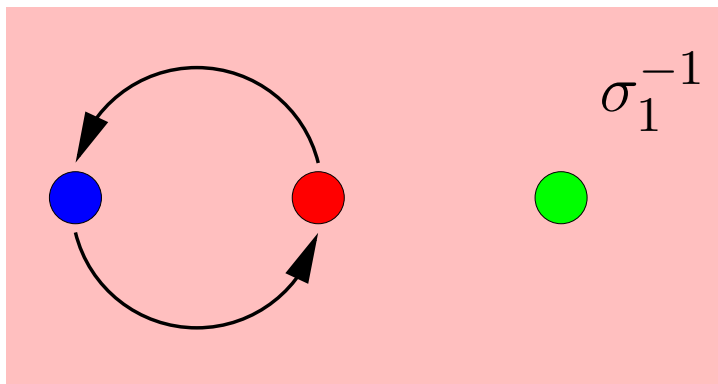
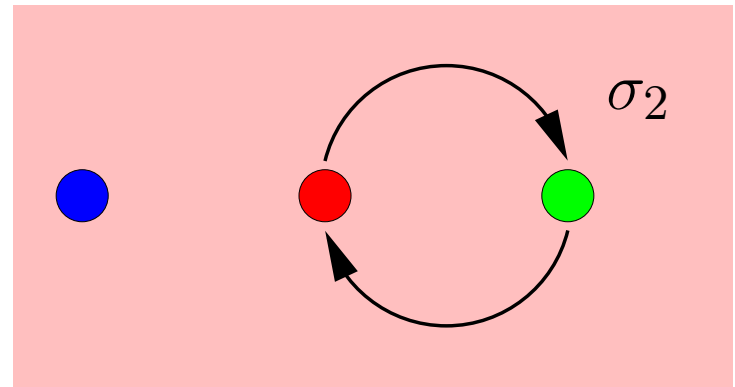
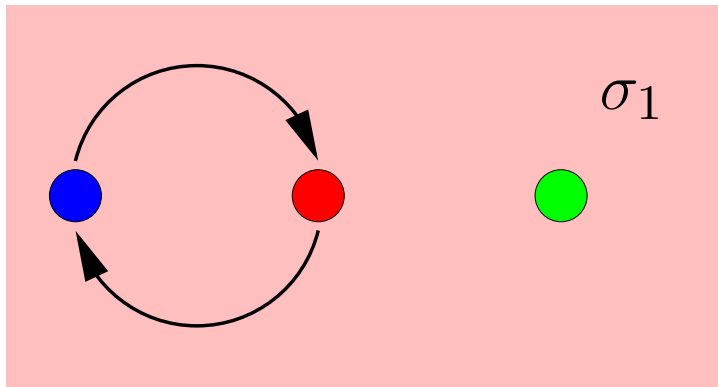
Imperial College London

Experiment of Boyland *et al.*



[P. L. Boyland, H. Aref, and M. A. Stremler, *J. Fluid Mech.* **403**, 277 (2000)]

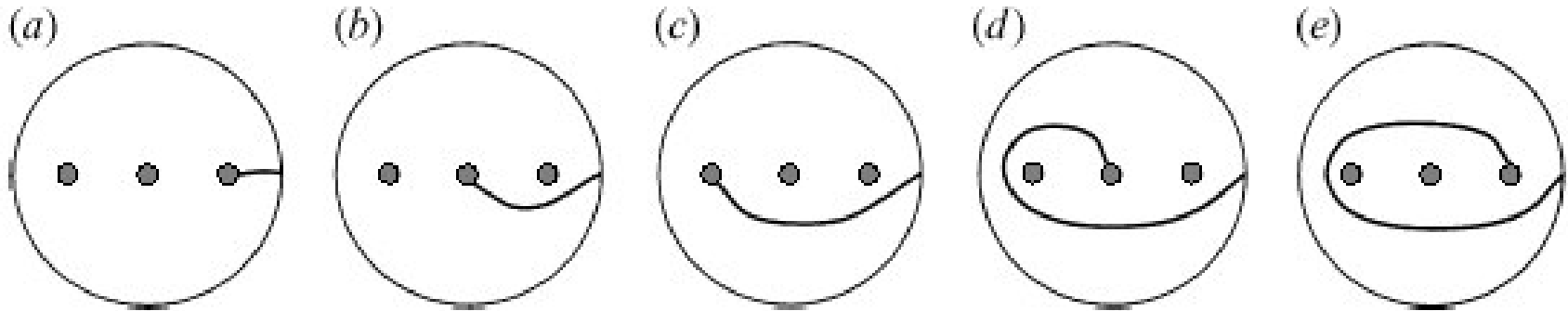
Four Basic Operations



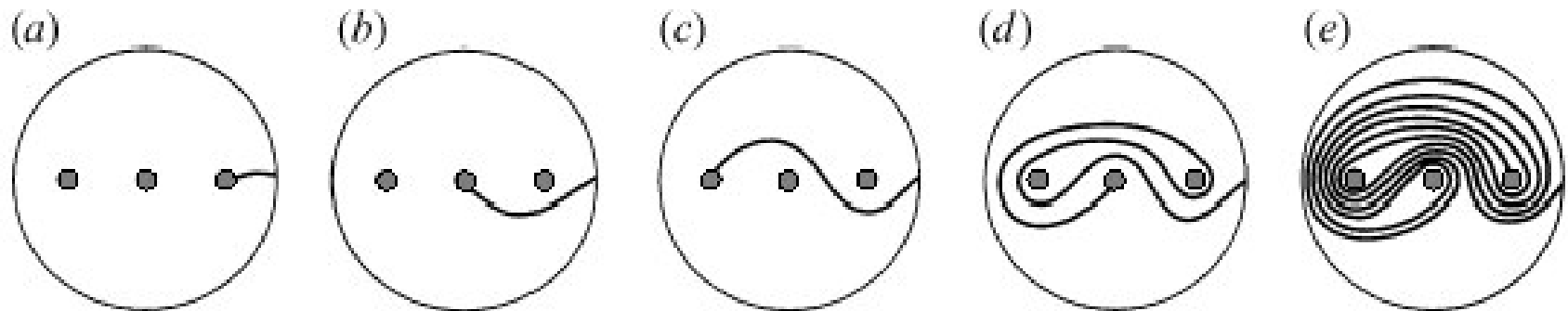
σ_1 and σ_2 are referred to as the **generators of the 3-braid group**.

