Modeling hagfish slime

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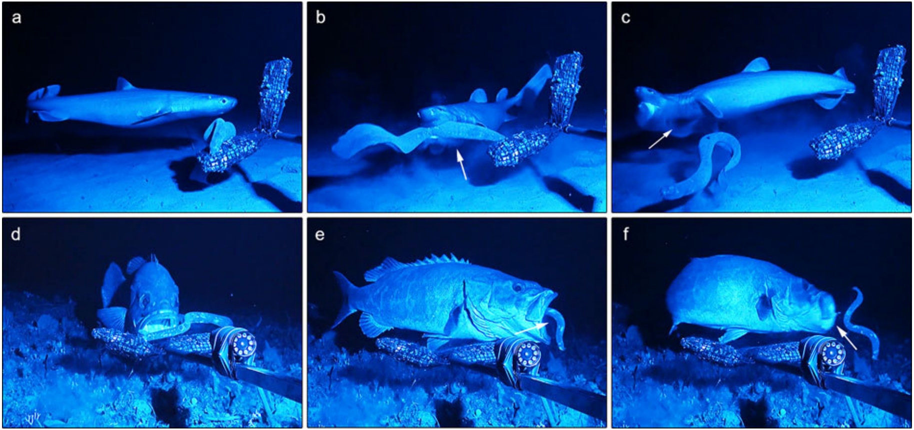
hagfish factoids

- Not the prettiest fish.
- An ancient fish: no teeth.
- Only known living animal that has a skull but not a vertebral column.
- 77 species, average 50 cm.
- Eats worms as well as dead fish, by burrowing into their carcass. They can feed through their own skin.

sliming predators



play movie

[Zintzen, V., Roberts, C. D., Anderson, M. J., Stewart, A. L., Struthers, C. D., & Harvey, E. S. (2011). *Scientific Reports,* **1**, 131]

knotting

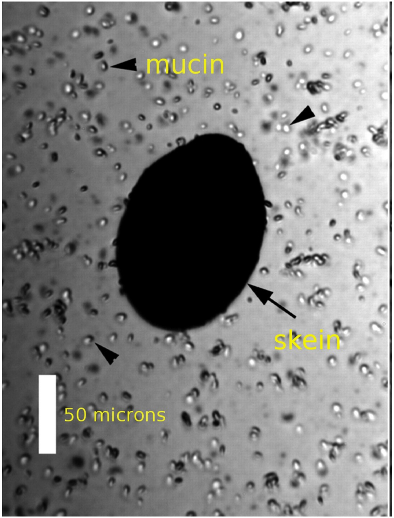
youtube movie (see around 1 min mark)

slime in the lab: a promising material

youtube movie

so what's inside the slime?

- .002% thread skein
- .0015% mucin
- 99.996% seawater (!)



[Fudge, D. S., Levy, N., Chiu, S., & Gosline, J. M. (2005). *J. Exp. Biol.* **208**, 4613–4625]

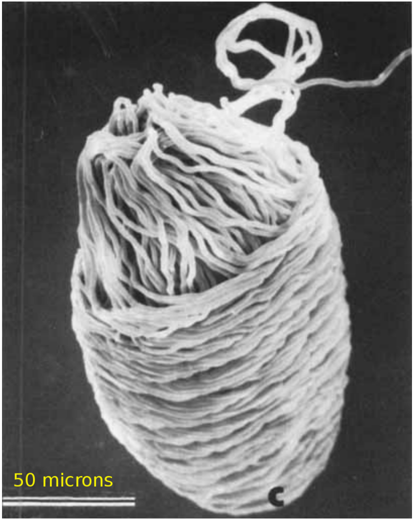
what's a skein?

A skein consists of thread rolled into a ball.

Skeins are about 0.1 mm in size.

