Do fish stir the ocean?

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Physics Colloquium, University of Alberta, 5 February 2010

Biomixing

A controversial proposition:

- There are many regions of the ocean that are relatively quiescent, especially in the depths (1 hairdryer/ km^3);
- Yet mixing occurs: nutrients eventually get dredged up to the surface somehow;
- What if organisms swimming through the ocean made a significant contribution to this?
- There could be a local impact, especially with respect to feeding and schooling;
- Also relevant in suspensions of microorganisms (Viscous Stokes regime).

Munk's Idea

Though it had been mentioned earlier, the first to seriously consider the role of biomixing was Walter Munk (1966):

Abyssal recipes

WALTER H. MUNK*

(*Received* 31 *January* 1966)

Abstract—Vertical distributions in the interior Pacific (excluding the top and bottom kilometer) are not inconsistent with a simple model involving a constant upward vertical velocity $w \approx 1{\cdot}2$ cm day^{-1} and eddy diffusivity $\kappa \approx 1{\cdot}3$ cm^2 sec^{-1}. Thus temperature and salinity can be fitted by exponential-like solutions to $[\kappa \cdot d^2/dz^2 - w \cdot d/dz]\, T, S = 0$, with $\kappa/w \approx 1$ km the appropriate "scale height." For Carbon 14 a decay term must be included, $[\quad]\,{}^{14}C = \mu\,{}^{14}C$; a fitting of the solution to the observed ^{14}C distribution yields $\kappa/w^2 \approx 200$ years for the appropriate "scale time," and permits w and

"…I have attempted, without much success, to interpret [the eddy diffusivity] from a variety of viewpoints: from mixing along the ocean boundaries, from thermodynamic and biological processes, and from internal tides."

Basic claims

The idea lay dormant for almost 40 years; then

- Huntley & Zhou (2004) analyzed the swimming of 100 (!) species, ranging from bacteria to blue whales. Turbulent energy production is $\sim 10^{-5}\ \mathrm{W\ kg^{-1}}$ for 11 representative species.
- Total is comparable to energy dissipation by major storms!
- Another estimate comes from the solar energy captured: 63 TeraW, something like 1% of which ends up as mechanical energy (Dewar *et al.*, 2006).
- Kunze *et al.* (2006) find that turbulence levels during the day in an inlet were 2 to 3 orders of magnitude greater than at night, due to swimming krill.

Rain on the parade

Visser (2007) debunks these claims:

Let the turbulence be generated at a scale L, with a rate of turbulent energy dissipation ε.

The buoyancy frequency N is defined as

$$N^2 = -\frac{g}{\rho}\frac{d\rho}{dz}$$

where g is the gravitational acceleration and $\rho(z)$ is the density.

The buoyancy length scale is

$$B = (\varepsilon/N^3)^{1/2}$$

Mixing efficiency

The mixing efficiency is defined as

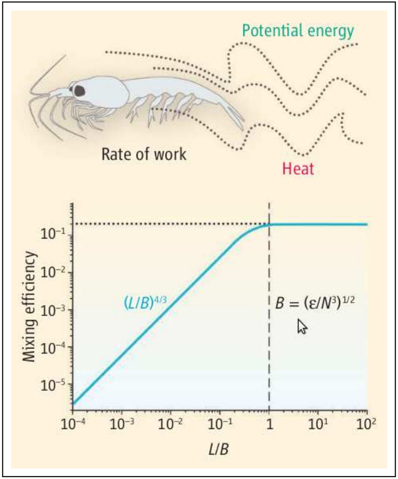
$$\Gamma = \frac{\text{change in potential energy}}{\text{work done}}$$

so $0 \le \Gamma \le 1$.

Visser's point is that Γ depends strongly on L/B.

For krill $L = 1.5\ \mathrm{cm}$, $B = 3$ to $10\ \mathrm{m}$, so $L/B = .005$ to $.0015$.