Two Stirring Protocols

$\sigma_1\sigma_2$ protocol



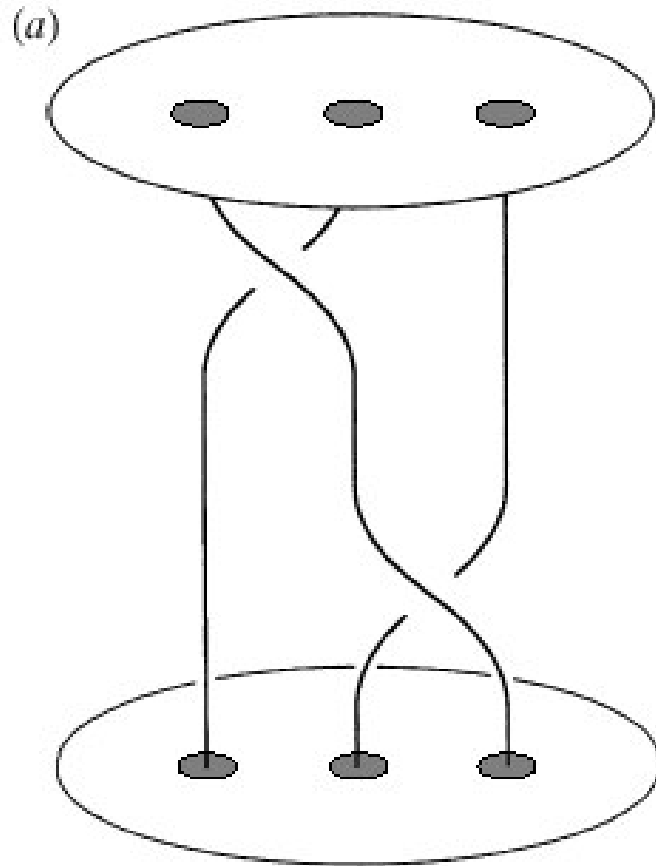
$\sigma_1^{-1}\sigma_2$ protocol



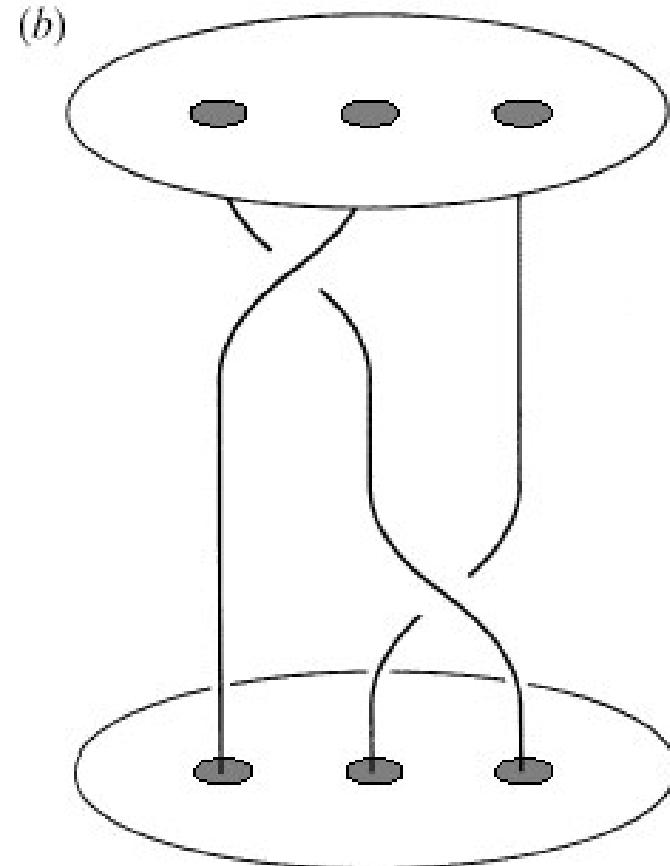
[P. L. Boyland, H. Aref, and M. A. Stremler, *J. Fluid Mech.* **403**, 277 (2000)]

Braiding

$\sigma_1\sigma_2$ protocol



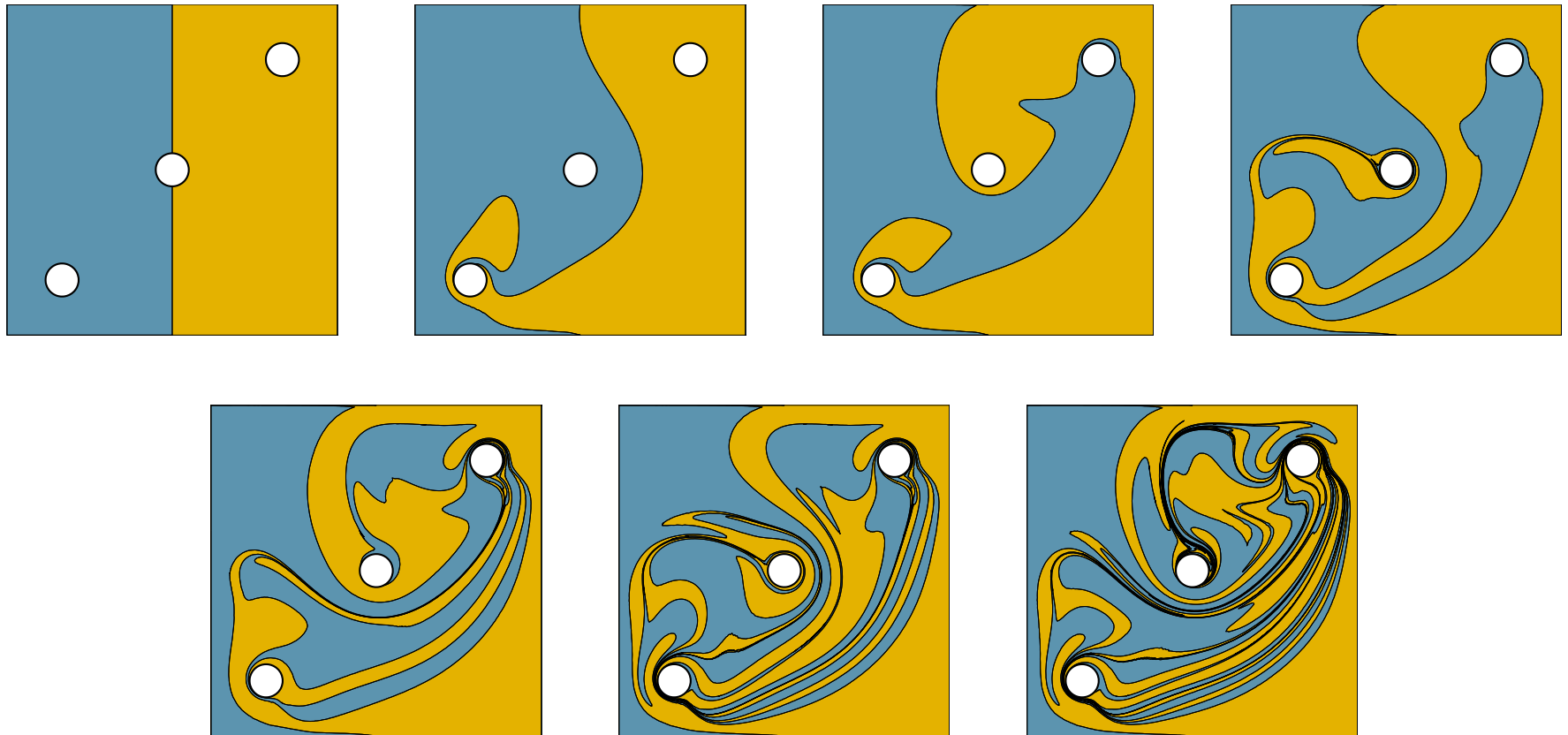
$\sigma_1^{-1}\sigma_2$ protocol



Time ↑

[P. L. Boyland, H. Aref, and M. A. Stremler, J. Fluid Mech. **403**, 277 (2000)]

Three-rod Mixer in a Bounded Domain



Three-rod Mixer in a Bounded Domain

[movie: bounded.mpg]

Computing the Line-stretching from a Braid

- How much are lines stretched by a given braid? What is the exponential rate? (could be zero)
- This rate is referred to as the braid's **topological entropy**.
- The T.E. is obtained from a **transition matrix**.
- The really high-powered algorithms are variations on an idea called “train-tracks”.
- We use both train-tracks and more prosaic methods.

