Braiding and Mixing

Periodic Boundary Conditions and Periodic Orbits

Jean-Luc Thiffeault

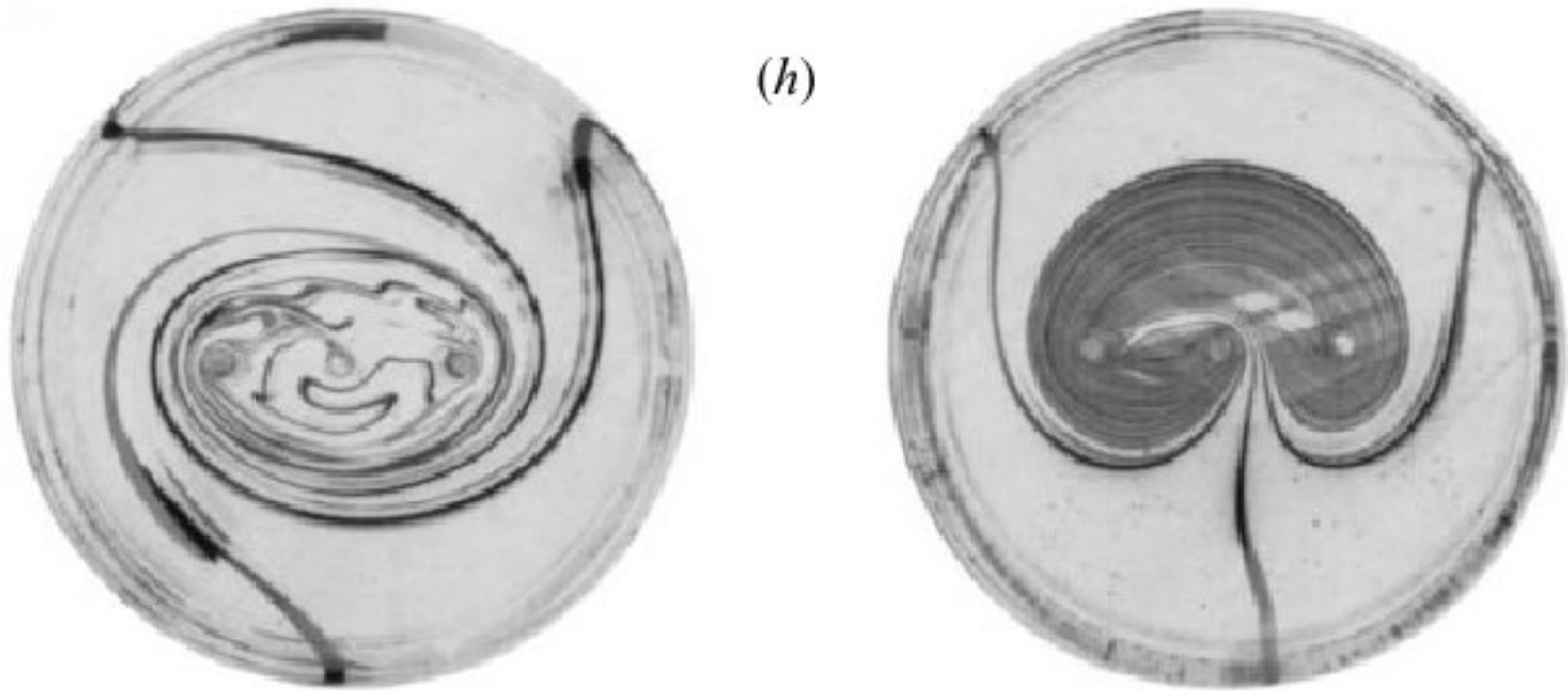Matthew Finn

Emmanuelle Gouillart

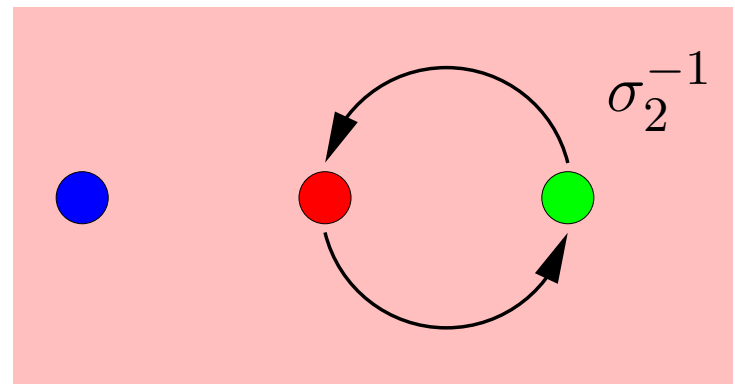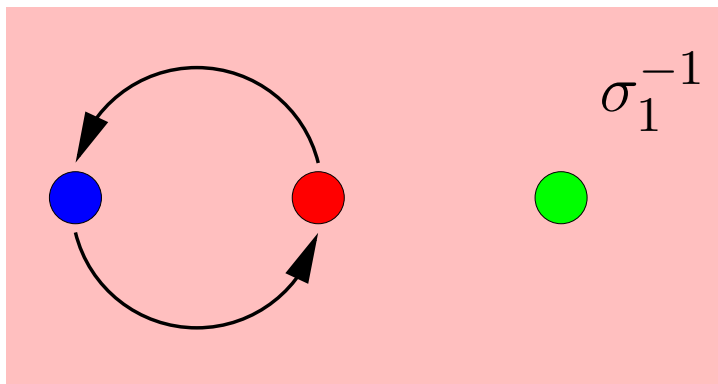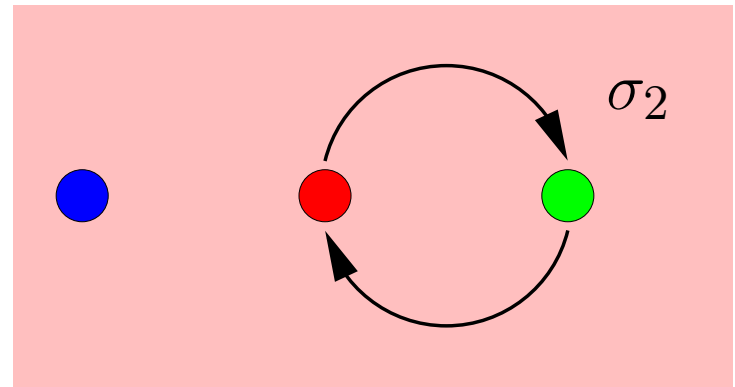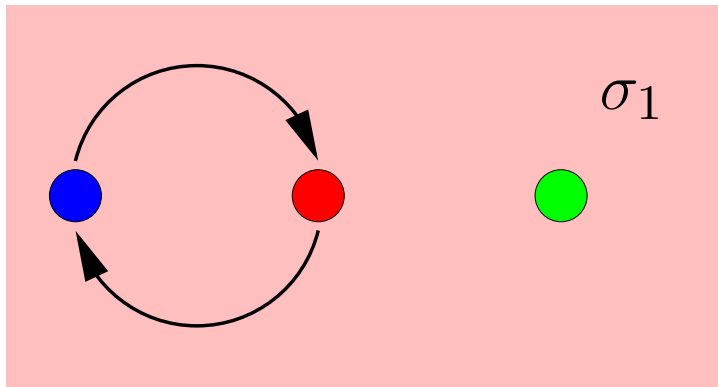http://www.ma.imperial.ac.uk/~jeanluc

