

# Moving walls accelerate mixing

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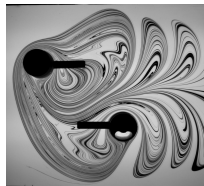
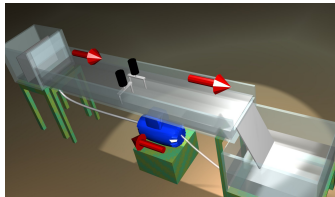
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## Stirring and Mixing of Viscous Fluids



- Viscous flows  $\Rightarrow$  no turbulence! (laminar)
- Open and closed systems
- Active (rods) and passive



Understand the **mechanisms** involved.

Characterise and optimise the **efficiency** of mixing.

## Stirring and Mixing: What's the Difference?

- **Stirring** is the mechanical motion of the fluid (**cause**);
- **Mixing** is the homogenisation of a substance (**effect, or goal**);
- Two extreme limits: **Turbulent** and **laminar** mixing, both relevant in applications;
- Even if turbulence is feasible, still care about energetic cost;
- For very viscous flows, use simple time-dependent flows to create **chaotic** mixing.
- Here we look at the impact of the vessel **walls** on mixing rates.

## A Simple Example: Planetary Mixers

In food processing, **rods** are often used for stirring.

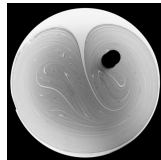
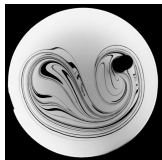


[movie 1] ©BLT Inc.

## The Figure-Eight Stirring Protocol



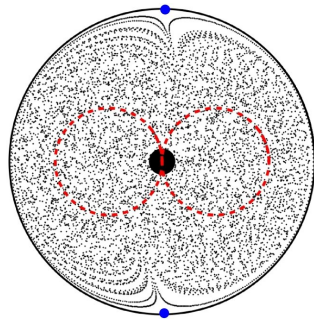
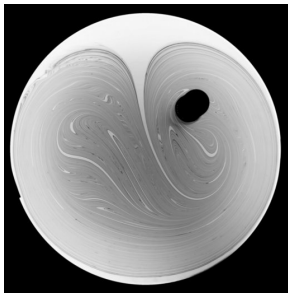
- Circular container of viscous fluid (sugar syrup);
- A rod is moved slowly in a 'figure-eight' pattern;
- Gradients are created by **stretching and folding**, the signature of chaos.



[movie 2] Experiments by E. Guillard and O. Dauchot (CEA Saclay).

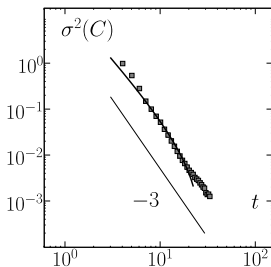
## The Mixing Pattern

- Kidney-shaped mixed region extends to wall;
- Two **parabolic points** on the wall, one associated with injection of material;
- Asymptotically self-similar, so expect an **exponential decay** of the concentration ('**strange eigenmode**' regime).  
(Pierrehumbert, 1994; Rothstein et al., 1999; Voth et al., 2003)

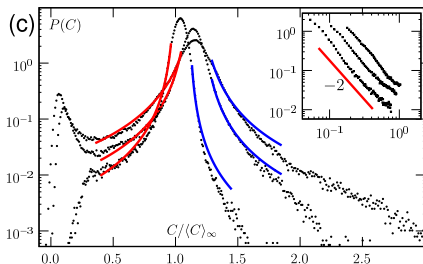


## Mixing is Slower Than Expected

Concentration field in a well-mixed central region



$$\text{Variance} = \int |\theta|^2 dV$$

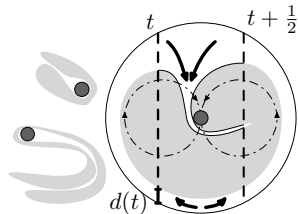
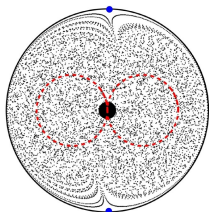


Concentration PDFs

⇒ Algebraic decay of variance  $\neq$  Exponential

The 'stretching and folding' action induced by the rod is an exponentially rapid process (**chaos!**), so why aren't we seeing exponential decay?