Conformal Map from the Cylinder to the Plane

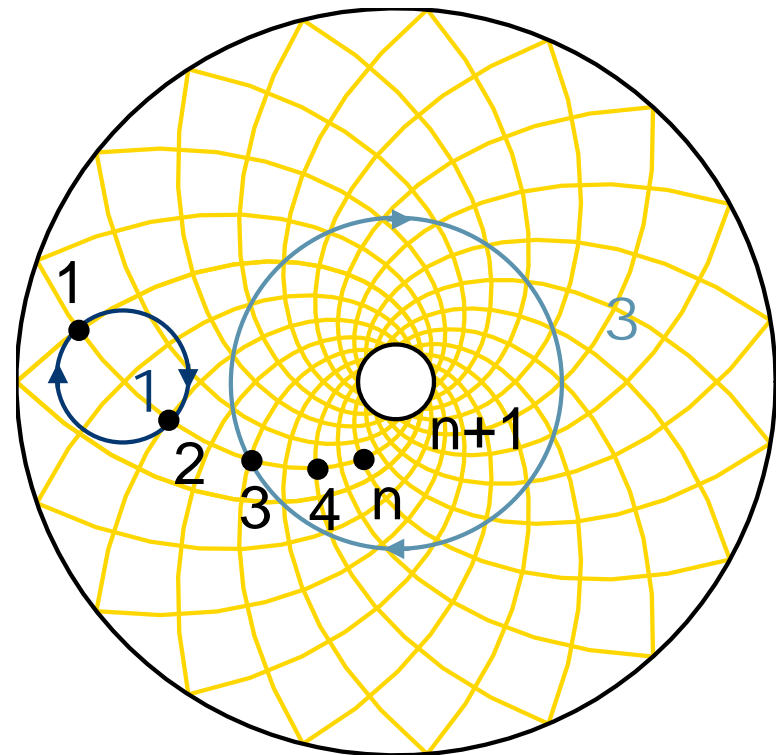
An interesting problem:
what about singly-periodic
boundary conditions?

Conformal map from cylinder
to punctured plane:

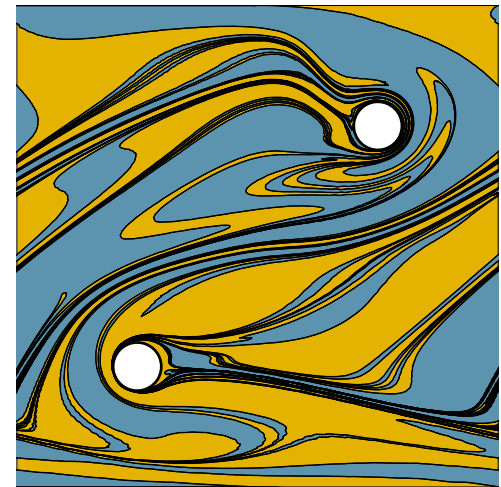
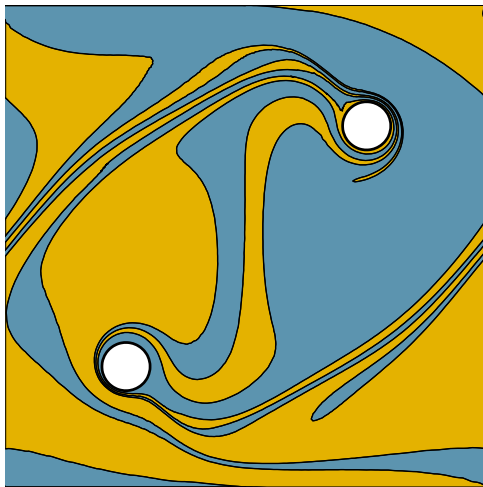
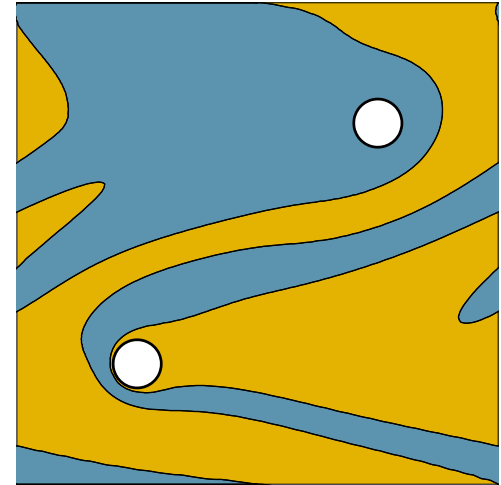
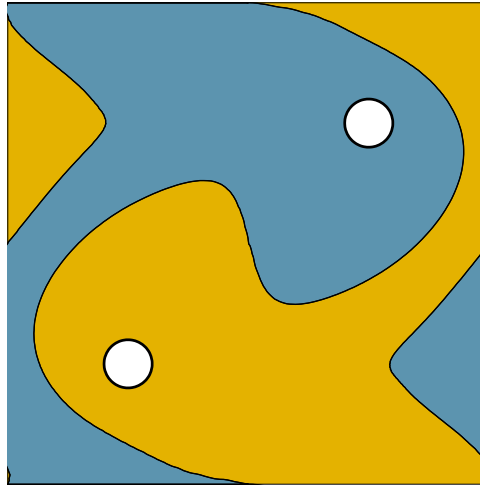
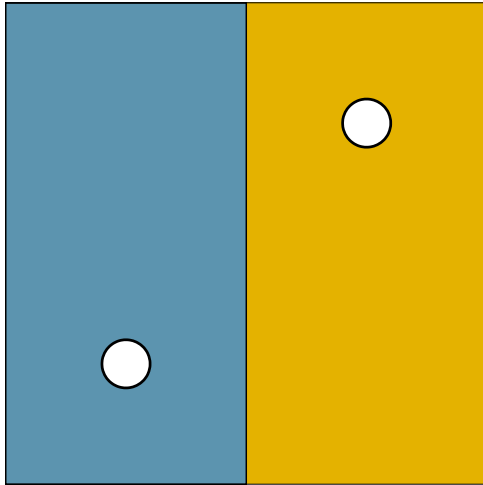
$$w = \exp(2\pi iz)$$

The origin in the w -plane
acts as an extra rod!