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Experiment of Boyland *et al.*

(h)

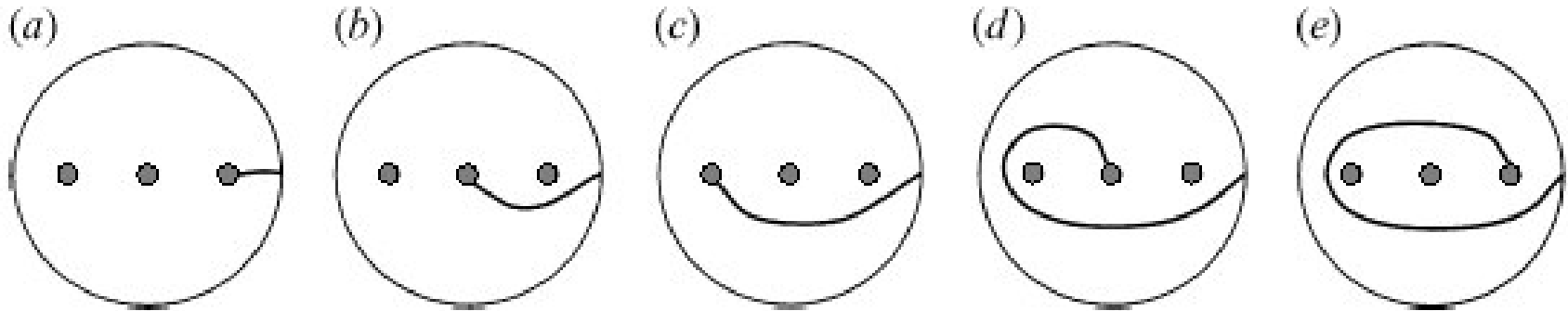[P. L. Boyland, H. Aref, and M. A. Stremler, J. Fluid Mech. **403**, 277 (2000)]
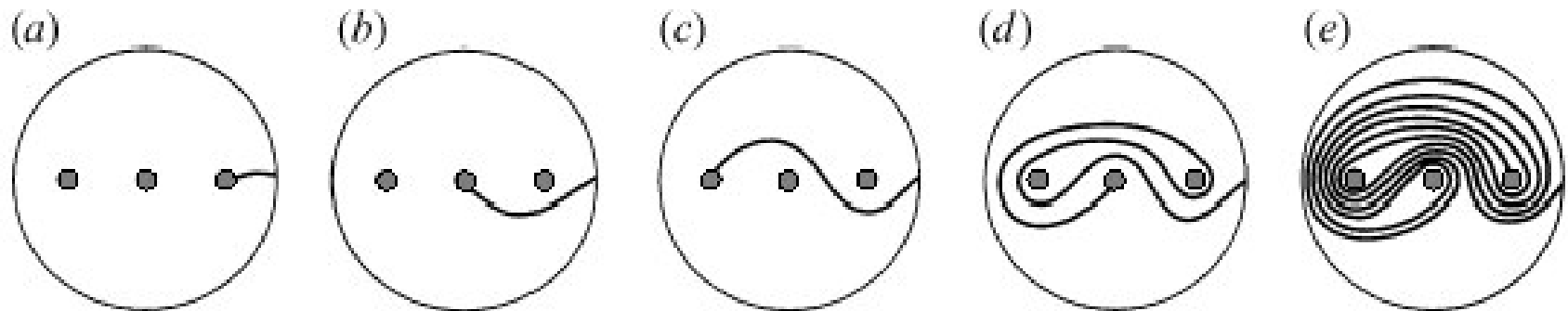
Four Basic Operations

σ_1 and σ_2 are referred to as the generators of the 3-braid group.

Two Stirring Protocols
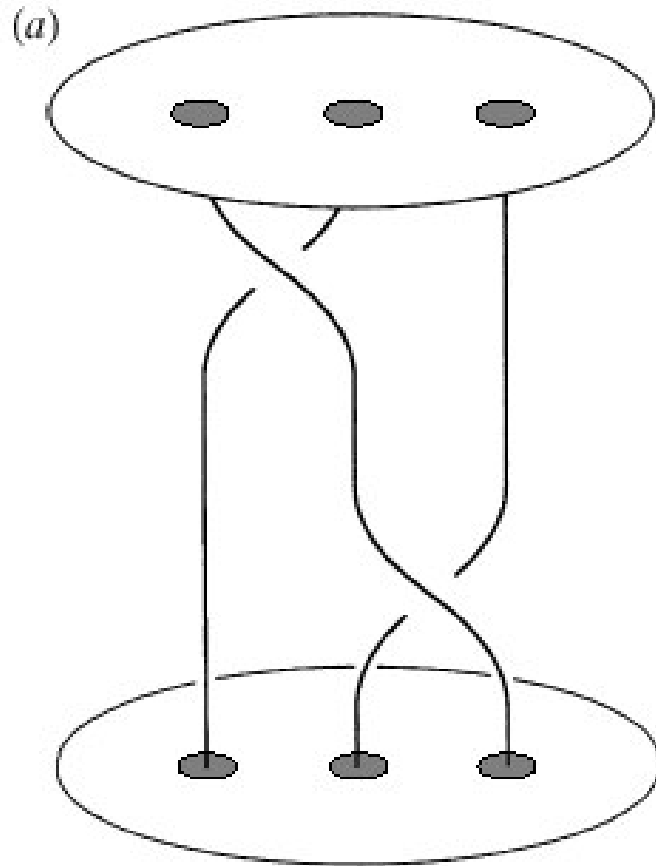
$\sigma_1\sigma_2$ protocol
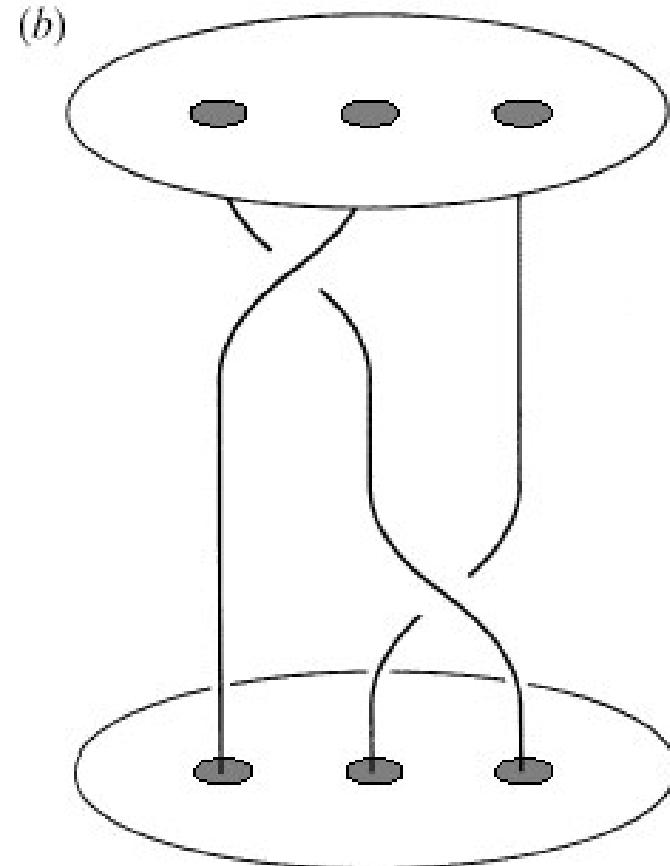

