The Role of Walls in Chaotic Mixing

Experimental Results

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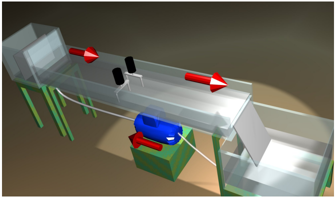
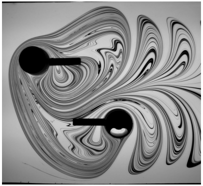
Stirring and Mixing of Viscous Fluids

- Viscous flows ⇒ no turbulence! (laminar)
- Open and closed systems
- Active (rods) and passive





Understand the mechanisms involved.
Characterise and optimise the efficiency of mixing.

Stirring and Mixing: What's the Difference?

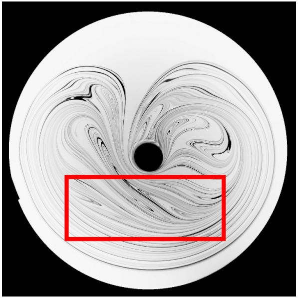
- Stirring is the mechanical motion of the fluid (cause);
- Mixing is the homogenisation of a substance (effect, or goal);
- Two extreme limits: Turbulent and laminar mixing, both relevant in applications;
- Even if turbulence is feasible, still care about energetic cost;
- For very viscous flows, use simple time-dependent flows to create chaotic mixing.
- Here we look at the impact of the vessel walls on mixing rates.

A Simple Example: Planetary Mixers

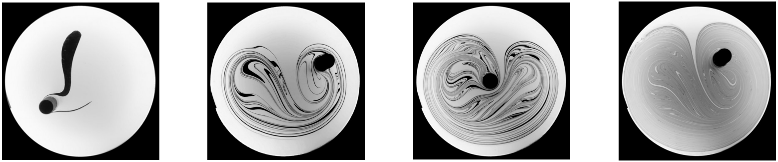
In food processing, rods are often used for stirring.

[movie 1] ©BLT Inc.

The Figure-Eight Stirring Protocol



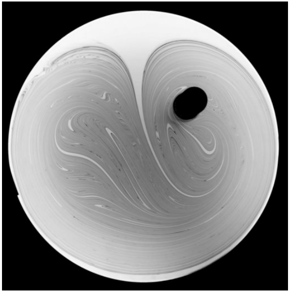
- Circular container of viscous fluid (sugar syrup);
- A rod is moved slowly in a ‘figure-eight’ pattern;
- Gradients are created by stretching and folding, the signature of chaos.

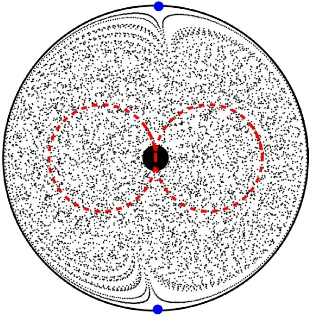


[movie 2] Experiments by E. Gouillart and O. Dauchot (CEA Saclay).

The Mixing Pattern

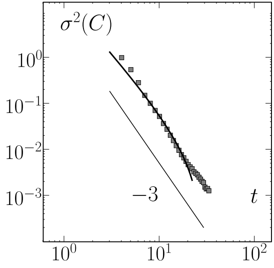
- Kidney-shaped mixed region extends to wall;
- Two parabolic points on the wall, one associated with injection of material;
- Asymptotically self-similar, so expect an exponential decay of the concentration ('strange eigenmode' regime).
  (Pierrehumbert, 1994; Rothstein et al., 1999; Voth et al., 2003)



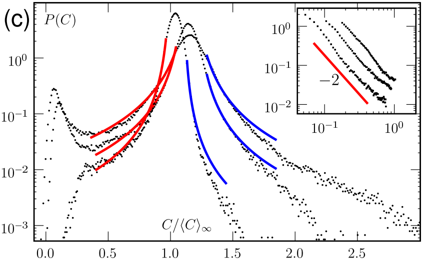


Mixing is Slower Than Expected

Concentration field in a well-mixed central region



Variance $= \int |\theta|^2 dV$

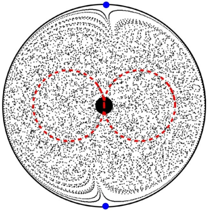


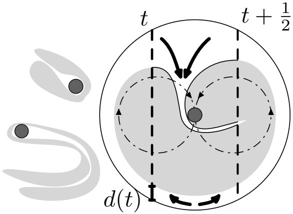
Concentration PDFs

$\Rightarrow$ Algebraic decay of variance $\neq$ Exponential

The 'stretching and folding' action induced by the rod is an exponentially rapid process (chaos!), so why aren't we seeing exponential decay?

Walls Slow Down Mixing





- Trajectories are (almost) everywhere chaotic
  $\Rightarrow$ but there is always poorly-mixed fluid near the walls;
- Re-inject unmixed (white) material along the unstable manifold of a parabolic point on the wall;
- No-slip at walls $\Rightarrow$ width of "white stripes" $\sim t^{-2}$ (algebraic);
- Re-injected white strips contaminate the mixing pattern, in spite of the fact that stretching is exponential in the centre.

