How Good is Your Mixer?

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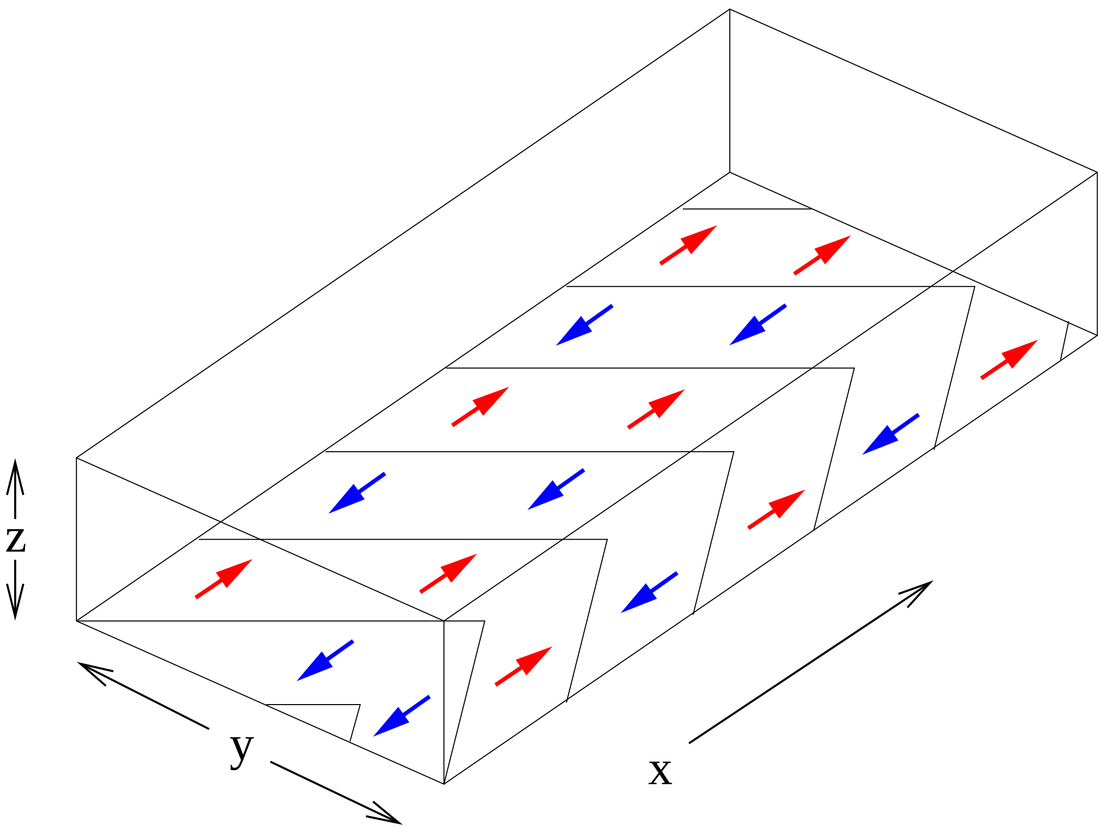
Introduction

- Mixing of a passive scalar by advection (stirring) and diffusion.
- Today: outline local theories, based on stretching of fluid elements.
- Calculation for a toy problem: a linear velocity field.
- The mixing rate depends on the rate of stretching of fluid elements.
- Show how this applies to a physical system (micromixer).
- Gives an indication of how efficient is the mixer.