$\sigma_1^{-1}\sigma_2$ protocol

[P. L. Boyland, H. Aref, and M. A. Stremler, J. Fluid Mech. **403**, 277 (2000)]
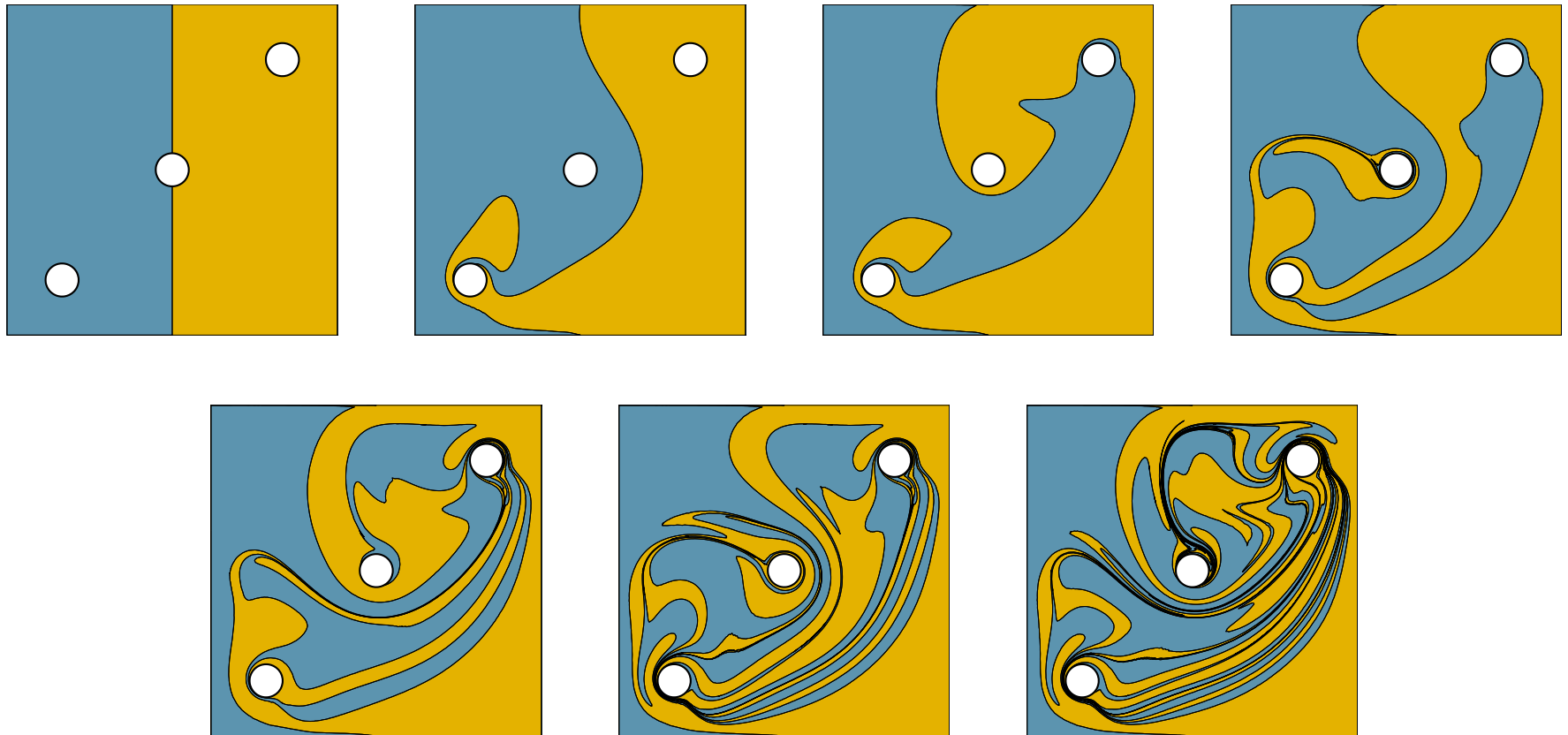
Braiding

$\sigma_1\sigma_2$ protocol

$\sigma_1^{-1}\sigma_2$ protocol

(a)

(b)

Time

[P. L. Boyland, H. Aref, and M. A. Stremler, J. Fluid Mech. **403**, 277 (2000)]

Three-rod Mixer in a Bounded Domain

Three-rod Mixer in a Bounded Domain

[movie: bounded.mpg]

Computing the Line-stretching from a Braid

- How much are lines stretched by a given braid? What is the exponential rate? (could be zero)

- This rate is referred to as the braid's topological entropy.

- The T.E. is obtained from a transition matrix.

- The really high-powered algorithms are variations on an idea called "train-tracks".

- We use both train-tracks and more prosaic methods.

Conformal Map from the Cylinder to the Plane

An interesting problem: what about singly-periodic boundary conditions?