So it should be possible to
make a nontrivial braid with
just two rods.



Two-rod Mixer on a Cylinder

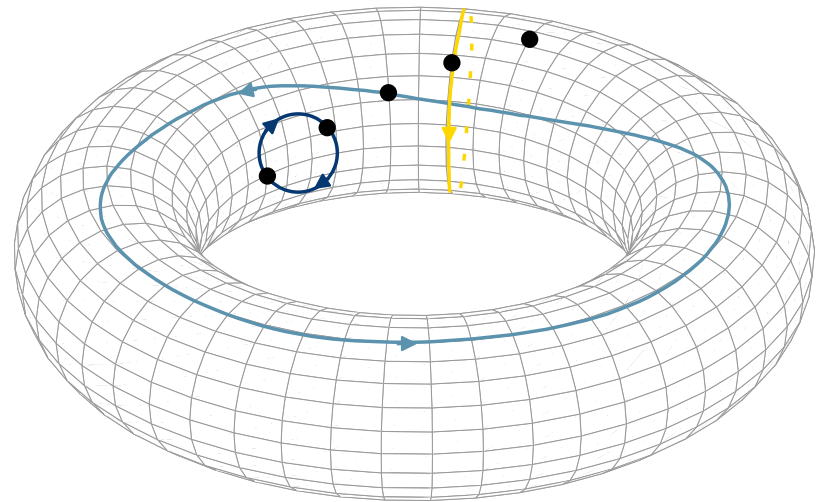
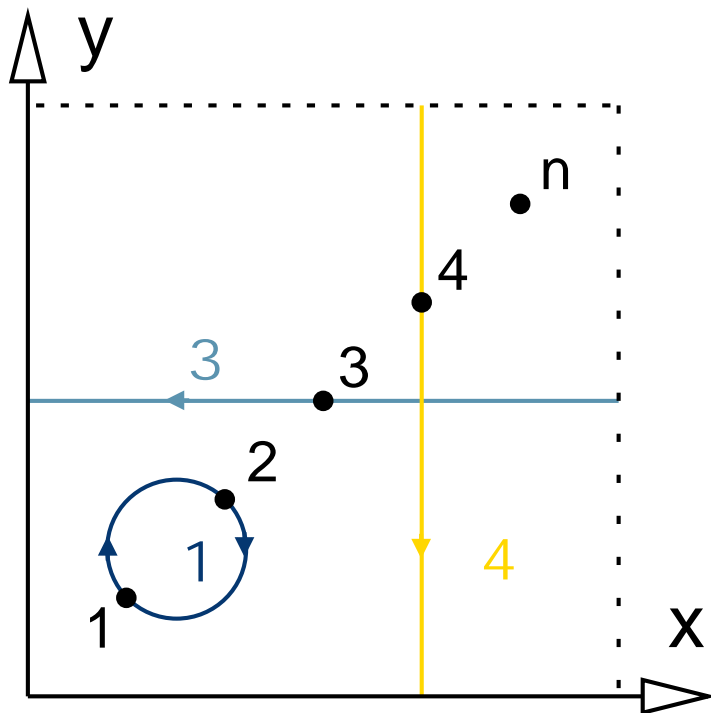


Two-rod Mixer on a Cylinder

[movie: singly.mpg]

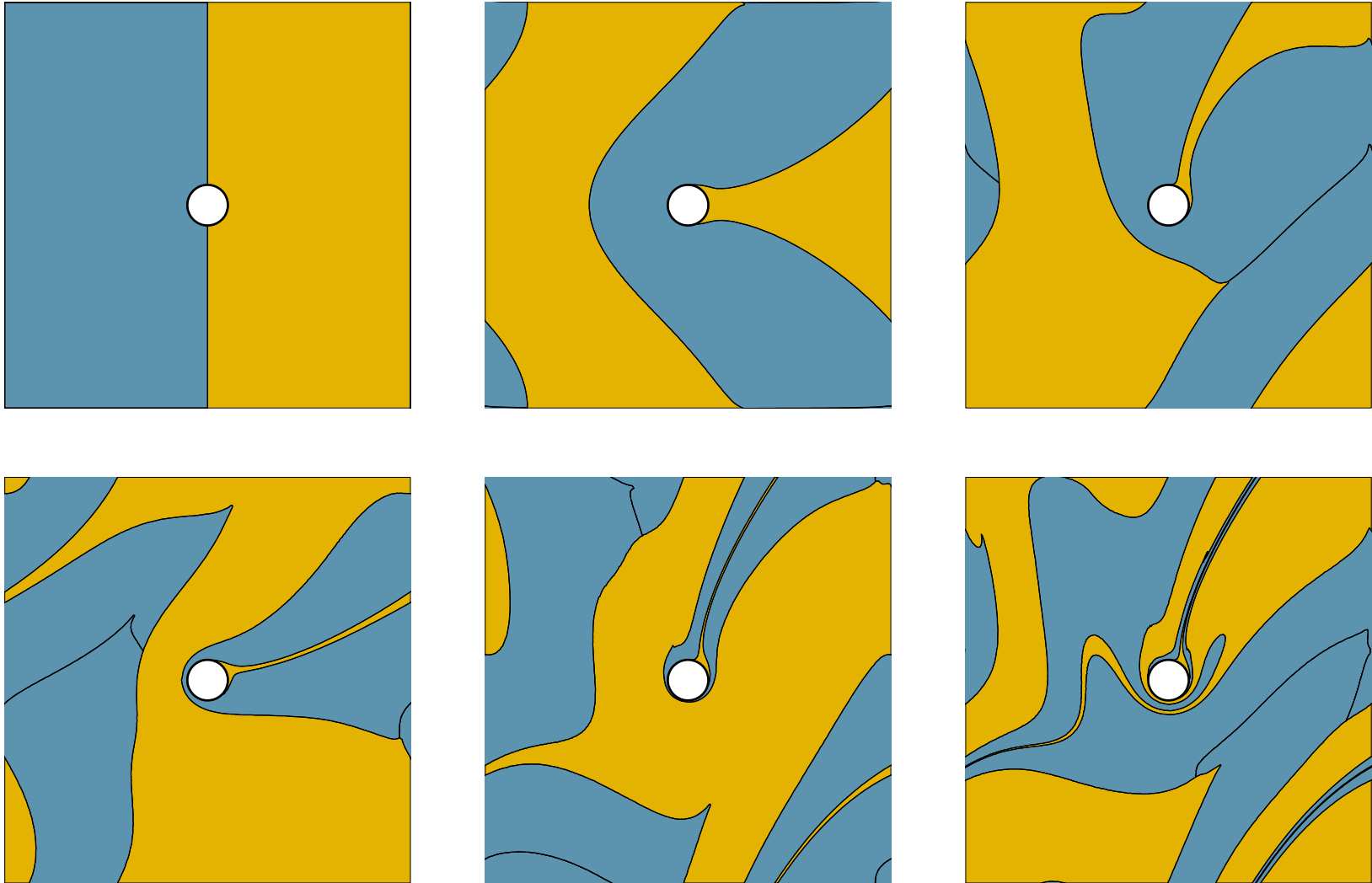
The Torus: Need New Operations

There is no corresponding conformal map for the torus.



