The Role of Walls in Chaotic Mixing : Experimental Results

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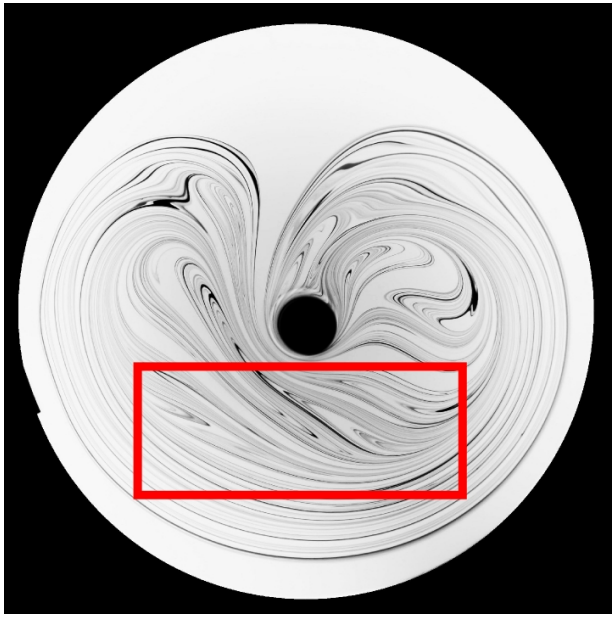
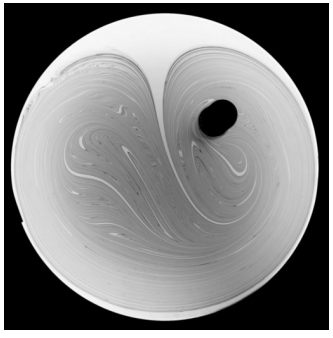
The Figure-Eight Stirring Protocol



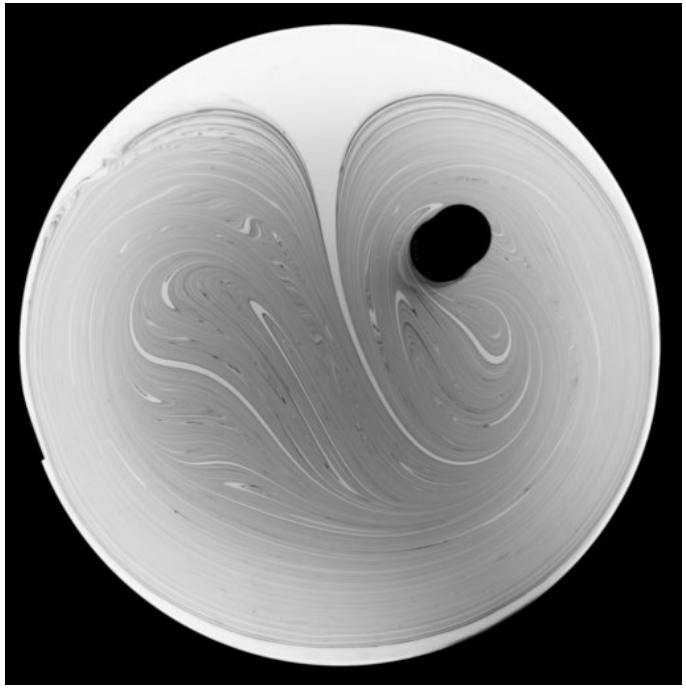
- Circular container of viscous fluid (sugar syrup) ;
- A rod is moved slowly in a ‘figure-eight’ pattern ;
- The flow is basically two-dimensional ;
- Chaotic advection by stretching and folding ;
- Measure concentration of blob of ink advected by the flow.