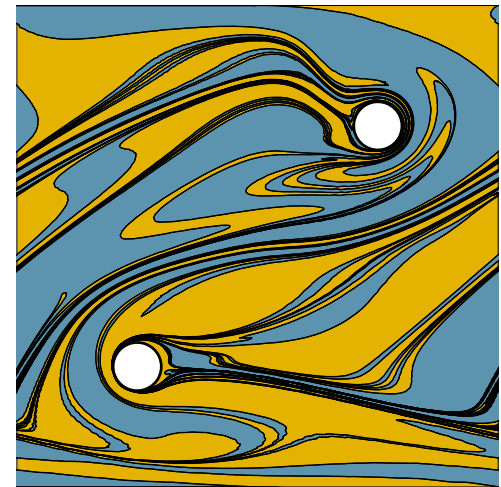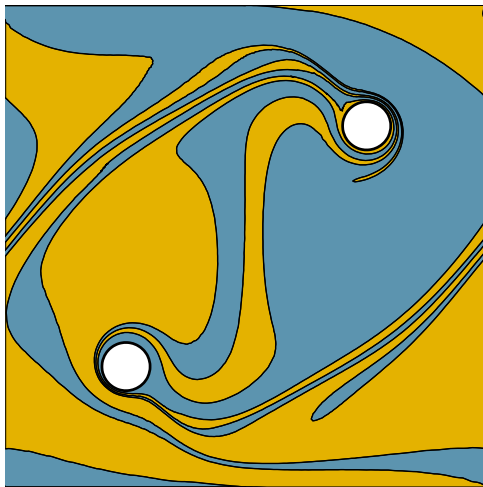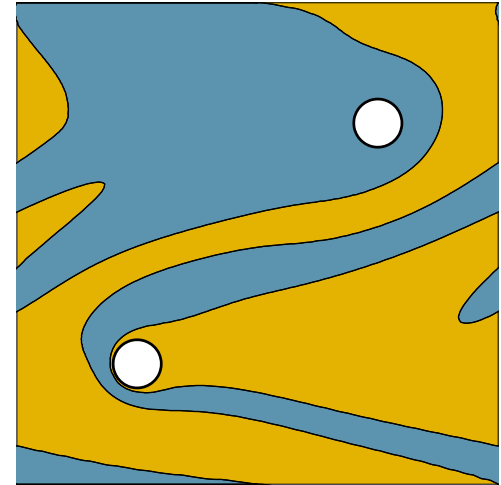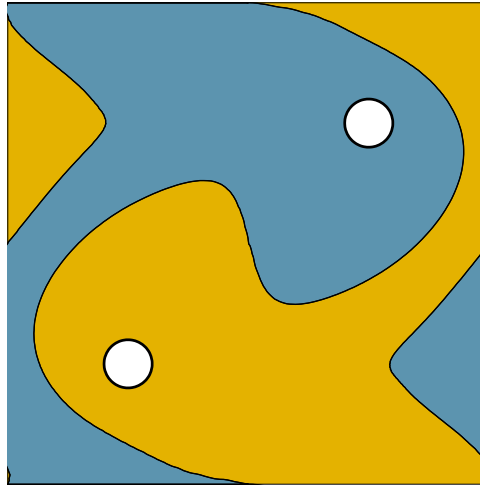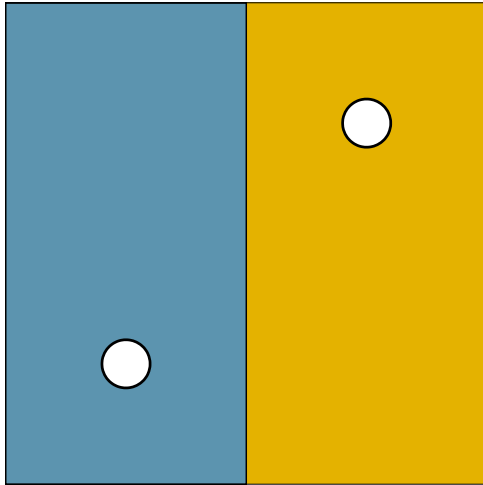Conformal map from cylinder to punctured plane:

$$w = \exp(2\pi i z)$$

The origin in the w-plane acts as an extra rod!

So it should be possible to make a nontrivial braid with just two rods.
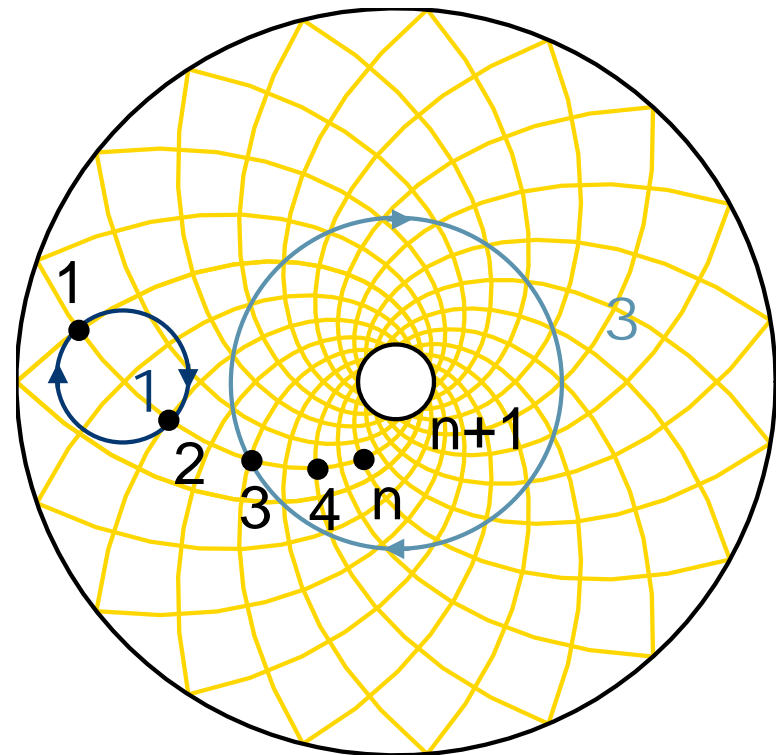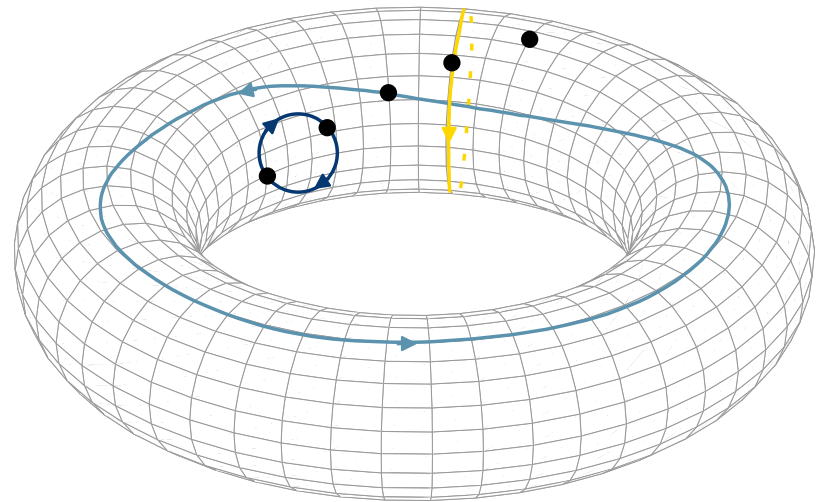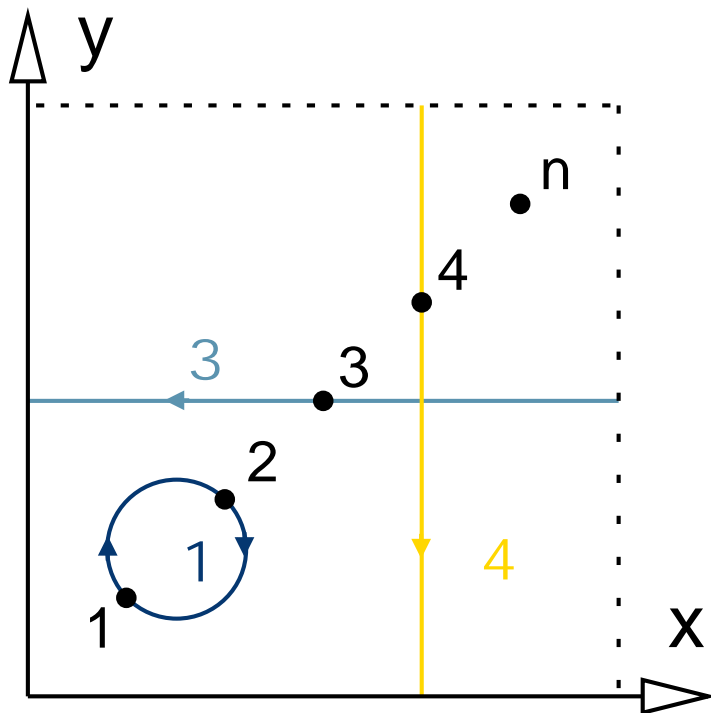
Two-rod Mixer on a Cylinder