So how do we compute entropies? Many chaotic systems live on doubly-periodic domains...

One-rod Mixer on a Torus: No Entropy



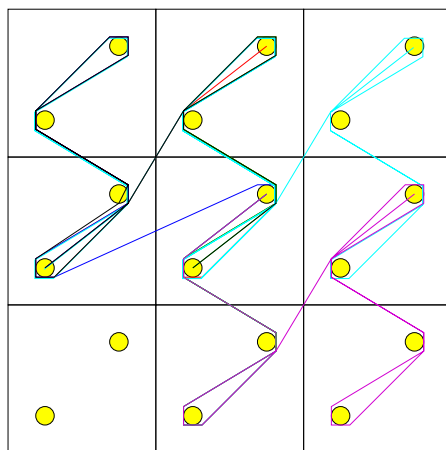
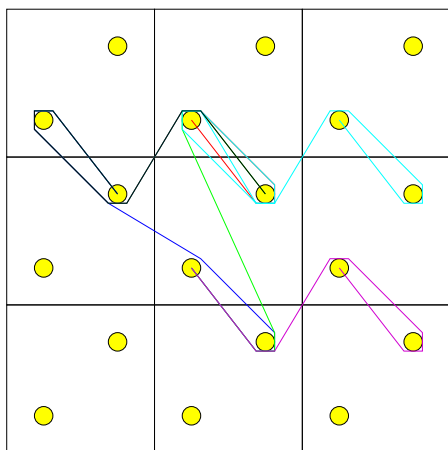
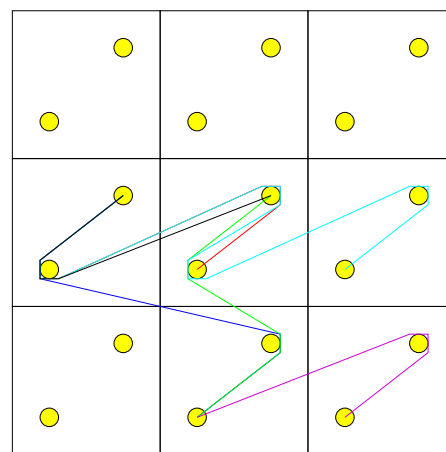
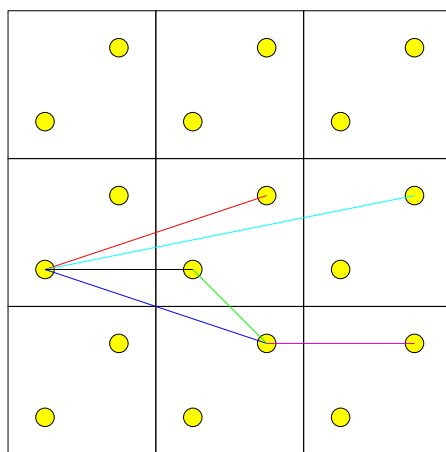
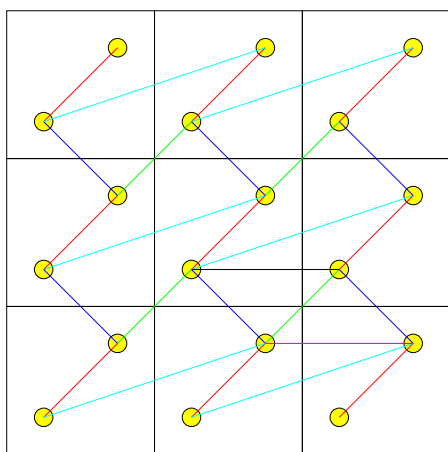
One-rod Mixer on a Torus: No Entropy

[movie: doubly.mpg]

Two-rod Mixer on a Torus: $\tau_1 \sigma_1 \rho_1^{-1} \sigma_1$

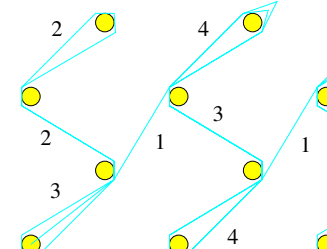
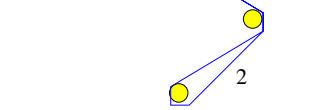
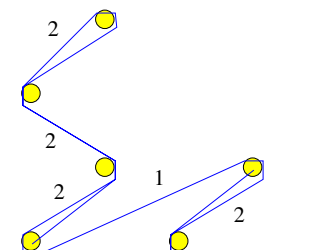
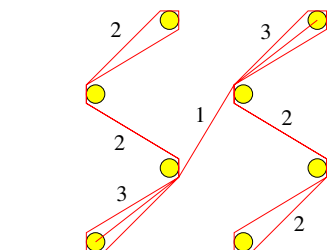
[movie: torus_braid.mov]