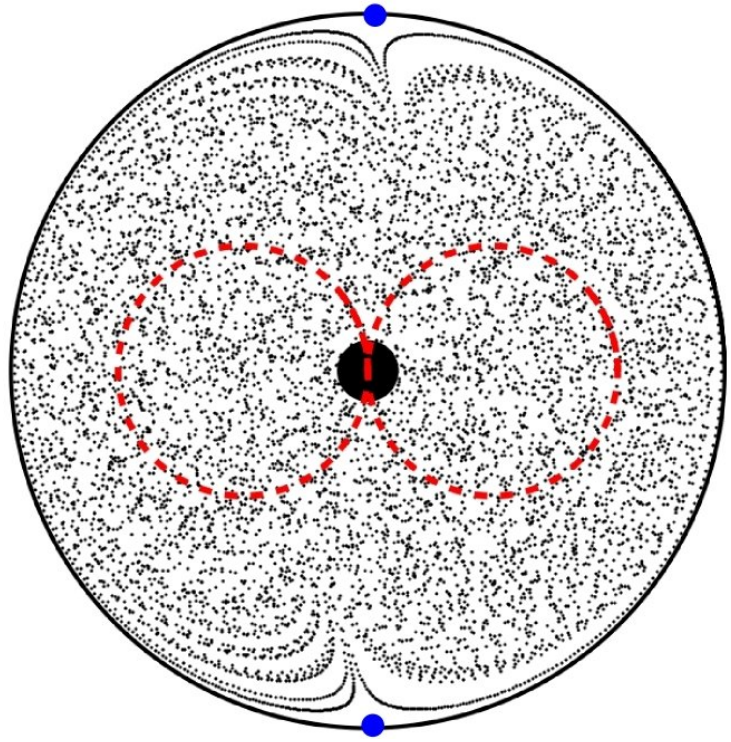


The Mixing Pattern

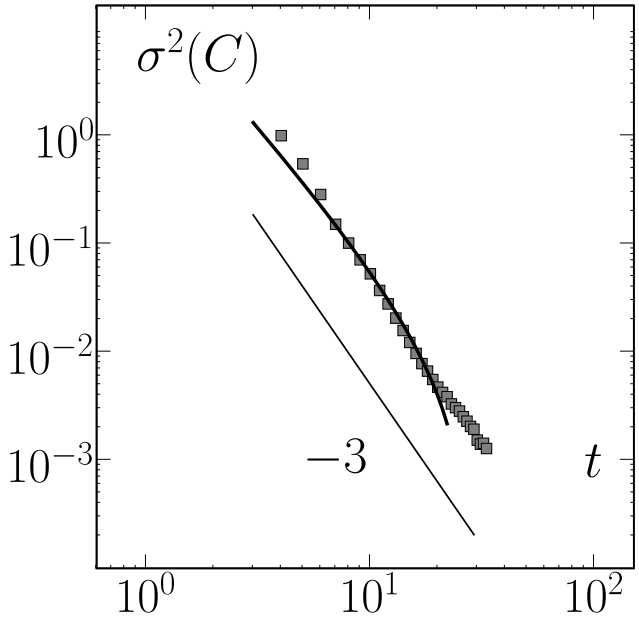
- The mixing pattern has a kidney shape ;
- Chaotic region extends to wall ;
- Two parabolic points on the wall, one associated with injection of material. ;
- Asymptotically self-similar, so expect a ‘strange eigenmode’ regime [Pierrehumbert 1994 ; Rothstein 1999].



