Bioconvection revisited

A progress report. . .

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AIMS Conference on Dynamical Systems, Orlando, FL
1 July 2012

'Traditional' bioconvection

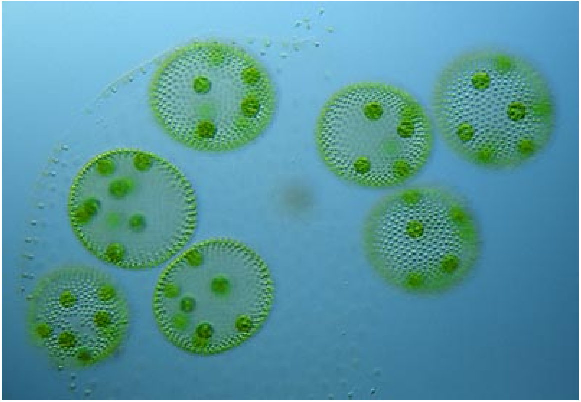
Platt (1961); Plesset & Winet (1974); Levandowsky et al. (1975); Pedley et al. (1988); Pedley & Kessler (1992); Childress & Spiegel (2004); Hill & Pedley (2005)

organisms swim up towards light

critical density is reached

fluid overturns

Volvox family



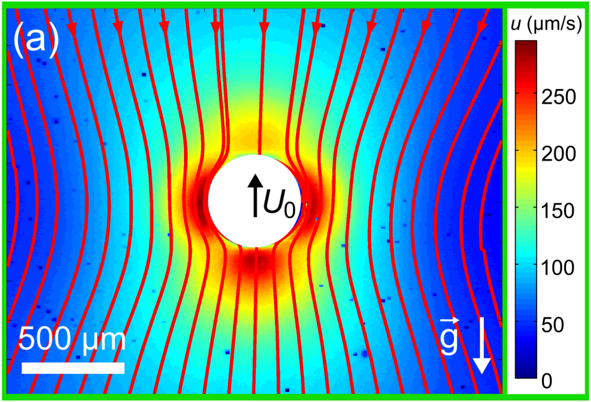
Volvox is a ciliated algae, about 200 μm in diameter.

It is really a colony of smaller cells.

Source: microscopy-uk.org

The flow field around *Volvox Carteri*

Drescher et al. (2010) measured the velocity field around a single organism:



An expression for the velocity field

Drescher et al. (2010) give the expression

$$\mathbf{u}(\mathbf{x}) = -U_0\hat{\mathbf{z}} - \frac{A_{\mathrm{St}}}{r}(\mathbb{I} + \hat{\mathbf{r}}\hat{\mathbf{r}})\cdot\hat{\mathbf{z}} - \frac{A_{\mathrm{str}}}{r^2}(1 - 3(\hat{\mathbf{z}}\cdot\hat{\mathbf{r}})^2)\hat{\mathbf{r}} \\ - \frac{A_{\mathrm{sd}}}{r^3}\left(\tfrac{1}{3}\mathbb{I} - \hat{\mathbf{r}}\hat{\mathbf{r}}\right)\cdot\hat{\mathbf{z}} + \frac{A_{\mathrm{sq}}}{r^4}\left(\hat{\mathbf{r}}\hat{\mathbf{z}} + \hat{\mathbf{z}}\hat{\mathbf{r}} + (\hat{\mathbf{z}}\cdot\hat{\mathbf{r}})(\mathbb{I} - 5\hat{\mathbf{r}}\hat{\mathbf{r}})\right)\cdot\hat{\mathbf{z}}$$

which are, respectively, the downward flow in the moving frame, Stokeslet, stresslet, source doublet, and source quadruplet.[1]

A typical value is $A_{\mathrm{St}} \sim 10^4\,\mu\mathrm{m}^2/\mathrm{s}$. This term would vanish for a neutrally buoyant swimmer, but is important for volvox, even though $\Delta\rho/\rho \simeq 0.3\%$.

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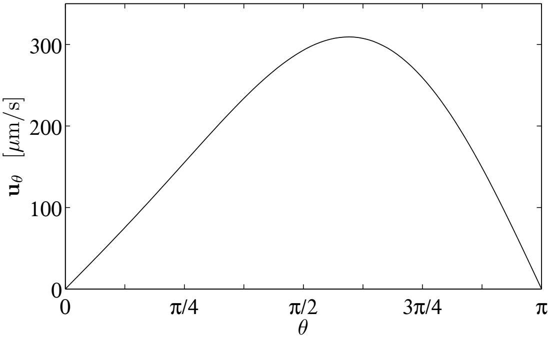
[1] This last one was not in their paper, but is required for the boundary condition at the swimmer's surface.

Volvox as squirmer

Drescher et al. (2010)'s measurerements fit nicely into the squirmer description — a spherical organism that swims by applying a tangential velocity (Lighthill, 1952; Blake, 1971; Ishikawa et al., 2006):



Note the asymmetry: *Volvox* swims more vigorously below its 'equator.'

Does *Volvox* bioconvect?

Of course not! Typical concentrations too low.

Mostly steady: the organisms remain at roughly fixed depths.

Aside: Sedimentation

The *Volvox* remaining at fixed depths means that they impose a point force on the fluid, not unlike sedimenting particles, except here the system is (statistically) steady!