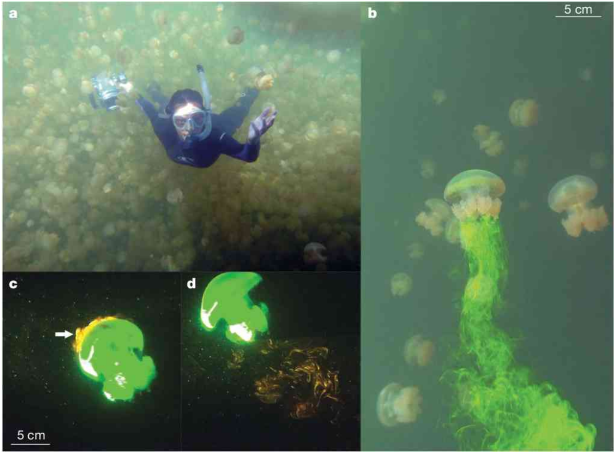
Conclude: $\Gamma = 10^{-4}$ to 10^{-3}: almost none of the turbulent energy goes into mixing.



(from Visser (2007))

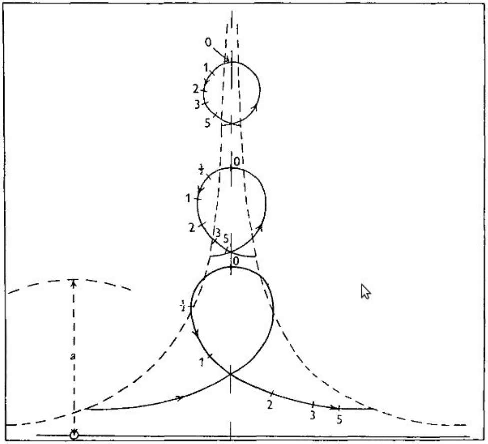
But it's not over...

Katija & Dabiri (2009) looked at jellyfish:



[movie 1]

Displacement by a moving body



(from Darwin (1953))

[movie 2] (movie from Katija & Dabiri (2009))

A sequence of kicks

The age-old paradigm for calculating an effective diffusivity consists of assuming a test particle undergoes uncorrelated "kicks": if a test particle initially at $\mathbf{x}(0) = 0$ undergoes N encounters with axially-symmetric swimming bodies, its position is

$$\mathbf{x}(t) = \sum_{k=1}^{N} \Delta(a_k)\,\hat{\mathbf{r}}_k$$

where $\Delta(a)$ is the displacement, a_k is the impact parameter, and $\hat{\mathbf{r}}_k$ is a direction vector.

After squaring and averaging, assuming isotropy:

$$\left\langle |\mathbf{x}|^2 \right\rangle = N \left\langle \Delta^2(a) \right\rangle$$

where a is treated as a random variable.

Assuming the swimmers move in a straight line at speed U, the number that will hit an "interaction disk" of radius R in time t is $2RUnt$, where n is the number density.

The approach from infinity means that a is distributed as da/R. Putting this together,

$$\langle |\mathbf{x}|^2 \rangle = 2Unt \int_0^\infty \Delta^2(a)\, da = 4\kappa t, \qquad \text{2D}$$

which defines the effective diffusivity κ.

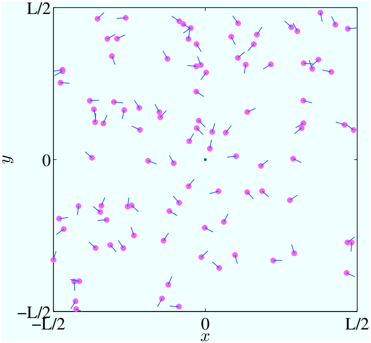
In 3D, the factors are modified slightly:

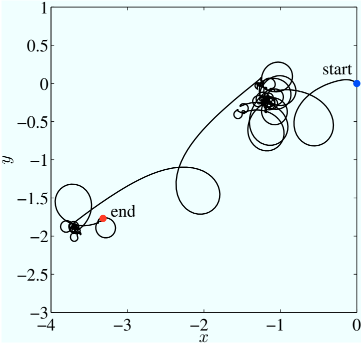
$$\langle |\mathbf{x}|^2 \rangle = 2\pi Unt \int_0^\infty a\, \Delta^2(a)\, da = 6\kappa t, \qquad \text{3D}$$