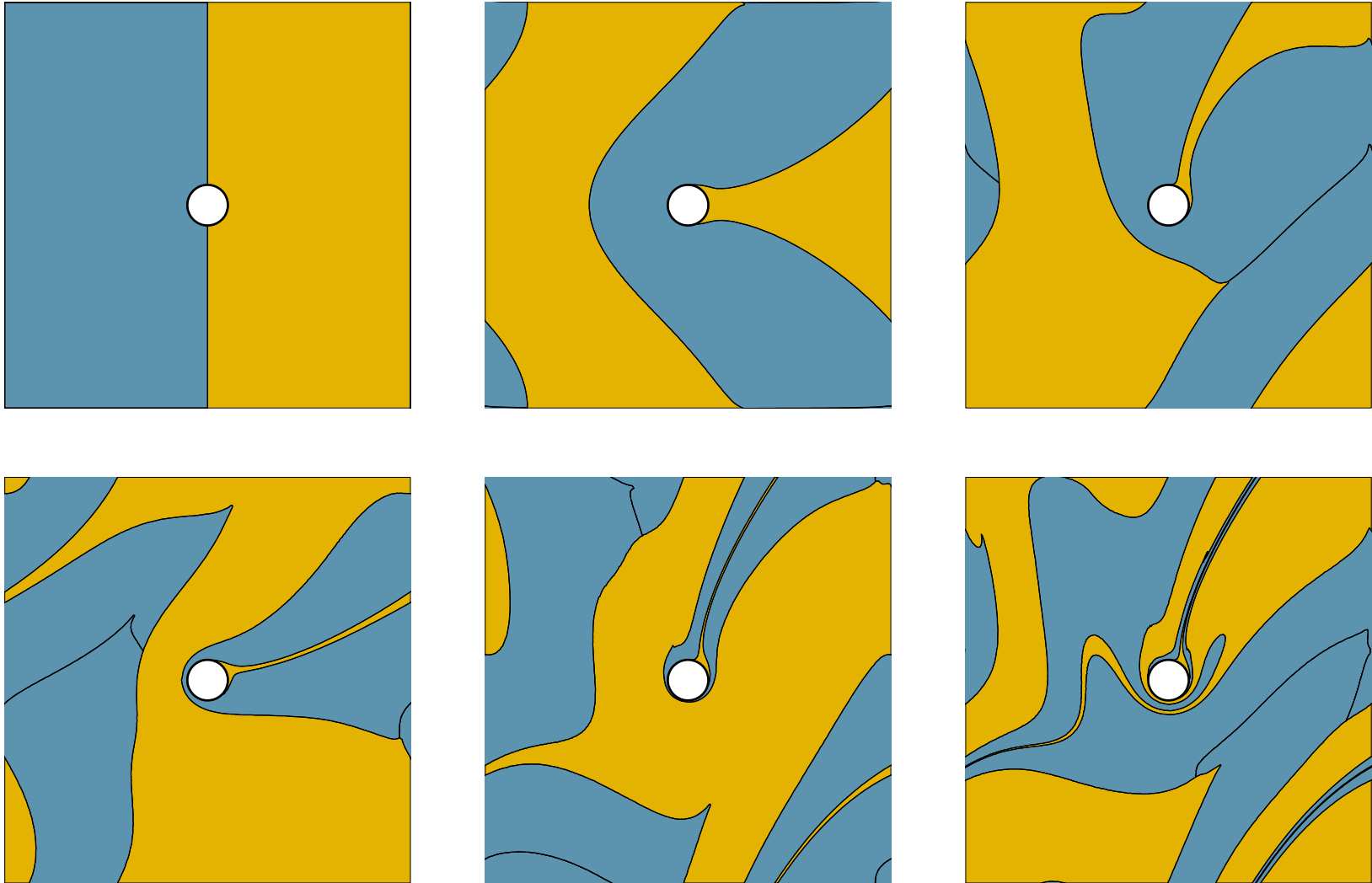# Two-rod Mixer on a Cylinder

[movie: singly.mpg]

The Torus: Need New Operations

There is no corresponding conformal map for the torus.