## Walls Slow Down Mixing



- Trajectories are (almost) everywhere chaotic  
⇒ but there is always poorly-mixed fluid near the walls;
- Re-inject unmixed (white) material along the unstable manifold of a parabolic point on the wall;
- No-slip at walls ⇒ width of “white stripes”  $\sim t^{-2}$  (algebraic);
- Re-injected white strips contaminate the mixing pattern, in spite of the fact that stretching is exponential in the centre.



## Hydrodynamics Near the Wall

We can characterize white strips in terms of hydrodynamics near the no-slip wall.  $x_{\parallel}$  and  $x_{\perp}$  denote respectively the distance along and  $\perp$  to the wall. No-slip boundary conditions impose

$$v_{\parallel} \sim x_{\perp}, \quad \text{near the wall: } x_{\perp} \ll 1.$$

Incompressibility

$$\frac{\partial v_{\parallel}}{\partial x_{\parallel}} + \frac{\partial v_{\perp}}{\partial x_{\perp}} = 0,$$

implies

$$v_{\perp} \simeq -a x_{\perp}^2.$$

Solve  $\dot{x}_{\perp} = v_{\perp}$ :

$$x_{\perp} \simeq \frac{x_0}{1 + at x_0}.$$

## Hydrodynamics Near the Wall (continued)

Hence, the distance between the wall and a particle in the lower part of the domain (where  $v_{\perp} < 0$ ) shrinks as

$$d(t) \simeq 1/at, \quad t \gg 1.$$

This scaling was derived in Chertkov & Lebedev (2003), and we verified it experimentally. [See also Lebedev & Turitsyn, 2004; Salman & Haynes, 2007; Chernykh & Lebedev, 2008.]

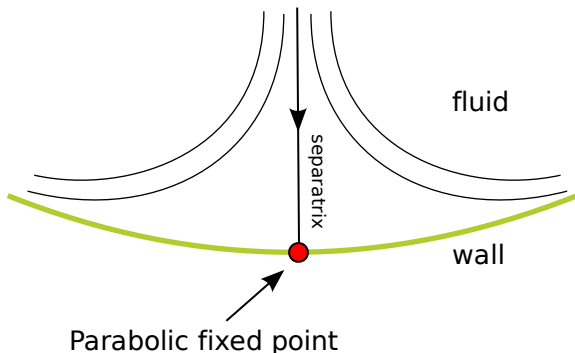
The amount of white that is 'shaved off' at each period is thus

$$\dot{d} \sim T/at^2, \quad t \gg 1,$$

where  $T$  is the period. This is the origin of the power-law decay. Corrections due to the stretch/fold action are described in [Gouillart et al., *Phys. Rev. Lett.* **99**, 114501 (2007)].

## The Problem: Separatrix at the Wall

The decay is algebraic near a reattachment point at the wall:

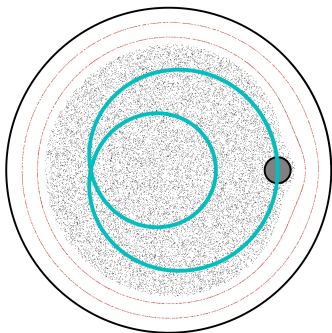


A fluid particle following the separatrix approaches the wall as  $1/t$ .

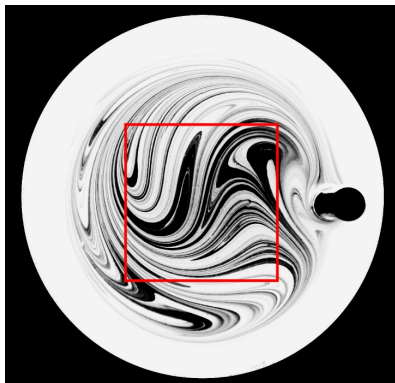
[Chertkov & Lebedev (2003); Lebedev & Turitsyn (2004); Salman & Haynes (2007); Gouillart et al. (2007, 2008, 2009); Chernykh & Lebedev (2008)]

## A Second Scenario

How do we mimic a slip boundary condition?



