Moving Walls Accelerate Mixing

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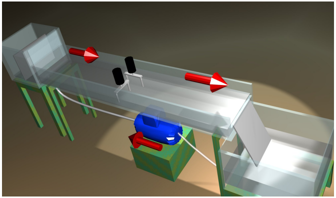
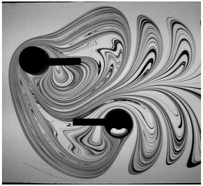
Stirring and Mixing of Viscous Fluids

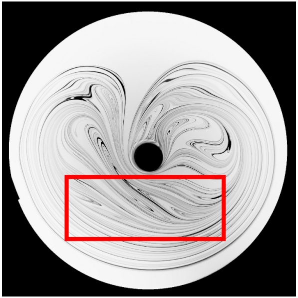
- Viscous flows $\Rightarrow$ no turbulence! (laminar)
- Open and closed systems
- Active (rods) and passive





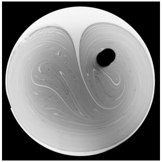
Understand the mechanisms involved.
Characterise and optimise the efficiency of mixing.

The Figure-Eight Stirring Protocol



- Circular container of viscous fluid (sugar syrup);
- A rod is moved slowly in a 'figure-eight' pattern;
- Gradients are created by stretching and folding, the signature of chaos.

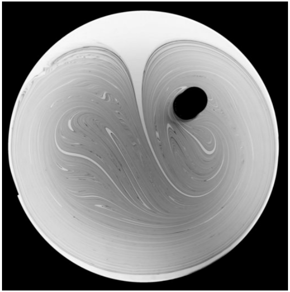


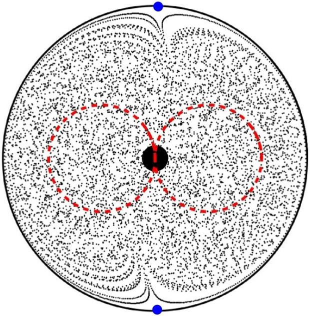



[movie 1] Experiments by E. Gouillart and O. Dauchot (CEA Saclay).

The Mixing Pattern

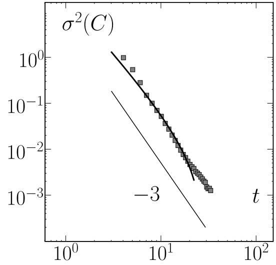
- Kidney-shaped mixed region extends to wall;
- Two parabolic points on the wall, one associated with injection of material;
- Asymptotically self-similar, so expect an exponential decay of the concentration ('strange eigenmode' regime).
  (Pierrehumbert, 1994; Rothstein et al., 1999; Voth et al., 2003)





Mixing is Slower Than Expected

Concentration field in a well-mixed central region



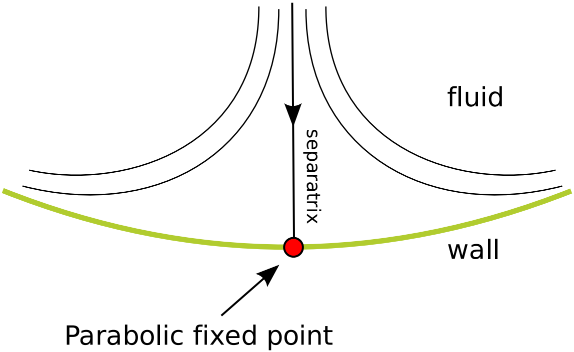
$$\text{Variance} = \int |\theta|^2 dV$$

$\Rightarrow$ Algebraic decay of variance $\neq$ Exponential

The ‘stretching and folding’ action induced by the rod is an exponentially rapid process (chaos!), so why aren’t we seeing exponential decay?

The Problem: Separatrix at the Wall

The decay is algebraic near a reattachment point at the wall:

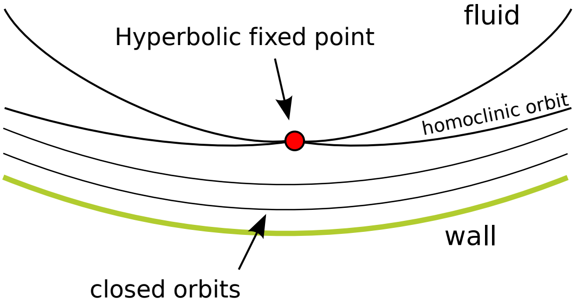


A fluid particle following the separatrix approaches the wall as $1/t$.