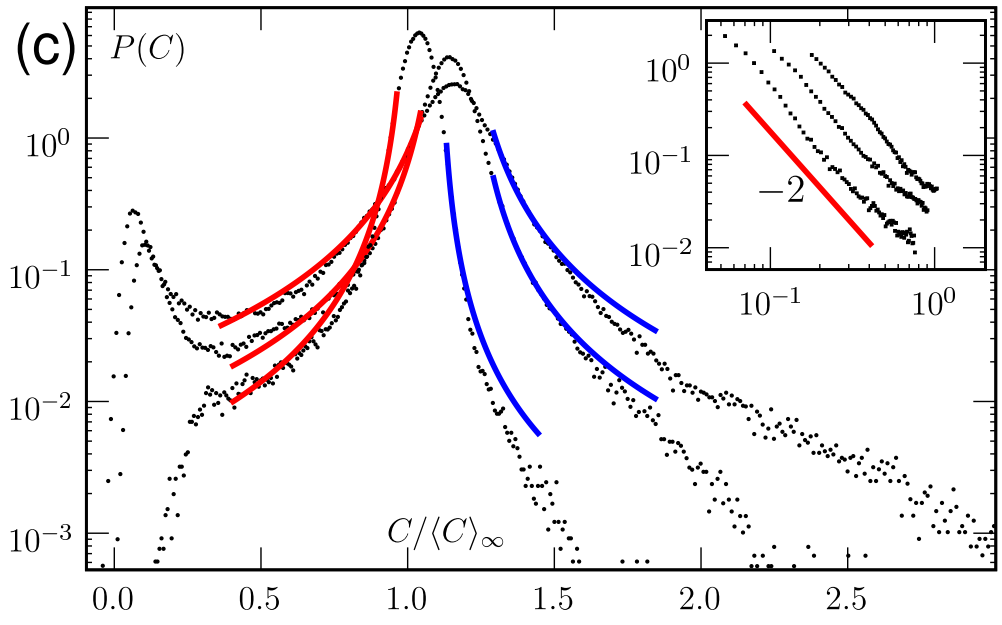


Mixing is Slower Than Expected

Concentration field in a **well-mixed central region**



Variance



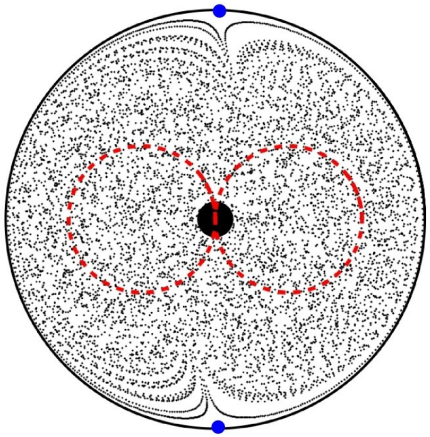
Concentration PDFs

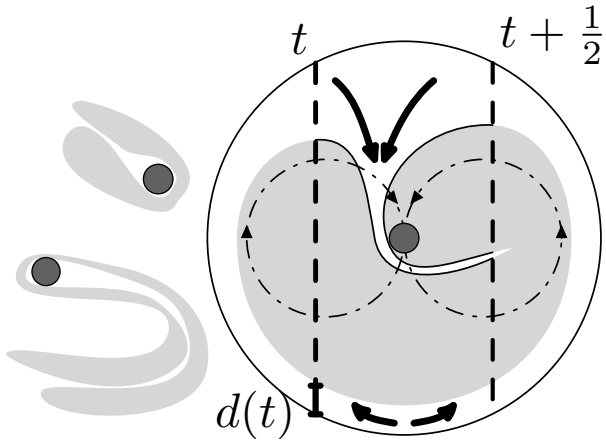
$\Rightarrow$ Algebraic decay of variance $\neq$ Exponential

Is this a ‘strange’ eigenmode of the A–D operator ?

Walls Slow Down Mixing...