A simple question is then: what kind of flow is driven by the steady *Volvox*?

Velocity $\sim 1/r$, number of organisms in a sphere $\sim r^3$

Hence, since the Stokeslets all point in the same direction, expect velocity at a point to scale as R^2 at fixed concentration, where R is the size of the container![2]

This would suggest that the induced velocity could get quite large...

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[2] Assuming the container remains smaller than the Oseen scale.

Or will it?

Now that I've convinced you that the velocity should diverge, let's look at a counterargument.

Consider Stokes flow with a suspension of fixed point forces:

$$-\nabla p + \mu \Delta \mathbf{u} = -\sum_{k=1}^{N} \mathbf{F}_\mathrm{e}\, \delta_{\mathbf{x}_k}(\mathbf{x}), \qquad \nabla \cdot \mathbf{u} = 0.$$

Ensemble averaging:

$$-\nabla \bar{p} + \mu \Delta \bar{\mathbf{u}} = -\mathbf{F}_\mathrm{e} \mathcal{P}(\mathbf{x}), \qquad \nabla \cdot \bar{\mathbf{u}} = 0,$$

where $\mathcal{P}(\mathbf{x})$ is the distribution of point forces.

Equilibrium solution

For a constant density $\mathcal{P}(\mathbf{x}) = c$,

$$-\nabla \bar{p} + \mu \Delta \bar{\mathbf{u}} = -\mathbf{F}_{\mathrm{e}} c, \qquad \nabla \cdot \bar{\mathbf{u}} = 0,$$

The solution to these equations depends on the boundary conditions, but in a closed container the unique solution is $\bar{\mathbf{u}} = 0$!

In other words, the point forces push down on the fluid, but the fluid pushes back and achieves hydrostatic balance.

Such global constraints are at the heart of the renormalization approaches to sedimentation developed by Batchelor (1972, 1976); Hinch (1977); Feuillebois (1984), as well as others.

Fluctuations

For swimmers, the mean velocity of a suspension thus might not depend on system size, though the calculation has proven difficult.

The velocity fluctuations, may or may not depend on system size. In sedimentation, this is still controversial even today (Caflisch & Luke, 1985; Koch & Shaqfeh, 1991; Nicolai & Guazzelli, 1995; Brenner, 1999).

It is now thought that the system-size dependent fluctuations are transient. What ultimately sets the amplitude of fluctutations is not understood (Guazzelli & Hinch, 2011).

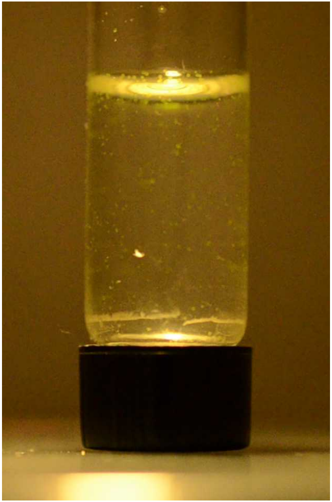
Fluctuations are crucial to address issues of mixing by the swimmers. (Dewar et al., 2006; Katija & Dabiri, 2009; Thiffeault & Childress, 2010; Lin et al., 2011)

The difficulty of obtaining robust theoretical predictions suggests that experiments are needed first.

Volvox Aureus lit from above

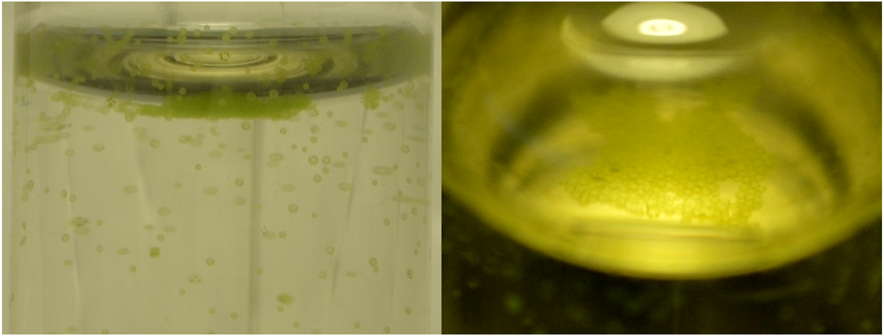
- Depth is about 3 cm;
- Volume fraction of *Volvox* is $\sim 1\%$;
- Movie is sped up 10 times.
- Corkscrew motion was mentioned by Mast (1907) (Interplay of gravity and photosensitivity).
- Does the rising 'column' in the center reflect large-scale flow?

[movie 1]



Organisms collect at the free surface

Organisms swim to the free surface and form a ‘carpet.’



Model as a ‘force sheet’ at the free surface?

Outlook

- We are only beginning to master experimenting with *Volvox*;
- Next steps: PIV, dye release. . .
- Goal is to study large-scale flows, but also to quantify induced biomixing, which can occur even at low concentrations (Thiffeault & Childress, 2010; Lin et al., 2011);
- Study interactions between large numbers of *Volvox* (extension of Ishikawa (2009); Drescher et al. (2009));
- Compare simple numerical models to experiments.

This work was supported by the Division of Mathematical Sciences of the US National Science Foundation, under grant DMS-1109315.

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