So how do we compute entropies? Many chaotic systems live on doubly-periodic domains...
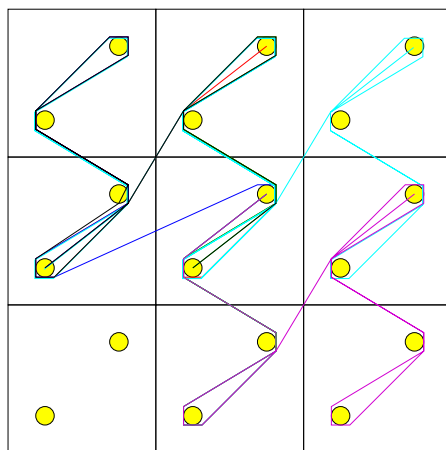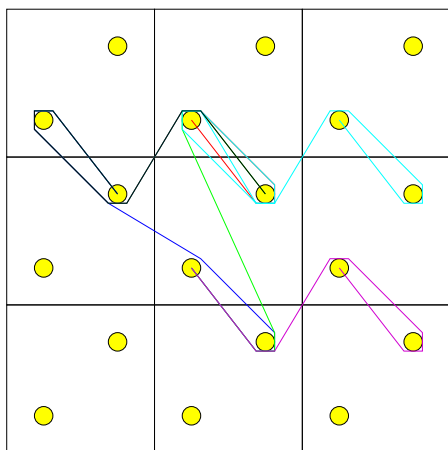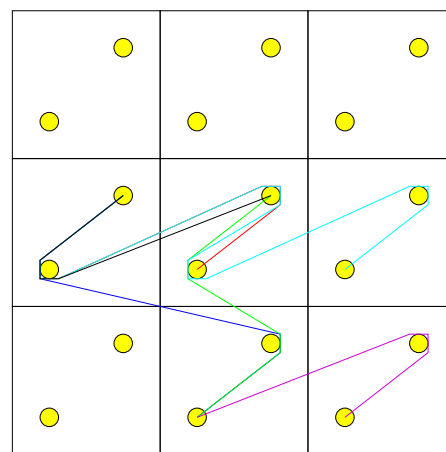
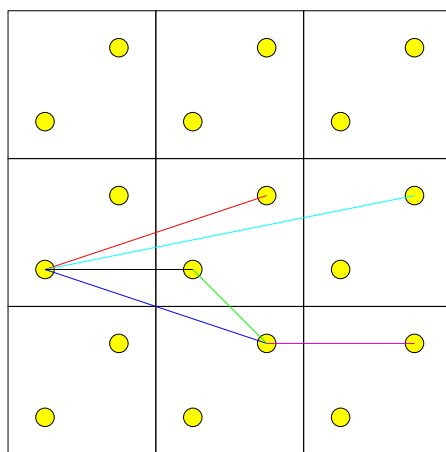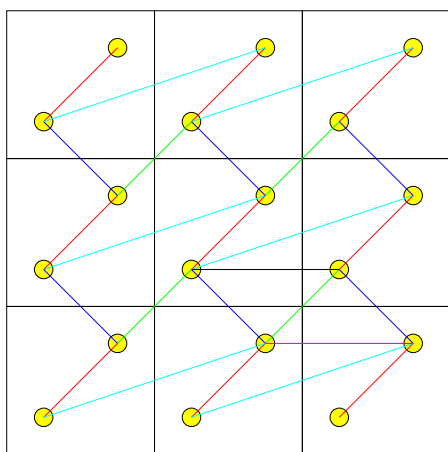# One-rod Mixer on a Torus: No Entropy

One-rod Mixer on a Torus: No Entropy

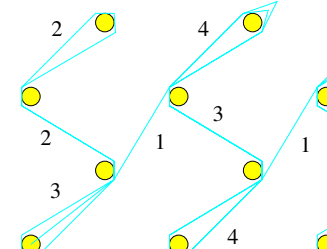[movie: doubly.mpg]

Two-rod Mixer on a Torus: $\tau_1 \sigma_1 \rho_1^{-1} \sigma_1$

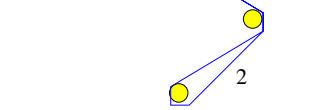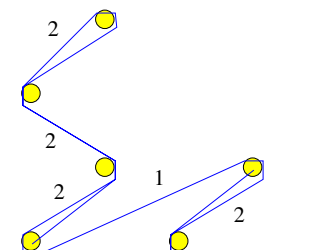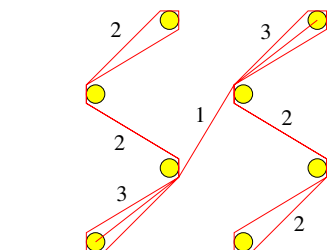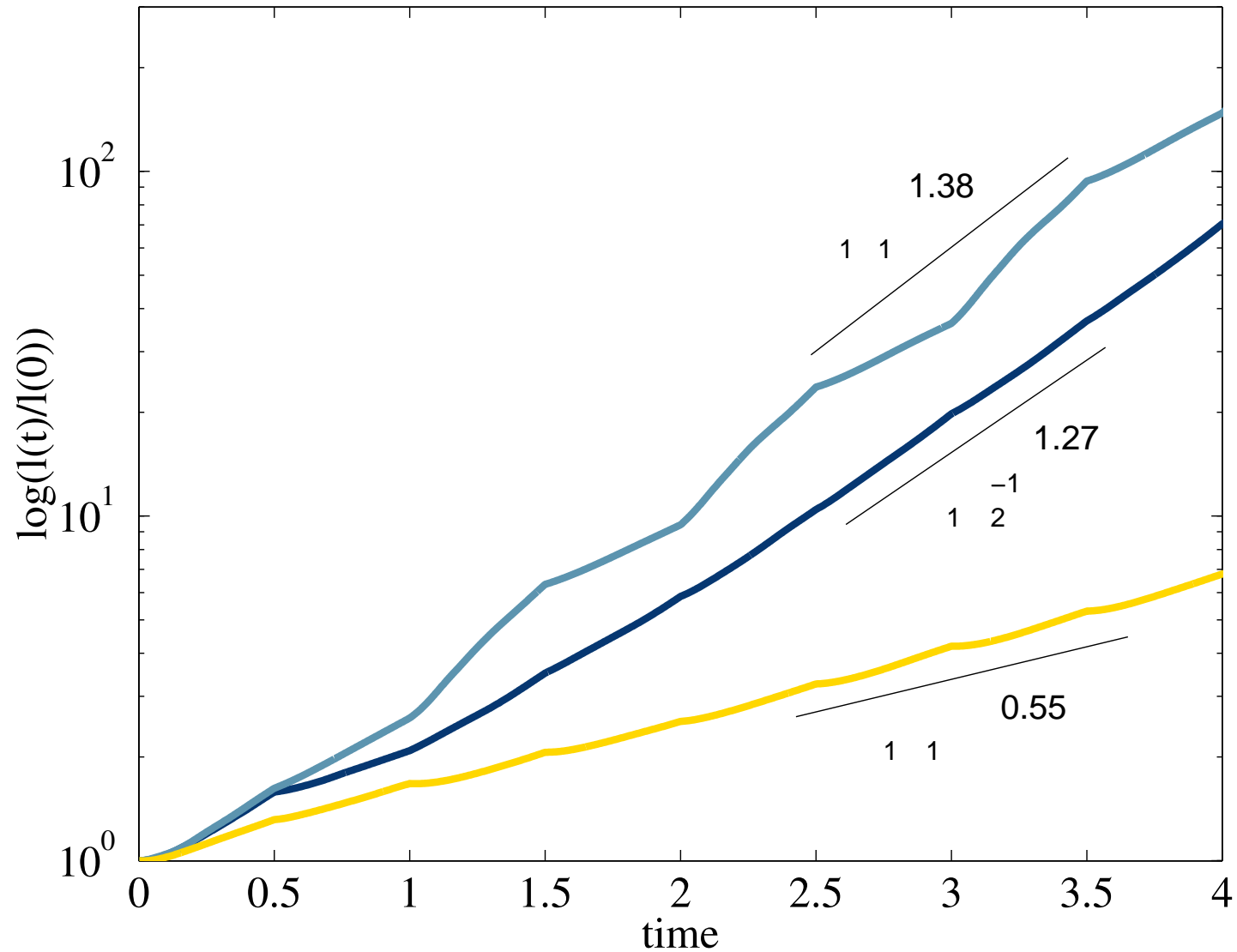[movie: torus_braid.mov]