Hydrodynamics Near the Wall

We can characterize white strips in terms of hydrodynamics near the no-slip wall. $x_\parallel$ and $x_\perp$ denote respectively the distance along and $\perp$ to the wall. No-slip boundary conditions impose

$$v_\parallel \sim x_\perp, \qquad \text{near the wall:} \quad x_\perp \ll 1.$$

Incompressibility

$$\frac{\partial v_\parallel}{\partial x_\parallel} + \frac{\partial v_\perp}{\partial x_\perp} = 0,$$

implies

$$v_\perp \simeq -a\, x_\perp^2\,.$$

Solve $\dot{x}_\perp = v_\perp$:

$$x_\perp \simeq \frac{x_0}{1 + at\, x_0}.$$

Hydrodynamics Near the Wall (continued)

Hence, the distance between the wall and a particle in the lower part of the domain (where $v_\perp < 0$) shrinks as

$$d(t) \simeq 1/at, \qquad t \gg 1.$$

This scaling was derived in Chertkov & Lebedev (2003), and we verified it experimentally.

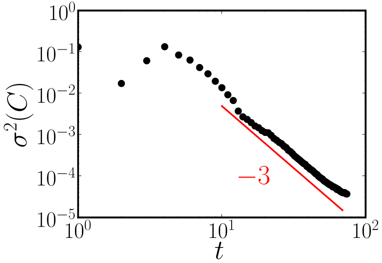
The amount of white that is 'shaved off' at each period is thus

$$\dot{d} \sim T/at^2, \qquad t \gg 1,$$

where T is the period. This is the origin of the power-law decay. Corrections due to the stretch/fold action are described in [Gouillart et al., *Phys. Rev. Lett.* **99**, 114501 (2007)].

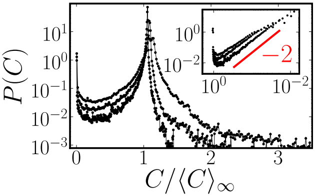
A Generic Scenario

- "Blinking vortex" (Aref, 1984) : numerical simulations



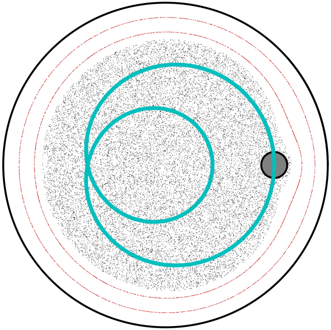
- 1-D Model: Baker's map + parabolic point

Reproduce statistical features of the concentration field;
Some analytical results possible.
(Gouillart et al., 2007)

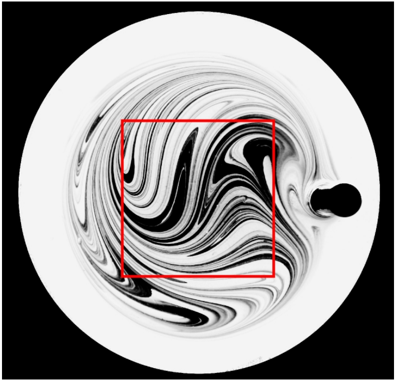


A Second Scenario

How do we mimic a slip boundary condition?

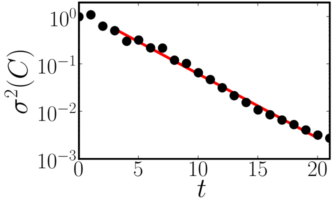


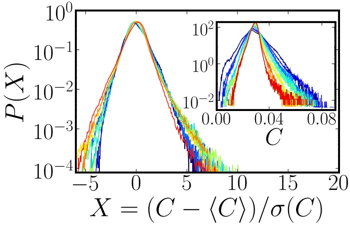
"Epitrochoid" protocol



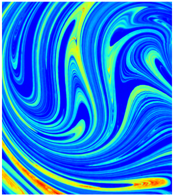
Central chaotic region + regular region near the walls.

Recover Exponential Decay

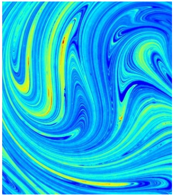




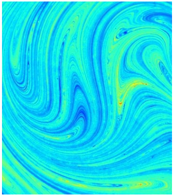
$t = 8$



$t = 12$



$t = 17$

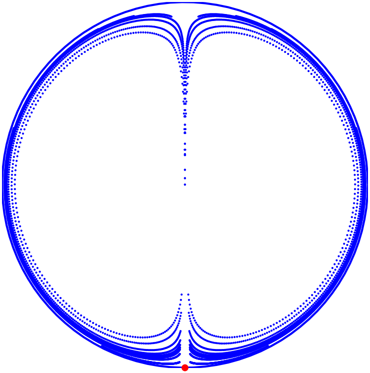


. . . as well as ‘true’ self-similarity.

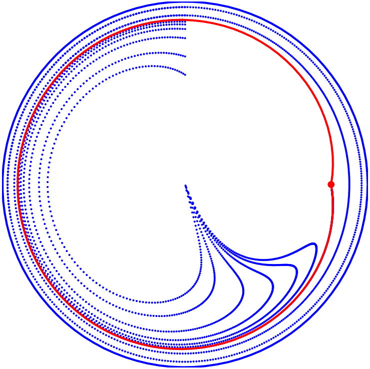
Another Approach: Rotate the Bowl!

Rotating the Wall

Can use a simplified 'edge map' to model the near-wall region:



Fixed wall: parabolic separation point (algebraic)



Moving wall: hyperbolic fixed point (exponential)

Conclusions

- If the chaotic region extends to the walls, then the decay of concentration is algebraic (typically t^{-3} for variance).
- The no-slip boundary condition at the walls is to blame.
- Would recover a strange eigenmode for very long times, once the mixing pattern is within a Batchelor length from the edge (not very useful in practice!).
- The decay is well-predicted by a baker's map with a parabolic point.
- We can shield the mixing region from the walls by wrapping it in a regular island.
- We then recover exponential decay.
- How to control this in practice? Is it really advantageous? Is scraping the walls better?
- See **[Gouillart et al., PRL 99, 114501 (2007)]**

References

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