[Chertkov & Lebedev (2003); Lebedev & Turitsyn (2004); Salman & Haynes (2007); Gouillart et al. (2007, 2008, 2009a); Chernykh & Lebedev (2008)]

How can we mimic a slip boundary condition?

Create closed orbits near the wall:

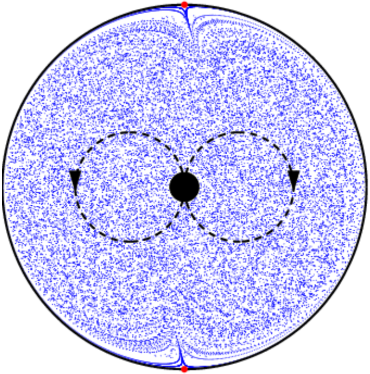


There will be a 'last closed orbit' followed by one or more fixed or periodic points and a separatrix, for example a hyperbolic orbit. Particles approach the hyperbolic fixed point exponentially fast.

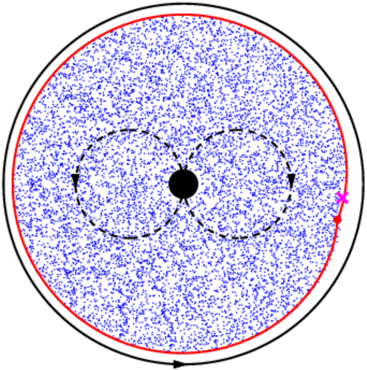
[Gouillart et al. (2009b)]

Rotating the Wall

We can create a hyperbolic fixed point by rotation:



Fixed wall: parabolic separation point (algebraic)

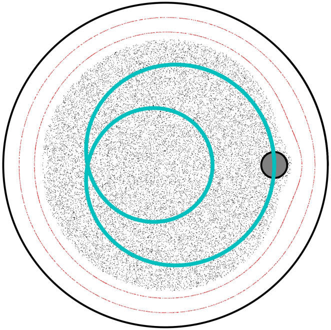


Moving wall: hyperbolic fixed point (exponential)

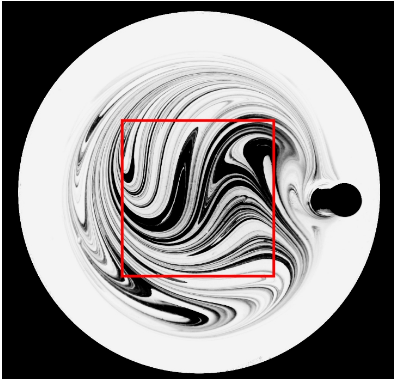
El Omari & Le Guer (2009) see exponential decay with a rotating wall.

A Second Experiment

Rotating the wall is not crucial: create closed orbits.

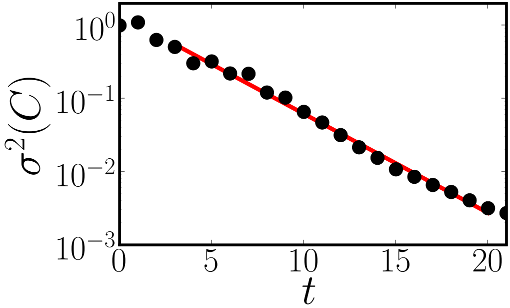


"Epitrochoid" protocol



Central chaotic region + regular region near the walls.

Recover Exponential Decay



Conclusions

- If the chaotic region extends to the walls, then the decay of concentration is algebraic (typically $(\log t)/t^{-2}$ for variance).
- The no-slip boundary condition at the walls is to blame.
- Would recover a strange eigenmode for very long times, once the mixing pattern is within a Batchelor length from the edge (not very useful in practice!).
- We can shield the mixing region from the walls by wrapping it in a regular island — rotate the wall!
- We then recover exponential decay.
- How to control this in practice? Is it really advantageous? Is scraping the walls better?
- See `http://arxiv.org/abs/0909.3888`.

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