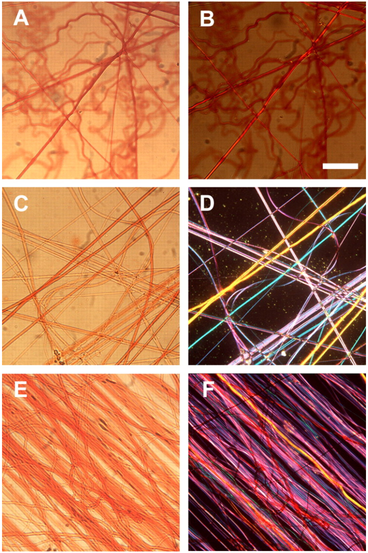
Thread length: about 15 cm!

The packing fraction is close to 1.



[Fernholm, B. (1981). *Acta Zool.* **62** (3), 137–145]

what happens when the skeins unravel?



The threads form a network, which gives the slime its properties.

The thread network determines the rheology of the slime.

[Fudge *et al.* (2005)]

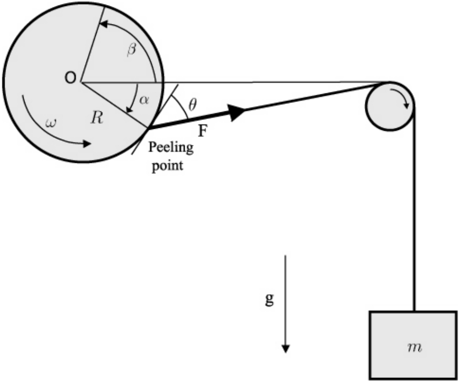
what happens when the skeins unravel?

Here the skein is stuck to a glass slide:

play movie

[experiment by Randy Ewoldt]

similar to experiments with tape



The dynamics at the peeling points can get very complicated and can even lead to triboluminescence.

[Figures from Cortet, P.-P., Ciccotti, M., & Vanel, L. (2007). *J. Stat. Mech.* **2007**, P03005 and Camara, C. G., Escobar, J. V., Hird, J. R., & Putterman, S. J. (2008). *Nature*, **455**, 1089–1092; see also Maugis & Barquins (1988); De *et al.* (2004).]

peeling tape: work-energy theorem

Work-energy theorem of Hong & Yue (1995):

$$\dot{U} = (T - F_0(V))V$$

- $\dot{U}$ is the total change in the energy of the system;
- T is the force drawing out the thread;
- $F_0(V)$ is a velocity-dependent peeling force.

Neglect changes to the elastic energy of the tape ($\dot{U} = 0$):

$$\boxed{T = F_0(V)}$$

peeling force

A simple model for the peeling force is

$$F_0(V) = \alpha V^m, \qquad 0 \leq m \leq 1$$

which we solve for the peeling velocity:

$$V = (T/\alpha)^{1/m}$$

The total length $L(t)$ of thread drawn out thus satisfies

$$\boxed{\dot{L} = (T/\alpha)^{1/m}}$$

Relate R, the skein radius, and L using mass conservation:

$$\frac{d}{dt}\left(\tfrac{4}{3}\pi\eta R^3 + \pi r^2 L\right) = 0 \quad \Longrightarrow \quad \dot{L} = -4\eta R^2 \dot{R}/r^2,$$

where r is the thread radius and $\eta \leq 1$ is the packing fraction of thread into the spherical skein.

Under constant tension T, we can easily solve for the depletion time

$$t_{\mathrm{dep}} = \frac{4\eta R_0^3}{3r^2}\left(T/\alpha\right)^{1/m}$$

to run out of thread, given an initial skein radius R_0.

So far there is no fluid. Assume the skein is immersed a simple 1D flow

$$\mathbf{u}(x, y, t) = u(x, t)\,\hat{\mathbf{x}}$$

at the origin. The thread remains straight and aligned with the horizontal.

Resistive force theory for a straight filament then says

$$8\pi\mu\delta\,(\mathbf{x}_t - \mathbf{u}) = 2T_s\mathbf{x}_s$$
$$2T_{ss} = -8\pi\mu\delta\mathbf{x}_s \cdot \mathbf{u}_s$$

where $\mathbf{x}(s, t)$ is the Eulerian position of a thread segment as a function of the Lagrangian label s (arc length).

[μ = viscosity, δ = slenderness parameter]

unraveling the skein

- The Lagrangian arc length parameter values $s \in [0, L_0]$ correspond to the 'initial' piece of thread.
- The thread added by unraveling is $s \in (L_0, L(t)]$.