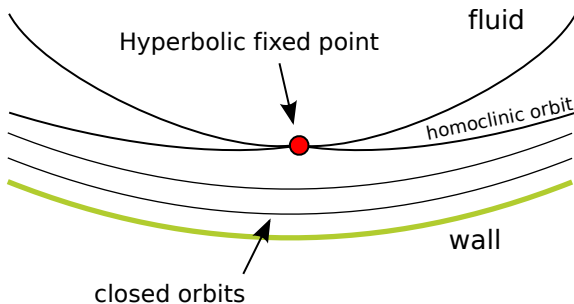
“Epitrochoid” protocol



Central chaotic region + regular region near the walls.

## Mimic a Slip Boundary Condition?

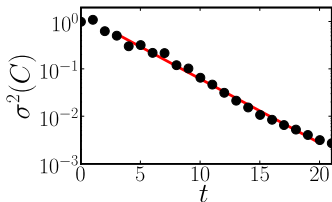
Create closed orbits near the wall:



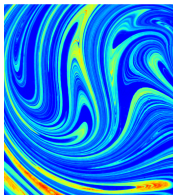
There will be a 'last closed orbit' followed by one or more fixed or periodic points and a separatrix, for example a hyperbolic orbit. Particles approach the hyperbolic fixed point **exponentially fast**.

[Thiffeault et al. (2011)]

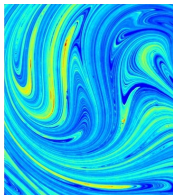
## Recover Exponential Decay



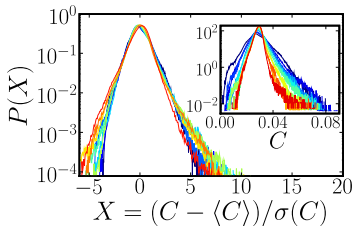
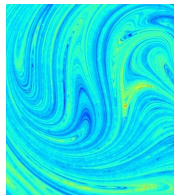
$t = 8$



$t = 12$



$t = 17$



... as well as 'true' self-similarity.

## Another Approach: Rotate the Bowl!



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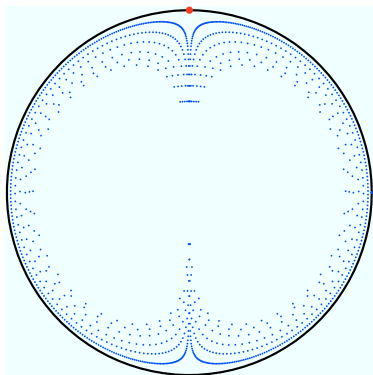
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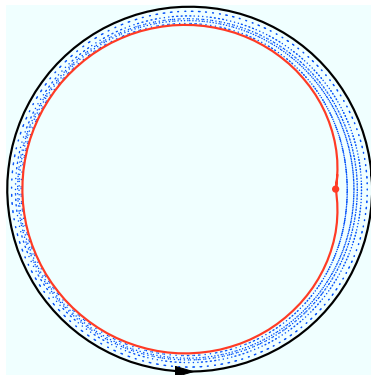
## Rotating the Wall

Can use a simplified 'edge map' to model the near-wall region:



Fixed wall: parabolic separation point (algebraic)

[Thiffeault et al. (2011)]



Moving wall: hyperbolic fixed point (exponential)



## Conclusions

- If the chaotic region extends to the walls, then the **decay of concentration is algebraic** (typically  $t^{-3}$  for variance).
- The **no-slip boundary condition** at the walls is to blame.
- Would recover a strange eigenmode for **very long times**, once the mixing pattern is within a Batchelor length from the edge (not very useful in practice!).
- The decay is well-predicted by a baker's map with a **parabolic point**.
- We can shield the mixing region from the walls by wrapping it in a **regular island**.
- We then recover **exponential decay**.
- How to control this in practice? Is it really advantageous? Is **scraping** the walls better?
- See Gouillart et al. (2007, 2008, 2009); Thiffeault et al. (2011).

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