... everywhere in the fluid domain



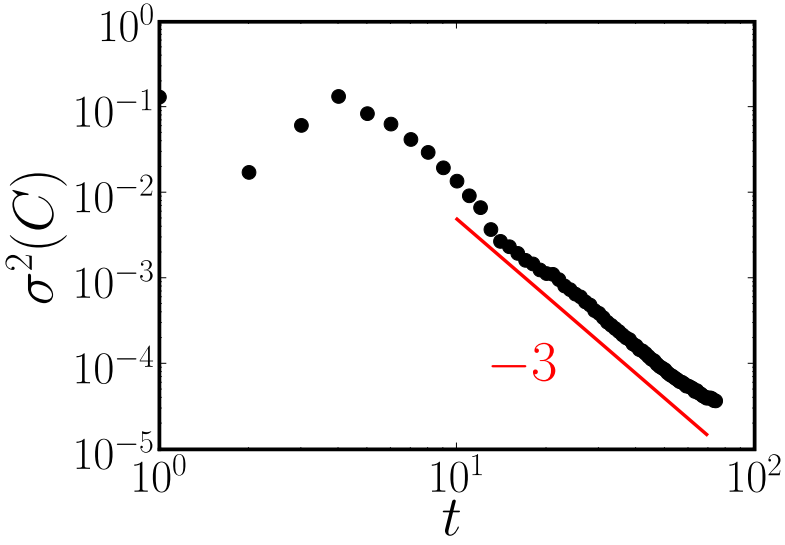


- **Trajectories are (almost) everywhere chaotic**
  $\Rightarrow$ but there is always poorly-mixed fluid near the walls.
- **Re-inject unmixed (white) material** along the unstable manifold of a parabolic point on the wall.
- **No-slip at walls**
  $\Rightarrow$ width of "white stripes" $\sim t^{-2}$ (algebraic).
- These re-injected white strips **contaminate** the mixing pattern, in spite of the fact that **stretching is exponential** in the centre.

A Generic Scenario

... When the Chaotic Region Extends to the Walls

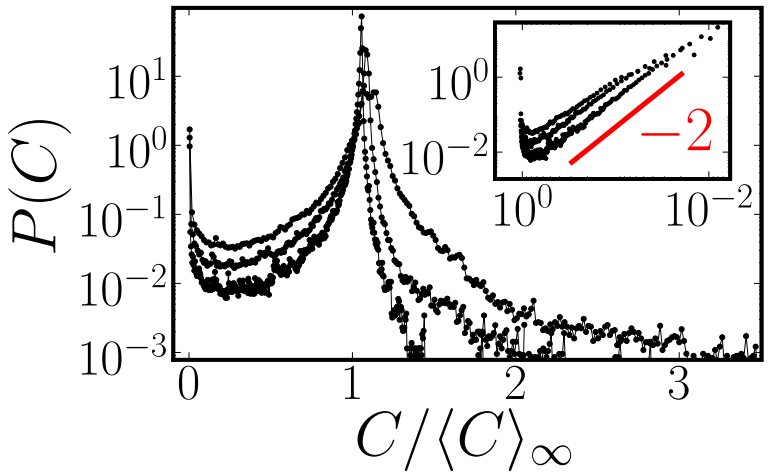
- "Blinking vortex" [Aref 1984] : numerical simulations



- 1-D Model : Baker's map + parabolic point

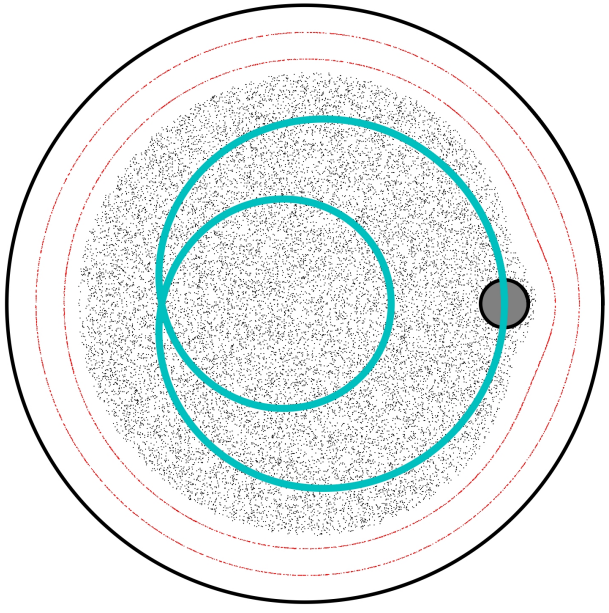
Reproduce statistical features of the concentration field ;
Some analytic results possible.

