Stirring by squirmers

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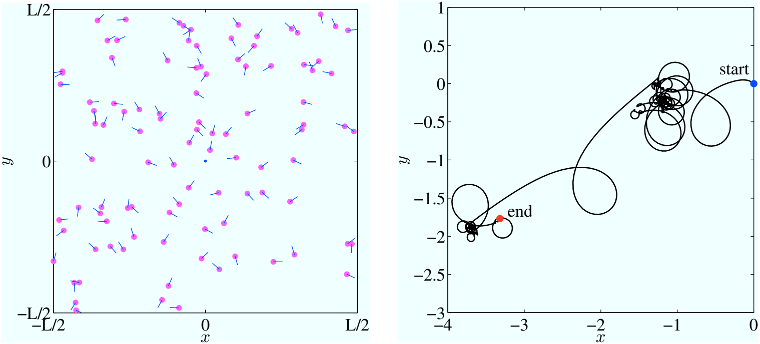
A ‘gas’ of swimmers



[movie 1] 100 cylinders, box size = 1000

Displacement by a moving body

86 Mr. J. Clerk-Maxwell *on* [Mar. 10,

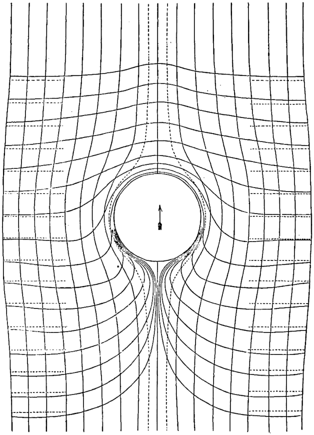


Fig. 1.
Fluid flowing past a fixed cylinder.

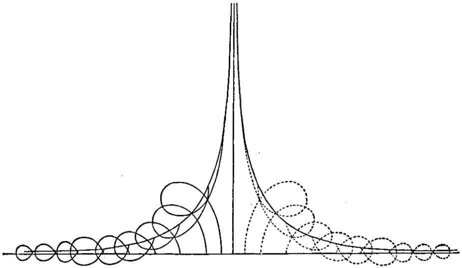


Fig. 2.
Paths of particles of the fluid when a cylinder moves through it.

Maxwell (1869); Darwin (1953); Eames *et al.* (1994)

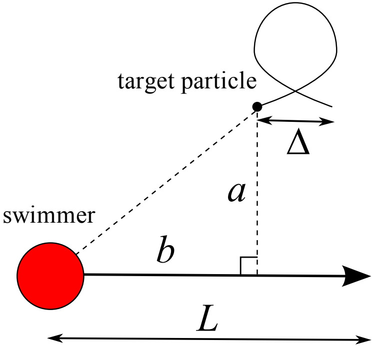
Suggests mechanism for stirring by swimming organisms. (Katija & Dabiri, 2009; Thiffeault & Childress, 2010)

A sequence of kicks

Inspired by Einstein's theory of diffusion (Einstein, 1905): a test particle initially at $\mathbf{x}(0) = 0$ undergoes N encounters with an axially-symmetric swimming body:

$$\mathbf{x}(t) = \sum_{k=1}^{N} \Delta_L(a_k, b_k)\,\hat{\mathbf{r}}_k$$

$\Delta_L(a, b)$ is the displacement, a_k, b_k are impact parameters, and $\hat{\mathbf{r}}_k$ is a direction vector.



($a > 0$, but b can have either sign.)

Effective diffusivity

Putting this together,

$$\left\langle |\mathbf{x}|^2 \right\rangle = \frac{2Unt}{L} \int \Delta_L^2(a,b)\,\mathrm{d}a\,\mathrm{d}b = 4\kappa t, \qquad \text{2D}$$

$$\left\langle |\mathbf{x}|^2 \right\rangle = \frac{2\pi Unt}{L} \int \Delta_L^2(a,b) a\,\mathrm{d}a\,\mathrm{d}b = 6\kappa t, \qquad \text{3D}$$

which defines the effective diffusivity κ.

Valid for low number density is low ($nL^d \ll 1$).

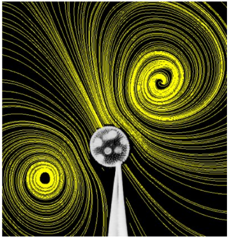
(Lin, Thiffeault & Childress, JFM, in press)

Squirmers

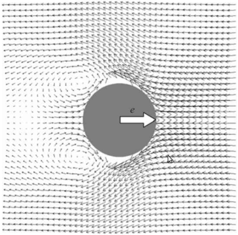
Considerable literature on transport due to microorganisms: Wu & Libchaber (2000); Hernandez-Ortiz *et al.* (2006); Saintillian & Shelley (2007); Underhill *et al.* (2008); Ishikawa (2009); Leptos *et al.* (2009)

Lighthill (1952), Blake (1971), and more recently Ishikawa *et al.* (2006) have considered squirmers:

- Sphere in Stokes flow;
- Steady velocity specified at surface, to mimic cilia;
- Steady swimming condition imposed (no net force on fluid).