Transition Matrix for Torus



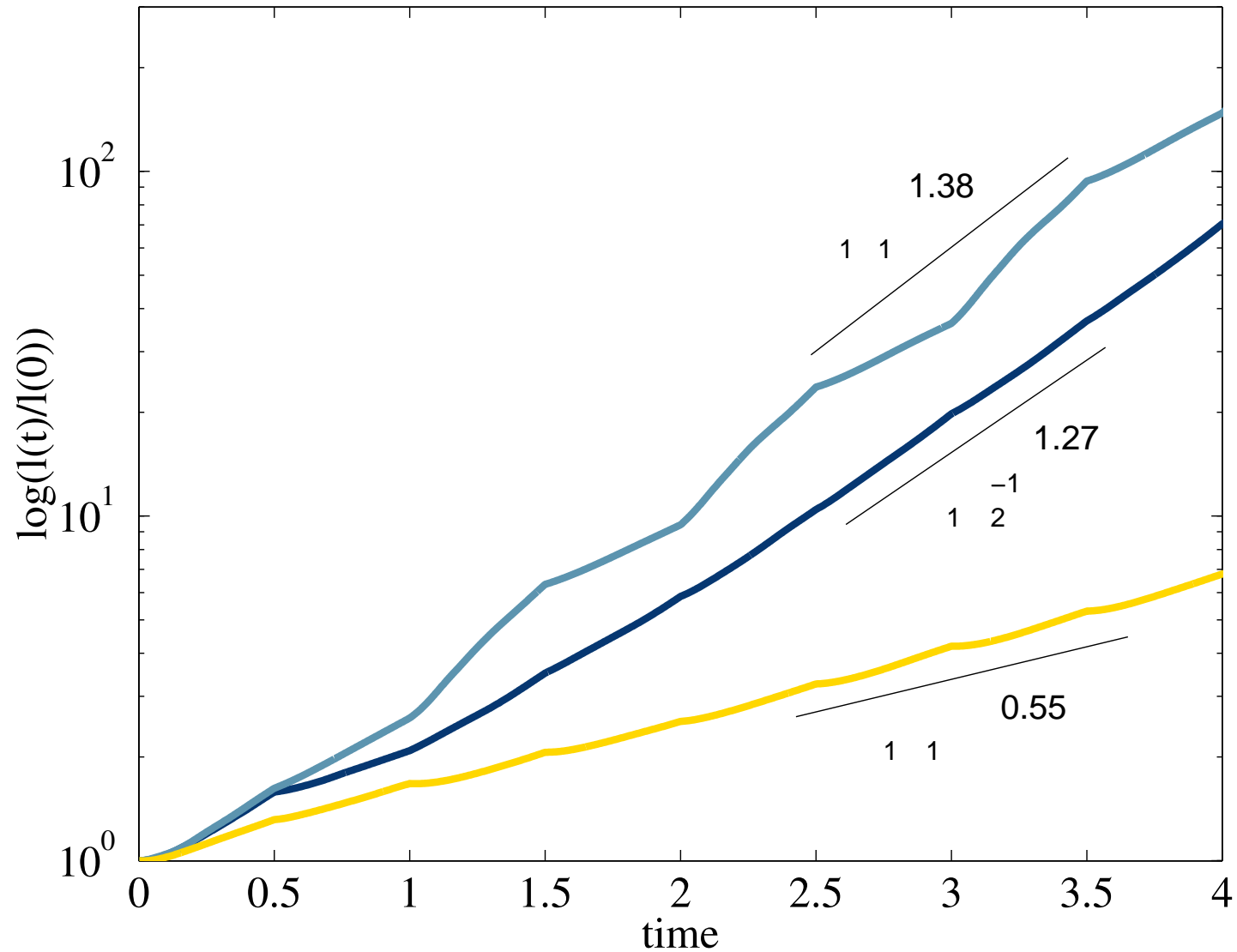
	4	2	4	7	
	8	10	7	18	
	0	1	0	2	
	1	0	0	0	

tau sigma rho-1 sig

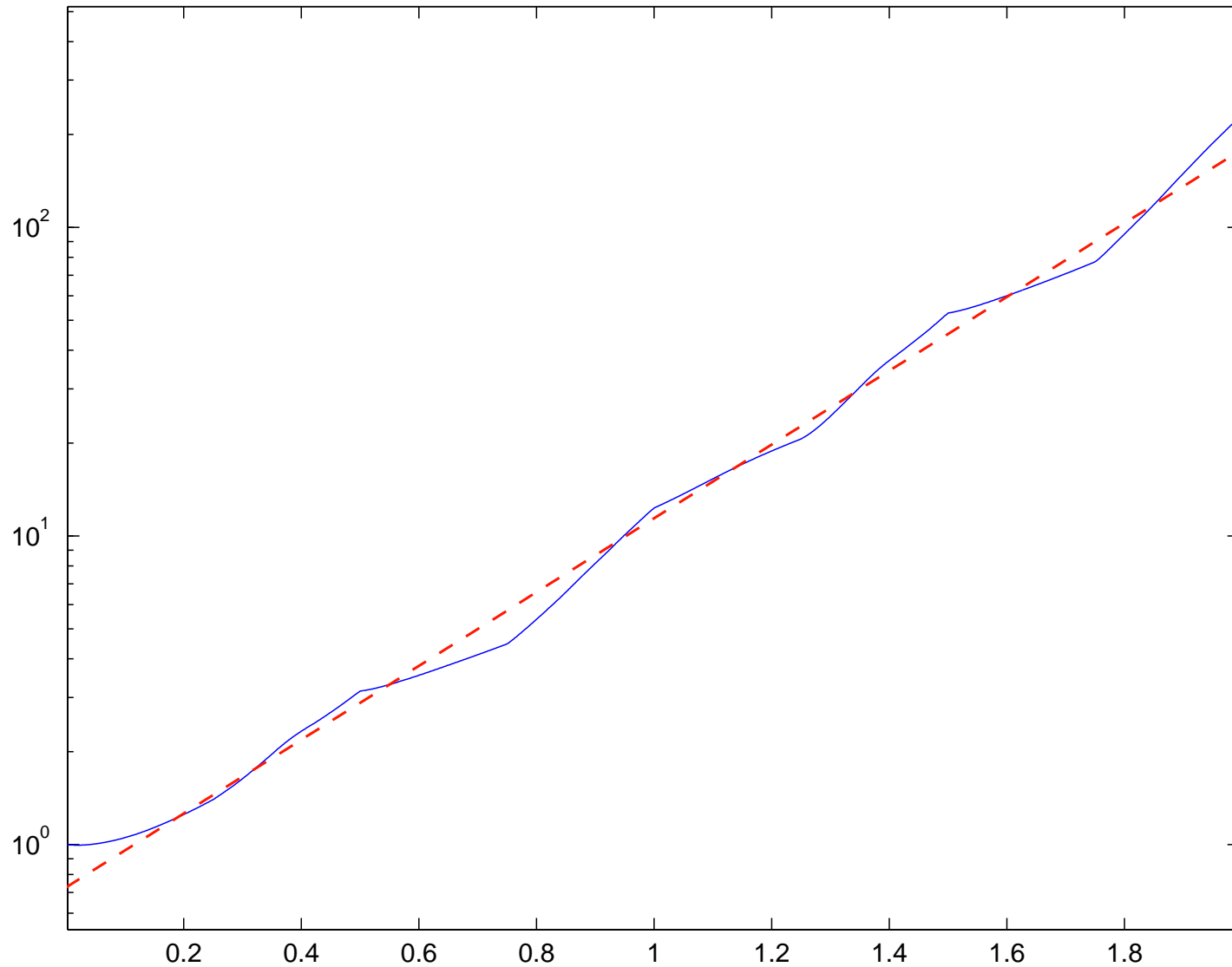


in each column put what that colour maps to

Growth Rates for Lines



Growth Rates for Lines on a Torus



One Rod Mixer: The Kenwood Chef



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at CHRISTMAS

and all through the year...