[Gouillart et al., PRL 99, 114501 (2007)]

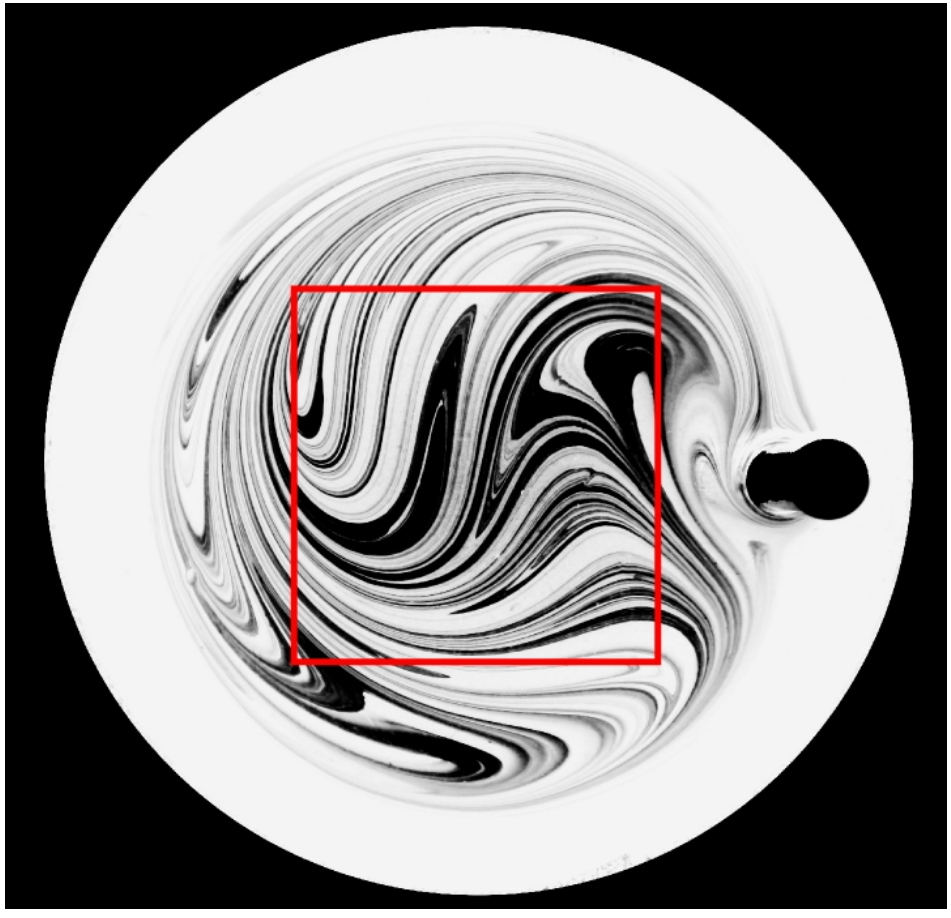


A Second Scenario

How do we mimic a slip boundary condition ?