Transition Matrix for Torus

tau sigma rho−1 sig

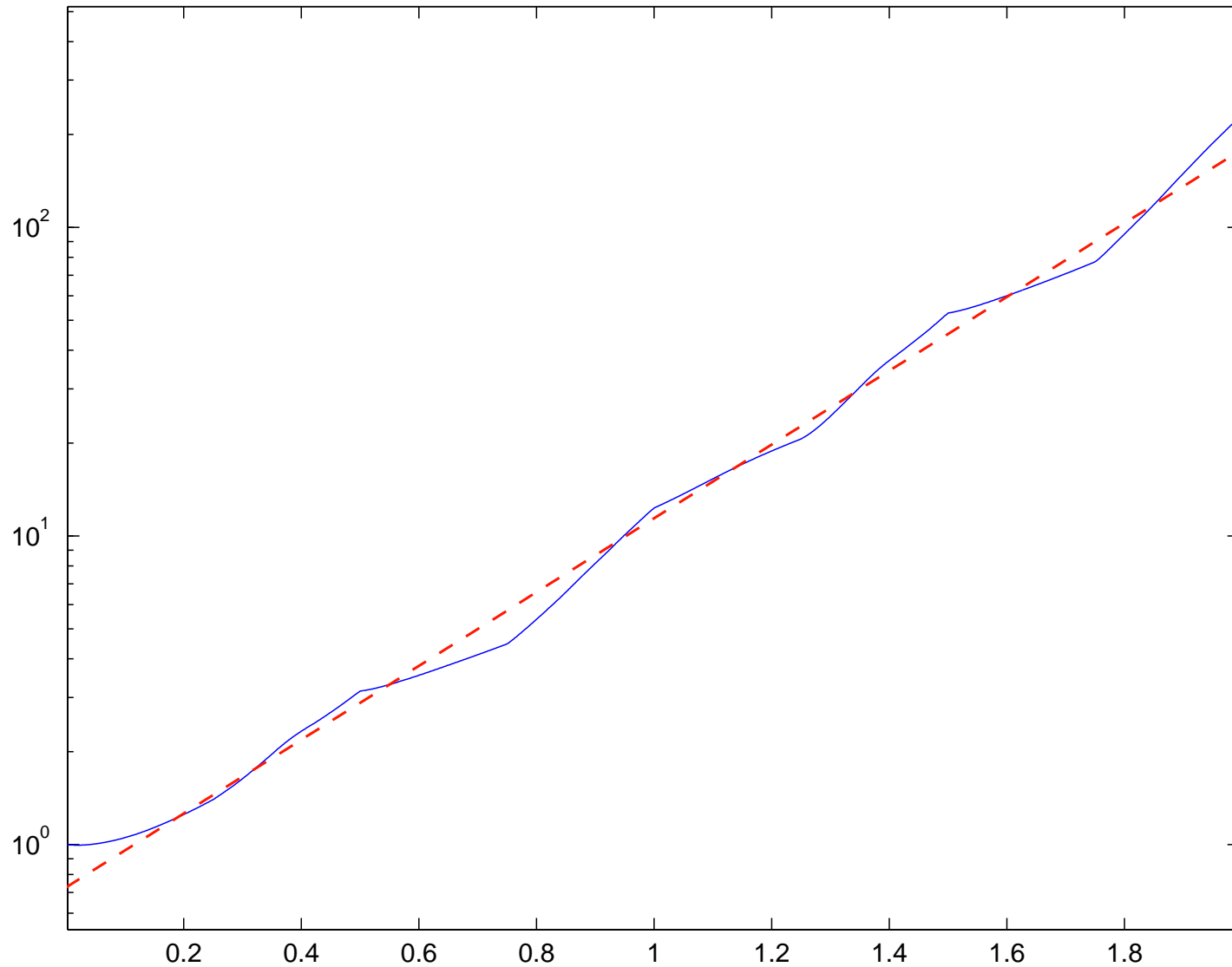|				
4	2	4	7	
8	10	7	18	
0	1	0	2	
1	0	0	0	

in each column put what that colour maps to

Growth Rates for Lines

Growth Rates for Lines on a Torus

One Rod Mixer: The Kenwood Chef
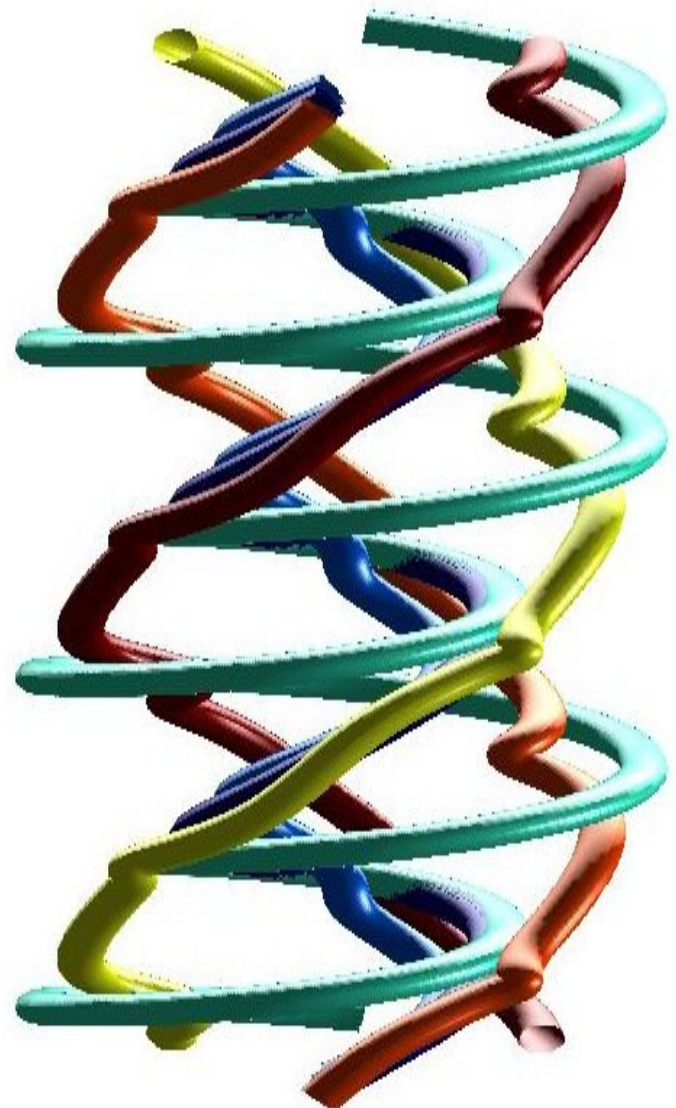
Poincaré Section