Channel Micromixer

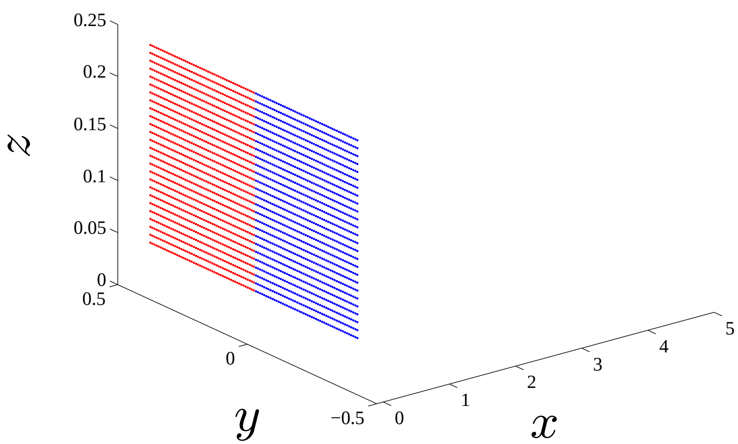


Typical parameters:

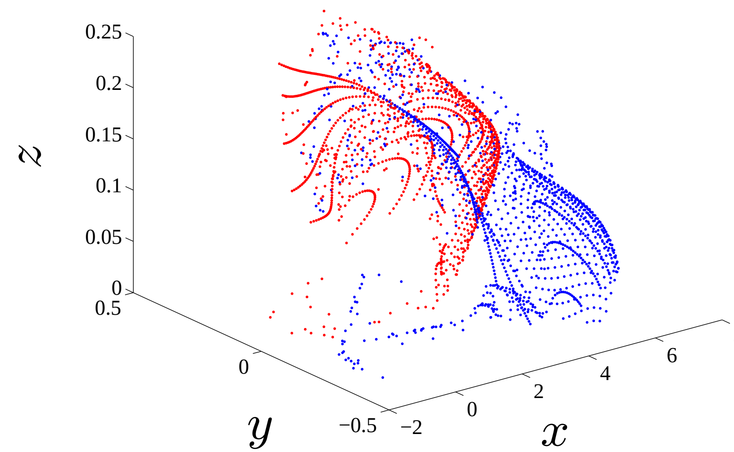
- width $\sim 100\ \mu$m, height ~ 10–$50\ \mu$m
- $U \sim 10^2$–$10^3\ \mu\text{m/s}$,
- Re $\sim$ 1–100, Pe $\sim 10^3$