(Drescher *et al.*, 2009)



(Ishikawa *et al.*, 2006)

Typical squirmer

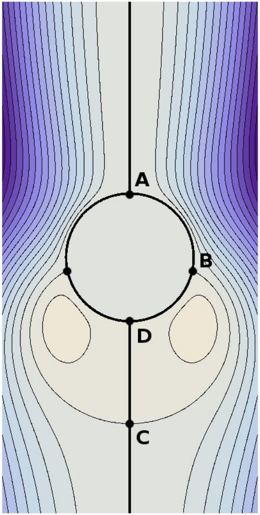
3D axisymmetric streamfunction for a typical squirmer, in cylindrical coordinates (ρ, z):

$$\psi = -\tfrac{1}{2}\rho^2 + \frac{1}{2r^3}\,\rho^2 + \frac{3\beta}{4r^3}\,\rho^2 z\left(\frac{1}{r^2} - 1\right)$$

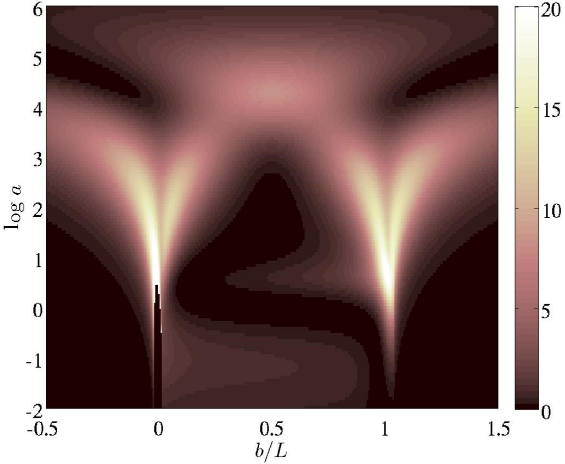
where $r = \sqrt{\rho^2 + z^2}$, $U = 1$, radius of squirmer $= 1$.

β is the amplitude of the stresslet (distinguises pushers/pullers).

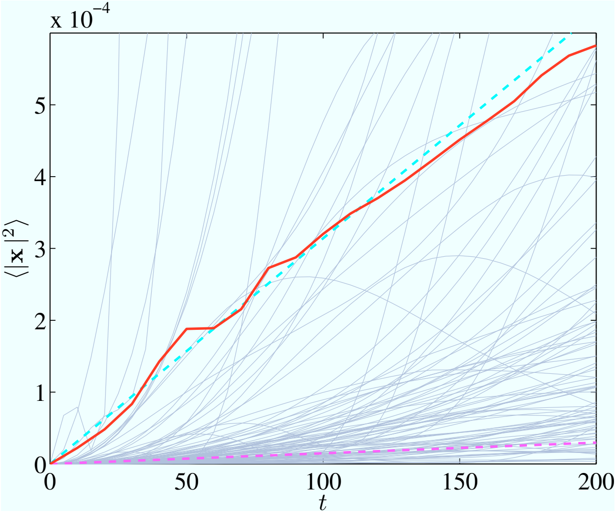
We will use $\beta = 5$ for most of the remainder.



Squirmer displacements $a^2\Delta_L^2(a,b)$

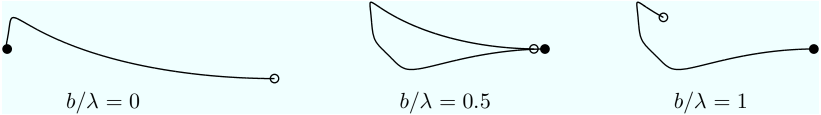


Squirmers: Transport



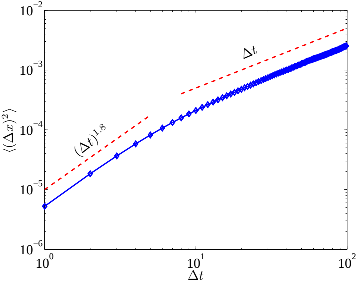
Squirmers: Trajectories

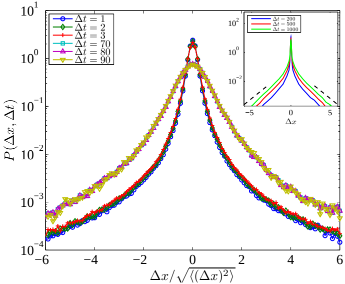
The two peaks in the displacement plot come from ‘incomplete’ trajectories:



For long path length, the effective diffusivity is independent of the swimming path length, and yet the dominant contribution arises from the finiteness of the path (uncorrelated turning directions).

Non-Gaussian PDFs of displacement





- Variance exhibits similar short-time anomalous scaling as in Wu & Libchaber (2000);
- PDF matches experiments of Leptos *et al.* (2009). In our case, exponential tails are due to sticking at the stagnation points on the squirmer's body.

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