Stretching of Lines
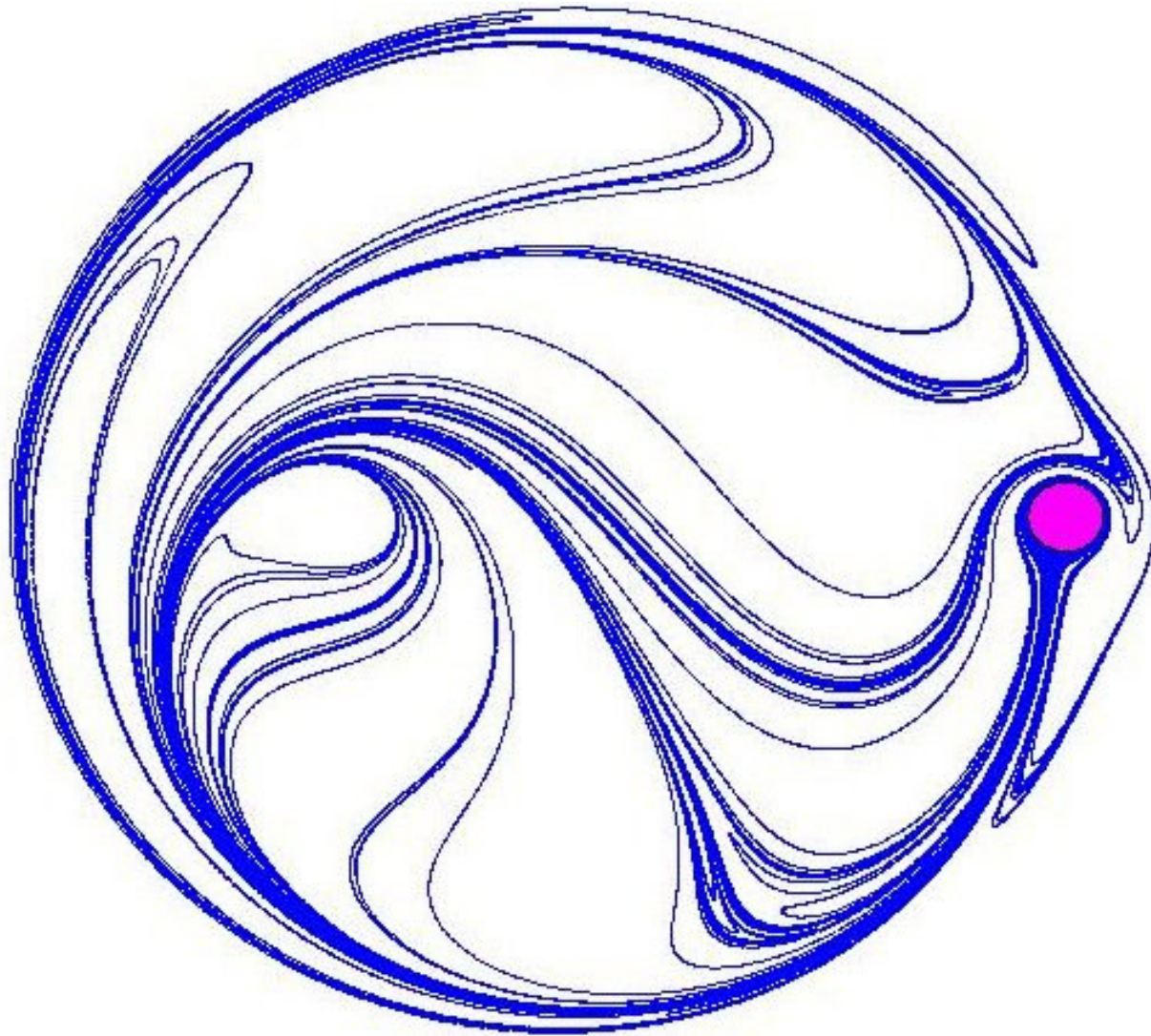
Motion of Islands

Make a braid from the motion of the rod and the periodic islands.
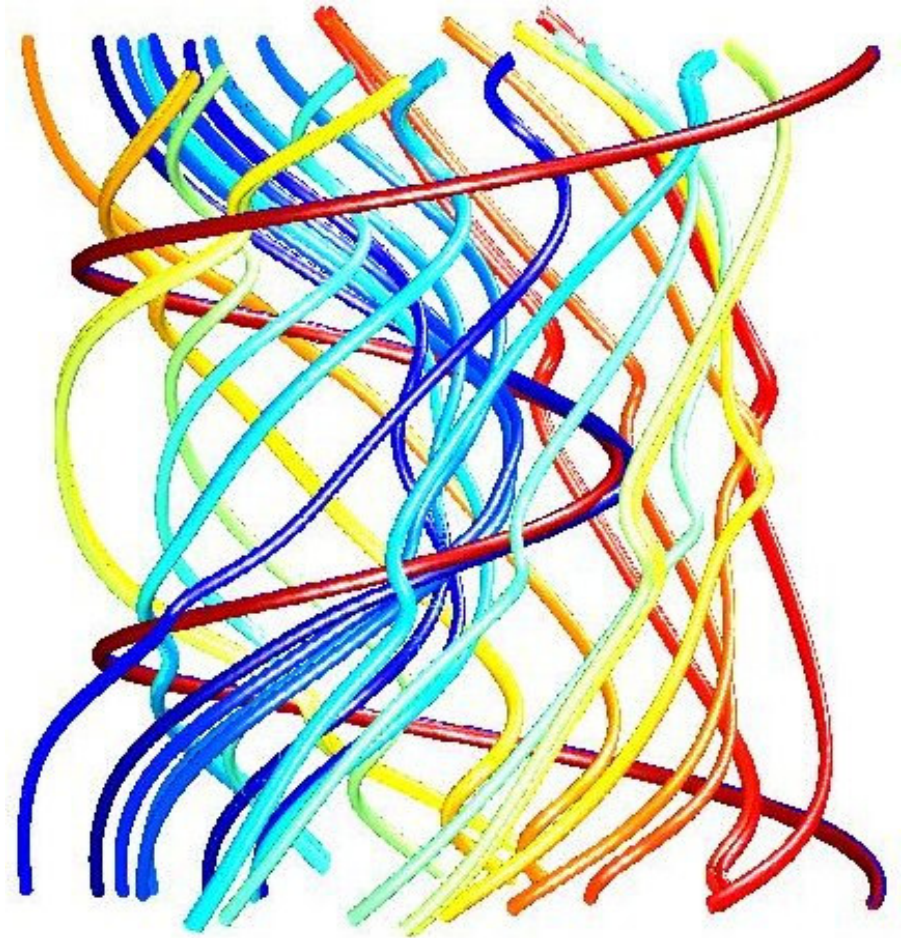
Most (74%) of the topological entropy is accounted for by this braid.

Motion of Islands and Unstable Periodic Orbits

Now we also include unstable periods orbits as well as the stable ones (islands).

Almost all (99%) of the topological entropy is accounted for by this braid.

Conclusion

All Chaos is Topological!