Numerical simulation

- Validate theory using simple simple simulations;
- Periodic box of size L;
- N swimmers (spheres of radius 1), initially at random positions, swimming in random direction with constant speed $U = 1$;
- Target particle initially at origin advected by the swimmers;
- Since dilute, superimpose velocities;
- Integrate for some time, compute $|\mathbf{x}(t)|^2$, repeat for a large number N_{real} of realizations and average.

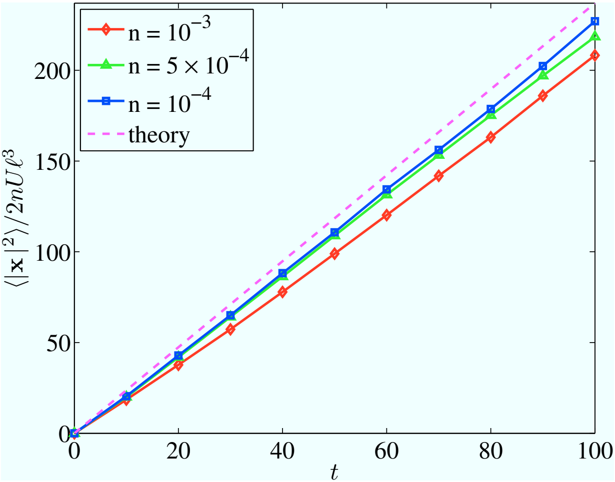
A ‘gas’ of swimmers



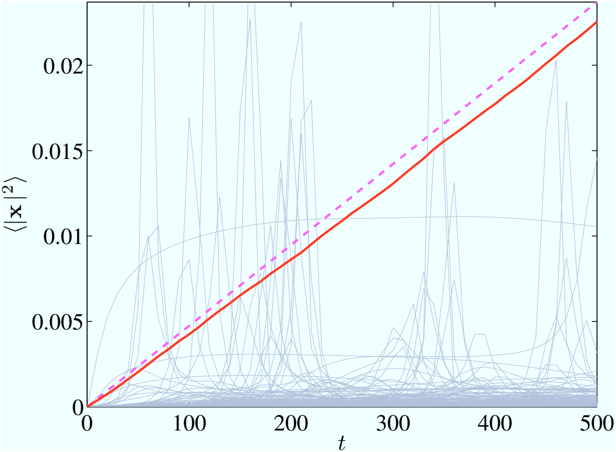


[movie 3] $N = 100$ swimmers, $L = 1000$

How well does the dilute theory work?



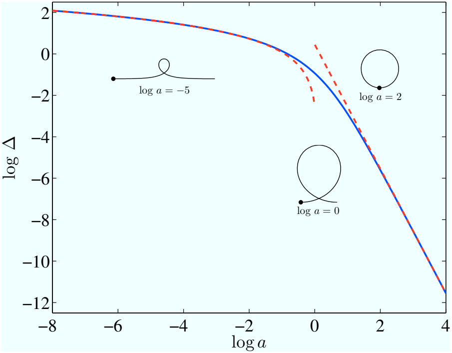
Diffusion is dominated by rare events



2×10^6 realizations of $N = 10$ cylinders, with $L = 1000$

Contribution to displacement

Small a: $\Delta \sim -\log a$, large a: $\Delta \sim a^{-3}$ (Darwin, 1953)



$\int_0^1 \Delta^2(a)da \simeq$ 2.31, whilst $\int_1^\infty \Delta^2(a)da \simeq$ .06.