“Epitrochoid” protocol

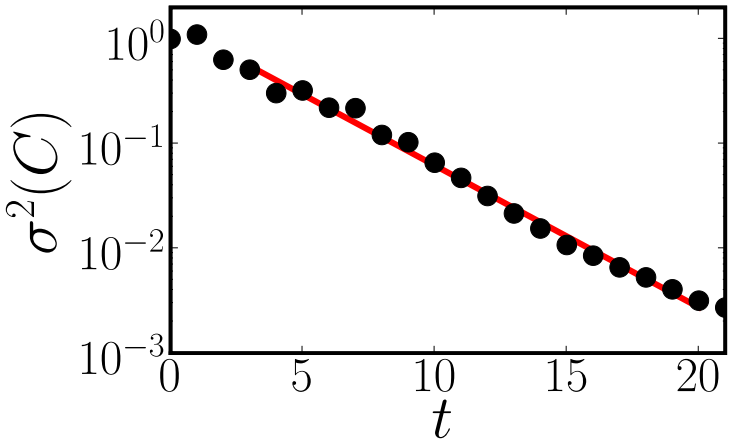


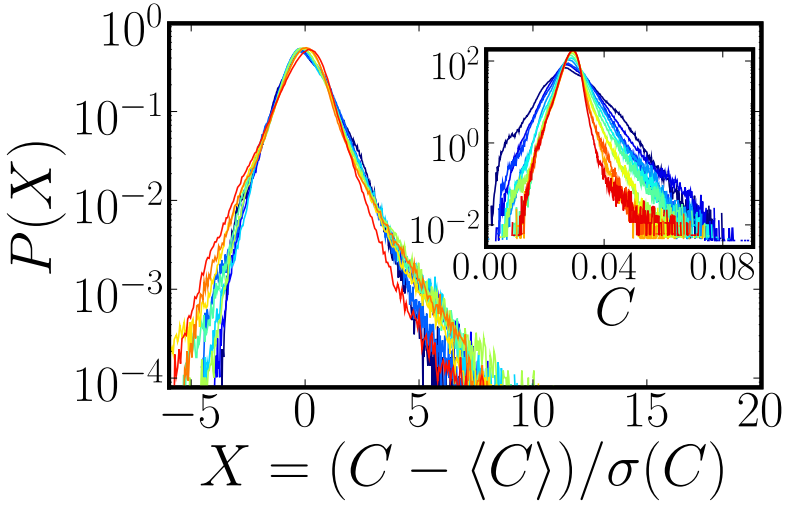
Central chaotic region + regular region near the walls.

(Another approach : rotate the wall ! – i.e., Journal Bearing Flow)

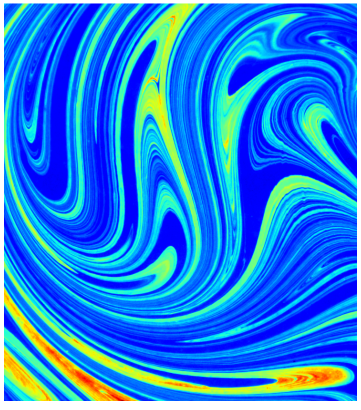
Recover a Strange Eigenmode !

... with Exponential Decay of Variance

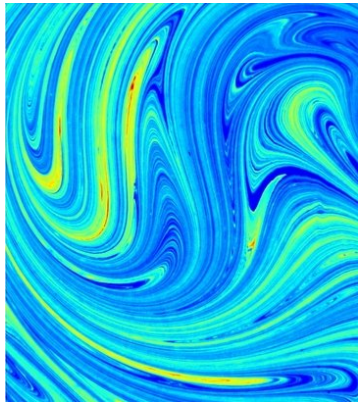




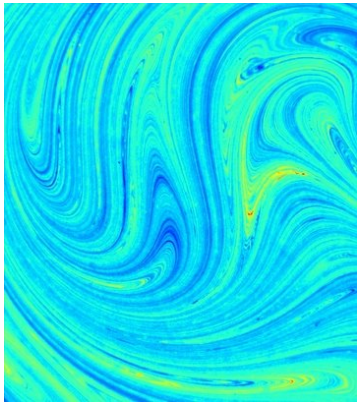
$t = 8$



$t = 12$



$t = 17$



Conclusions

- If the chaotic region extends to the walls, then the **decay of concentration is algebraic** (typically t^{-3} for variance).
- The **no-slip boundary condition** at the walls is to blame.
- Would recover a strange eigenmode for **very long times**, once the mixing pattern is within a Batchelor length from the edge (not very useful in practice !).
- The decay is well-predicted by a baker's map with a **parabolic point**.
- We can shield the mixing region from the walls by wrapping it in a **regular island**.
- We then recover **exponential decay**.
- How to control this in practice ? Is it really advantageous ? Is **scraping** the walls better ?
- See **[Gouillart et al., PRL 99, 114501 (2007)]**