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★ LIQUIDISES, BLENDS, PUREES,
EXTRACTS JUICES
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★ Even OPENS CANS

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Standard Pack: £28 7 6
complete with Mixing Bowl, K-Beater, Whisk, Dough Hook, Rubber Spatula and Plastic Dust Cover.
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H.P. TERMS AVAILABLE

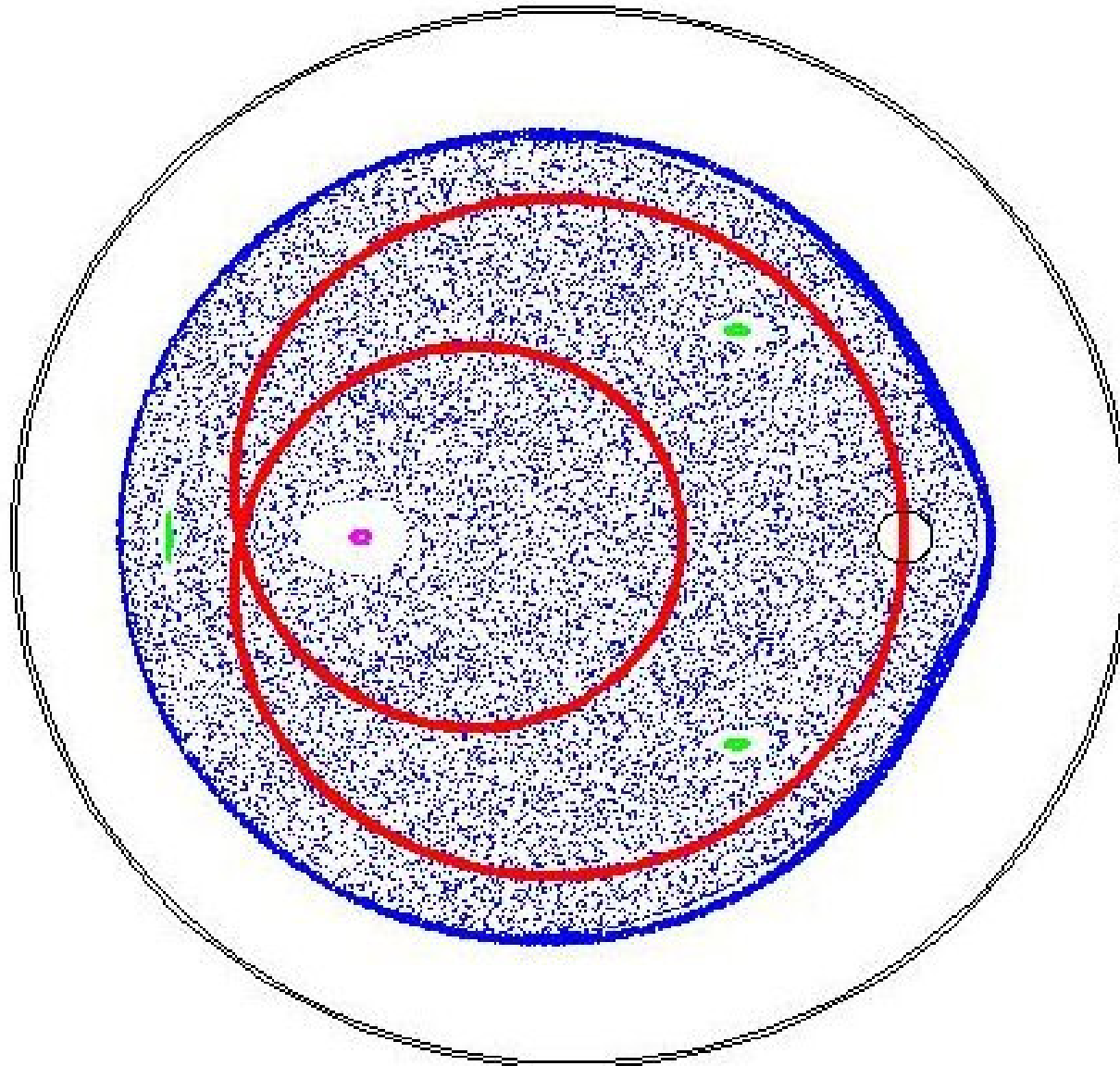
For illustrated literature of all Kenwood products and name of nearest dealer, write to

Other delightful Kenwood Gifts

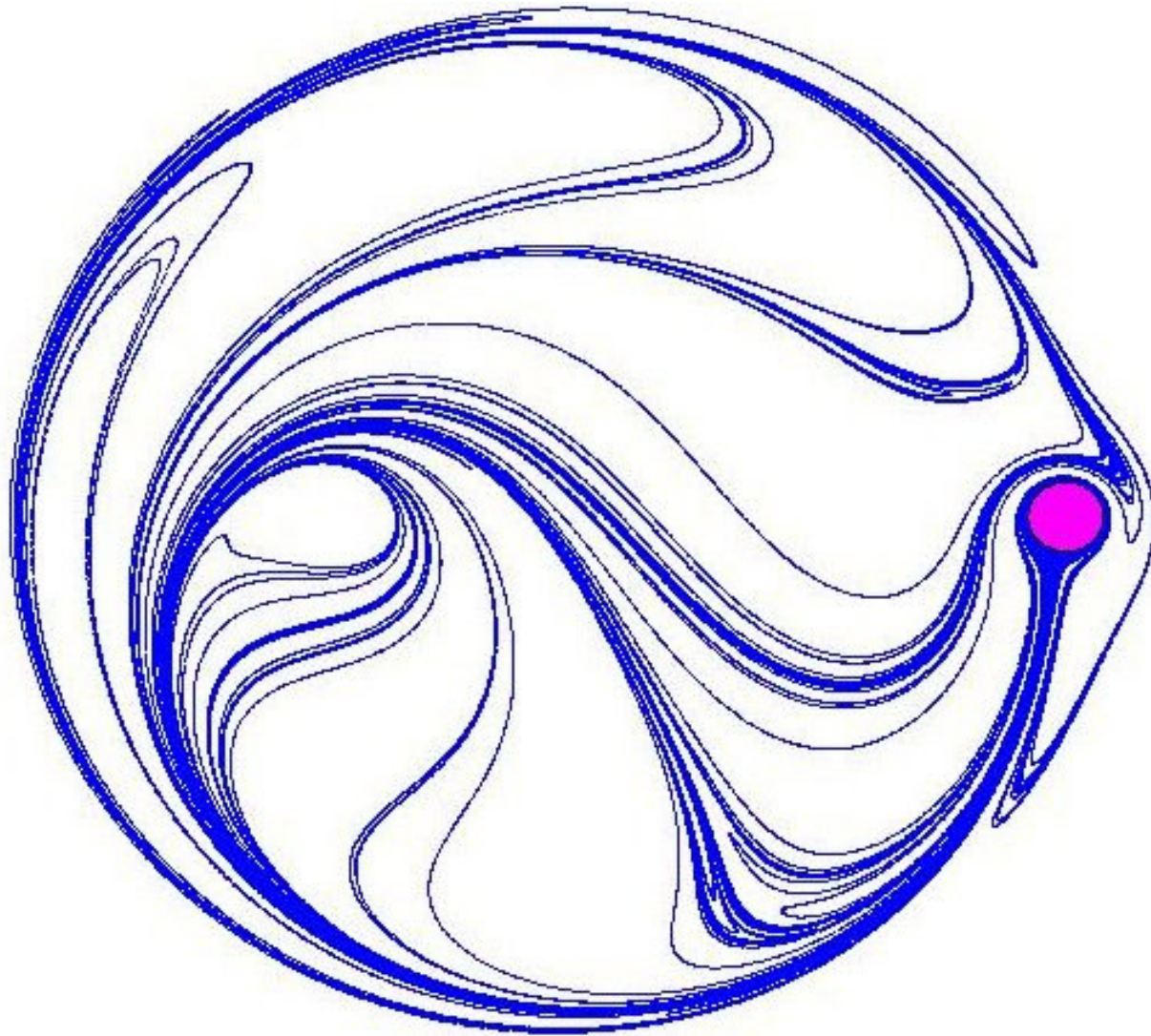
<p>The ROTO-BLEND Gives you Wealth — Why? Because it gives you health and vitamins in seconds. This versatile machine, retains all the natural goodness of fruit and vegetables, giving you juices unobtainable without it. Versatile because you can add to the Roto-Blend a multi-purpose Blender attachment and high-speed Slicer and Shredder attachment. Standard Unit: £14 11 2</p>	<p>The KENMIX '55' The Continental Wonder! Does the work of hours in a few seconds, retaining vitamins and full flavours. So simple to use; blends fruits and vegetables, drinks cocktails and eggs, pulverises nuts and dries cereals, mixes drinks and confections. £14 12 7</p>	<p>The MINOR A really efficient Portable Food Mixer for only £9 18 0. It mixes, whips, beats and blends—in fact does everything a Food Mixer should, saving hours of wrist-aching work. Weighs only 3½ lbs., light enough to be used anywhere, in the kitchen—at the stove, over the sink or in the table. £9 18 0</p>
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KENWOOD ELECTRICS LTD., 26 North Audley Street, London, W.1