$\implies$ 97% dominated by "head-on" collisions

Origin of the singularity

At the leading and trailing 'edges' of a body, there is a hyperbolic point. Locally,

$$\dot{x} = -\lambda x, \quad \dot{y} = \lambda y$$

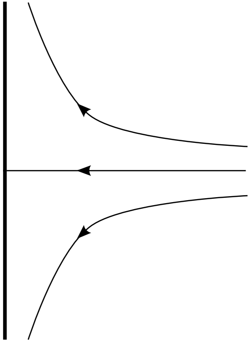
so that $y(t) = y_0 \exp(\lambda t)$. The time it takes to go from $y_0 = a$ to $y > a$ is

$$t = \lambda^{-1} \log(y/a) \sim -\lambda^{-1} \log a$$

which is the source of the logarithmic divergence of the displacement:

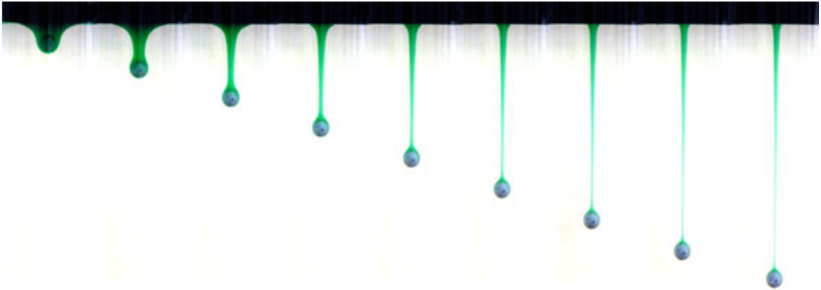
$$\boxed{\Delta \sim -2U\lambda^{-1} \log a, \qquad a \ll 1}$$

The factor of 2 is for leading+trailing edges.



Sphere in Viscous (Stokes) flow

A natural question is what happens in the presence of viscosity, which greatly increases the "sticking" to the swimmer's surface?



(from Camassa et al., *Sphere Passing Through Corn Syrup*)

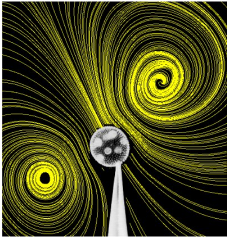
Considerable literature on transport due to microorganisms: Wu & Libchaber (2000); Hernandez-Ortiz *et al.* (2006); Saintillian & Shelley (2007); Ishikawa & Pedley (2007); Underhill *et al.* (2008); Ishikawa (2009)

Squirmers

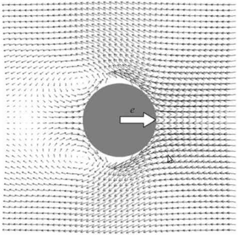
One problem with the Stokesian spheres is that they are an awful model for swimming: there is a net force on the fluid. It's as if the spheres are pulled by invisible threads.

Lighthill (1952), Blake (1971), and more recently Ishikawa *et al.* (2006) have considered squirmers:

- Sphere in Stokes flow;
- Steady velocity specified at surface, to mimic cilia;
- Steady swimming condition imposed (no net force on fluid).



(Drescher *et al.*, 2009)



(Ishikawa *et al.*, 2006)

Typical squirmer

3D axisymmetric streamfunction for a typical squirmer, in cylindrical coordinates (ρ, z):

$$\psi(\rho, z) = -\tfrac{1}{2}\rho^2 + \frac{1}{2r^3}\,\rho^2 + \frac{3\beta}{4r^3}\,\rho^2 z \left(\frac{1}{r^2} - 1\right)$$

where $r = \sqrt{\rho^2 + z^2}$, $U = 1$, radius of squirmer $= 1$.

Note that $\beta = 0$ is the sphere in potential flow!

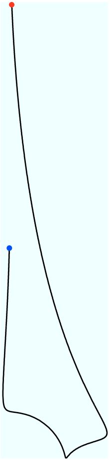
We will use $\beta = 5$ for most of the remainder.

Particle motion for squirmer

A particle near the squirmer's swimming axis initially (blue) moves towards the squirmer.

After the squirmer has passed the particle follows in the squirmer's wake.

(The squirmer moves from bottom to top.)