The tension at $s = L(t)$ — the end unraveling from the skein — is equal to the drag force on a sphere of radius R:

$$T = 6\pi\mu R\, \mathbf{x}_s \cdot (\mathbf{u} - \mathbf{x}_t), \qquad s = L(t).$$

The other end is free:

$$T = 0, \qquad s = 0.$$

the unraveling rate equation

The Eulerian position of a thread element is

$$x = X(t) - L(t) + s$$

where $X(t)$ is the position of the skein.

Hence, using the tension in our peeling law:

$$(\dot{L})^m = 6\pi\mu R\alpha^{-1}(\dot{L} - \dot{X} + u(X,t))$$

This is not closed: we need to find a separate equation for $\dot{X}$. We do this by solving the equations of resistive force theory.

the closed system

Skipping some algebraic details, we eventually obtain the system of differential equations

$$(\dot{L})^m = 6\pi\mu\alpha^{-1} R\, \bar{u}(X, L, t)$$
$$\dot{X} = \dot{L} + u(X, t) - \bar{u}(X, L, t)$$

where

$$\bar{u}(X, L, t) = \frac{1}{L + (3R/2\delta)} \int_{X-L}^{X} \{u(X, t) - u(x, t)\}\, \mathrm{d}x$$

Note that R and L are related by mass conservation.

extensional flow

For an extensional flow

$$u(x,t) = \lambda(t)x$$

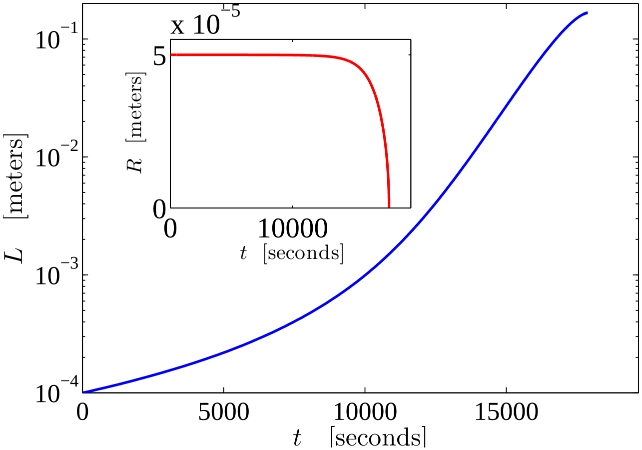
we can reduce the system to one ODE:

$$\boxed{(\dot{L})^m = 3\pi\mu\alpha^{-1}\lambda\, RL^2/(L + (3R/2\delta))}$$

where again $R = R(L)$, and also $\delta = \delta(L)$ (slenderness parameter).

[Recall: $\delta = -1/\log(\varepsilon^2 \mathrm{e})$ with $\varepsilon = r/L$ the slenderness ratio.]

This nonlinear ODE cannot be solved analytically, except in some asymptotic limits for special choices of m.



Numerical solution for $r = 1\,\mu\mathrm{m}$, $R_0 = 50\,\mu\mathrm{m}$, $L_0 = 2R_0$, $\mu = 1.5 \times 10^{-3}\,\mathrm{Pa\,s}$, $m = 1/3$, $\alpha = 8 \times 10^{-4}\,\mathrm{N\,(m/s)^{-1/3}}$, $\eta = 1$, $\lambda = 1\,\mathrm{s}^{-1}$.

numerical solution: discussion

- The depletion time is about $10^3\,\mathrm{s}$.
- Problem: this is 3 orders of magnitude too long!
- Possible issues:
  - No real idea what peeling force parameter values to use. (Here used educated guess based on tape.) Changing these can radically alter the results.
  - Maybe adjust the drag force if the filament doesn't remain straight: go beyond resistive force to full slender-body theory.
  - Are the mucins important? Experiments suggest so but their role is unclear. They might catalyze the peeling somehow, or stick to the filament and increase the drag force.
  - Proper rheological experiments needed (Randy Ewoldt)!
  - Use a more 'mixing' flow, closer to turbulence.
- Future research: network created by threads.

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