Poincaré Section



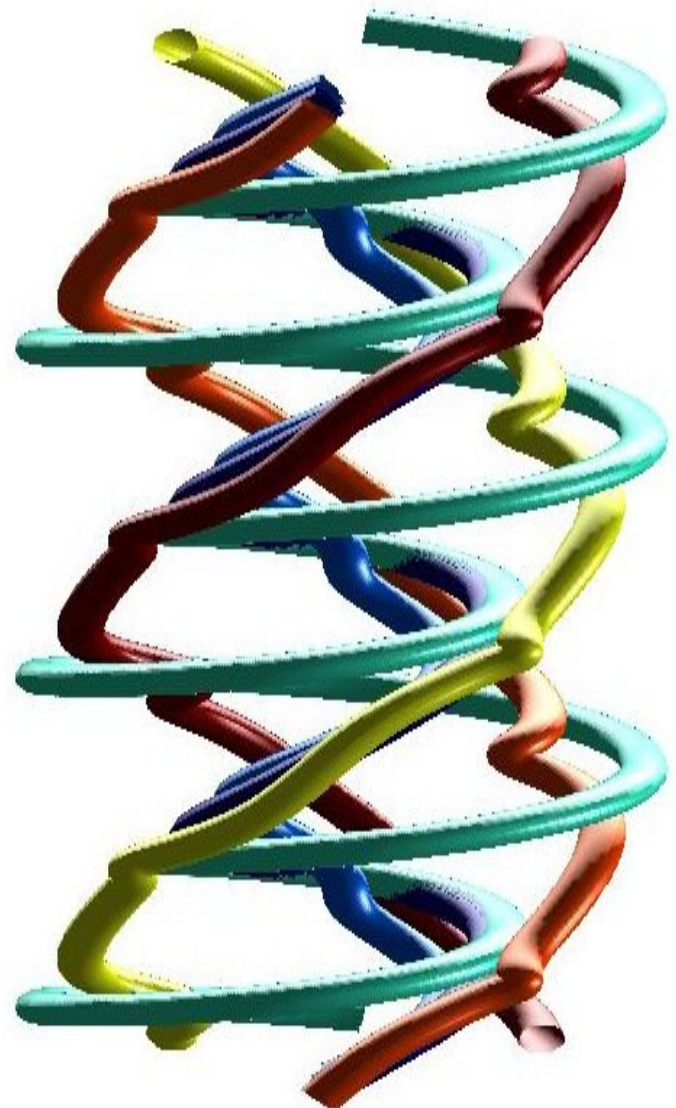
Stretching of Lines



Motion of Islands

Make a braid from the motion of the rod and the **periodic islands**.

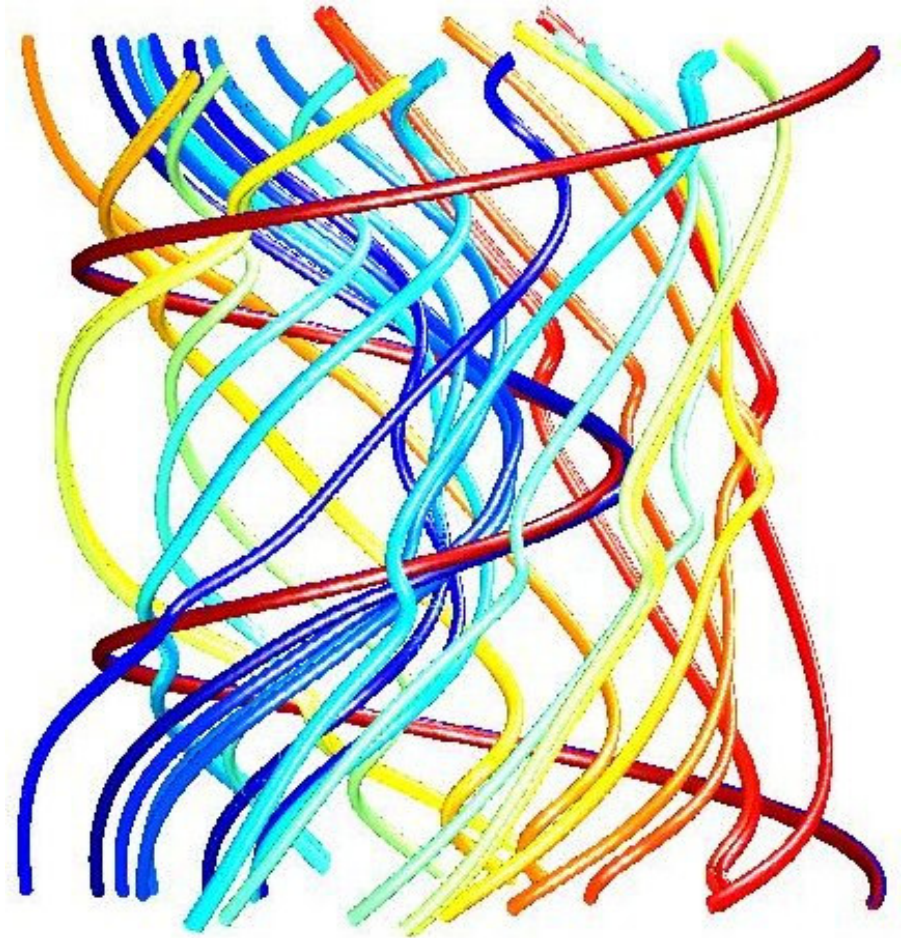
Most (74%) of the topological entropy is accounted for by this braid.



Motion of Islands and Unstable Periodic Orbits

Now we also include **unstable** periodic orbits as well as the stable ones (islands).

Almost all (99%) of the topological entropy is accounted for by this braid.



Conclusion

All Chaos is Topological!