[movie 4]

Small a asymptotics for squirmer

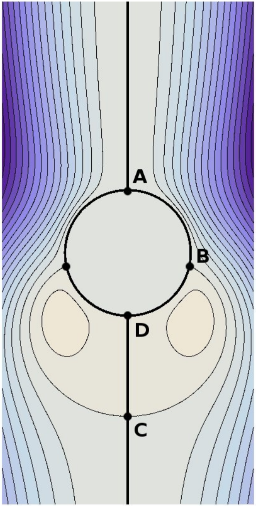
Four stagnation points for the squirmer (**B** is a "ring" around the squirmer). A particle coming close to the axis from $z = \infty$ will encounter **A**, **B**, **C** in turn, but never come near the trailing edge stagnation point **D**.

The relative contribution of each point is proportional to $-\lambda^{-1} \log a$, where λ is the coefficient of the linearized flow:

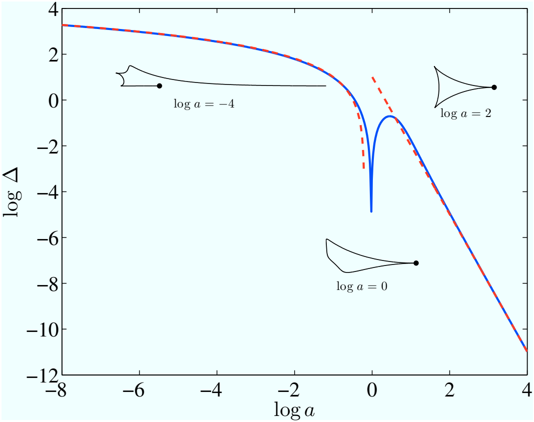
$$\lambda_{\mathbf{A}}^{-1} = \frac{2}{3(\beta+1)} \simeq 0.1111$$

$$\lambda_{\mathbf{B}}^{-1} = \frac{4\beta}{3(\beta^2-1)} \simeq 0.2778$$

$$\lambda_{\mathbf{C}}^{-1} = (\text{mess}) \simeq 3.0095$$

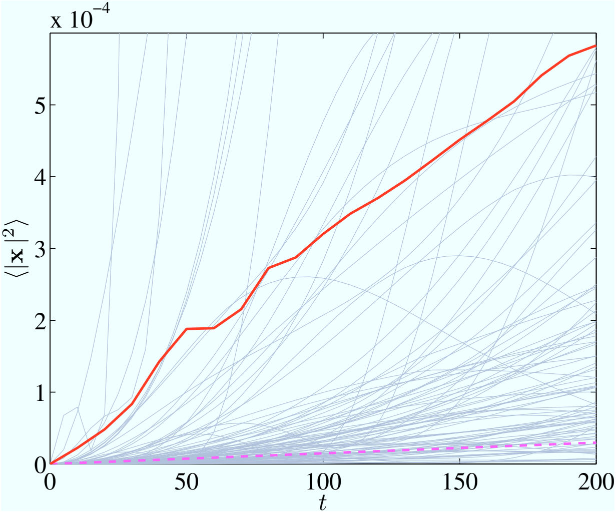


Displacement for squirmer



$\Longrightarrow$ 81% dominated by "head-on" collisions,
or 92% if we use the wake radius, 1.96.

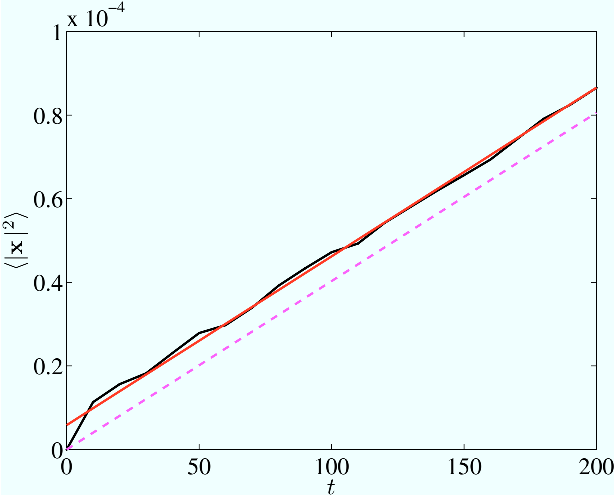
Squirmers: Transport



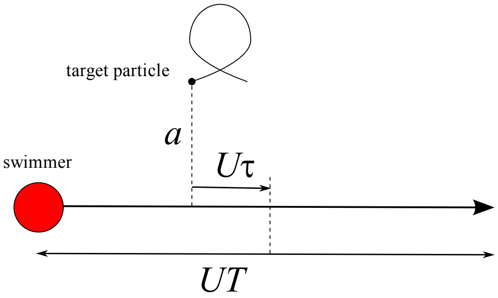
Measured slope is 20 times larger than theory predicts! Oops!

'Shielded' squirmers

Hint: if we artificially cut off the squirmer's long-range velocity field, the theory works.

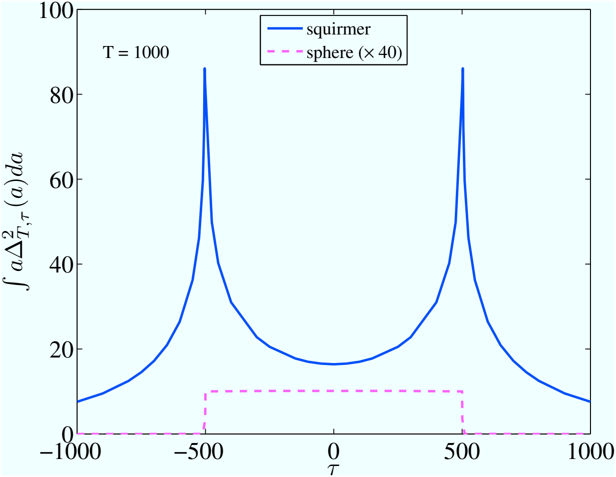


Correlation time/length of swimming



- T is the correlation time of straight swimming;
- UT is the path length;
- τ is the 'phase': $\tau = 0$ means a symmetric setup;
- a is the impact parameter, as before.

Cannot use infinite path length



The integral for $T \to \infty$ is 2.37, so this is much larger! No such problem for the sphere in potential flow.

Why does the theory fail?

- Assumes swimmers come from $-\infty$ and recede to ∞.
- The far-field displacement $\Delta(a)$ decay very fast ($1/a^3$ for squirmers).
- However, there are corrections that decay more slowly and don't vanish if the 'travel length' (or correlation length) is not infinite.
- This means that a new lengthscale is introduced in the problem: the correlation length of swimming. Typically much larger than body size!
- This can increase the effective diffusivity by a large factor (20 for our example).
- No proper theory just yet...

So, do the fish stir the ocean?

- Sphere in potential flow: $\kappa \simeq .0635\, U\ell n_V$ in terms of a volume fraction n_V.
- If we assume spheres that are $1\ \mathrm{cm}$ (the size of typical krill) moving at $1\ \mathrm{cm}/\,\mathrm{sec}$, with $n_V = 10^{-3}$, we get an effective diffusivity of $6 \times 10^{-5}\ \mathrm{cm}^2/\,\mathrm{sec}$.
- This is well below the thermal molecular value $1.5 \times 10^{-3}\ \mathrm{cm}^2/\,\mathrm{sec}$, but about four times larger than the molecular value $1.6 \times 10^{-5}\ \mathrm{cm}^2/\,\mathrm{sec}$ for salt.
- Could a factor of 20 save us? Maybe... but we need to understand the dependence on swimming correlation length.

Conclusions

- Biomixing: no verdict yet;
- Simple dilute model works well, at least for potential flow;
- Potential flow dominated by "sticking";
- Viscous flow dominated by finite correlation length;
- An important moral: scaling arguments/order of magnitude don't tell you much.

Future work:

- Wake models and turbulence;
- PDF of scalar concentration;
- Buoyancy effects;
- Schooling: longer length scale?

This work was supported by the Division of Mathematical Sciences of the US National Science Foundation, under grant DMS-0806821.

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