Dispersion of Particles

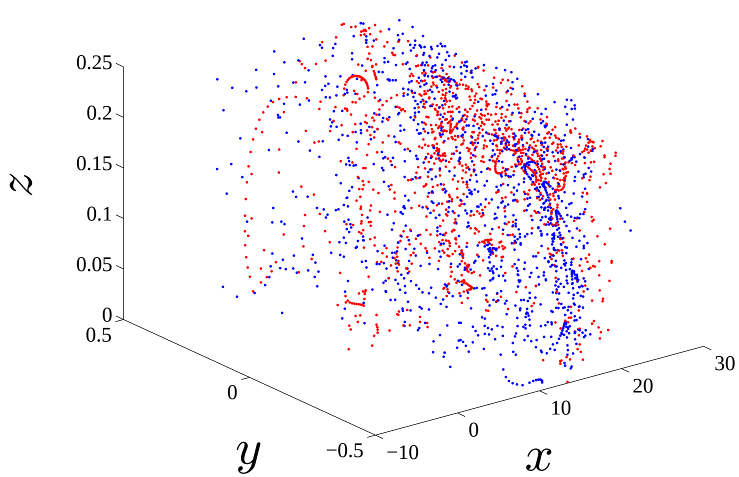
Initially



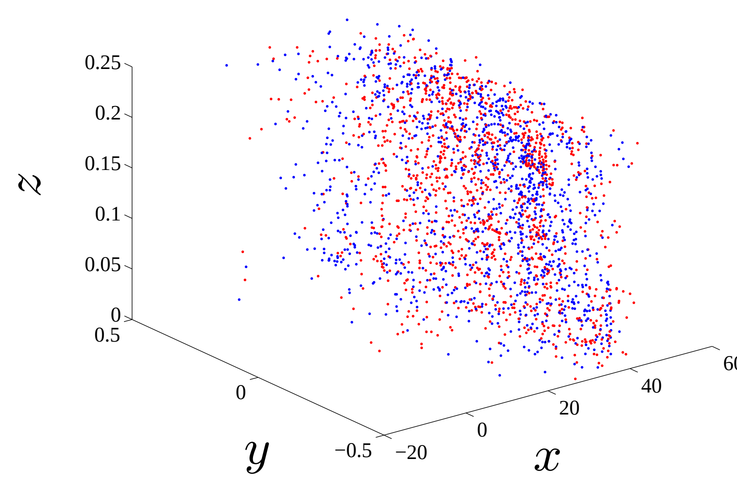
After 1 Period



After 5 Periods

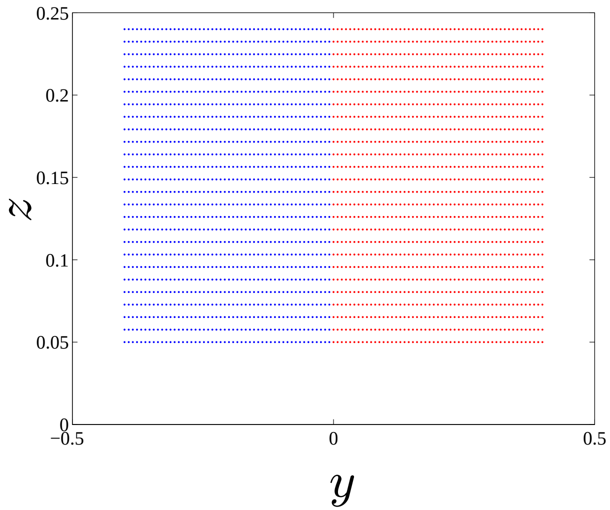


After 10 Periods

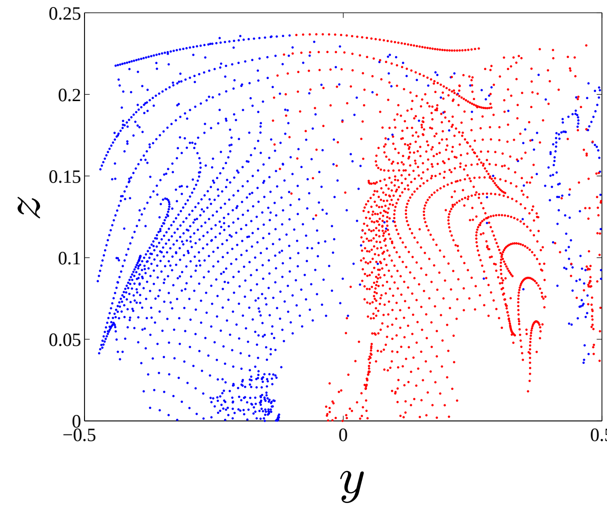


Dispersion of Particles: Downchannel View

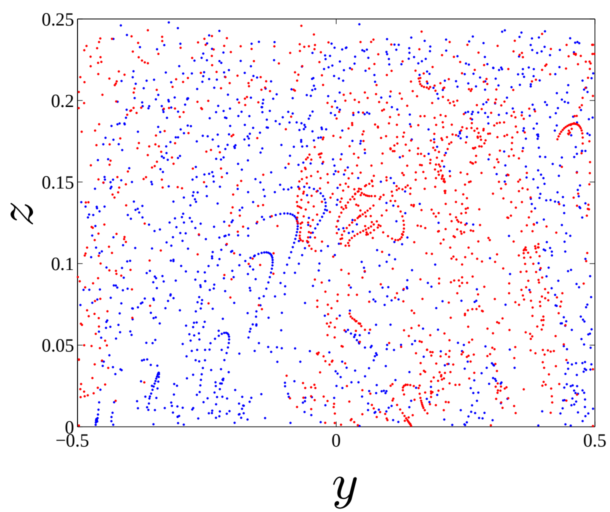
Initially



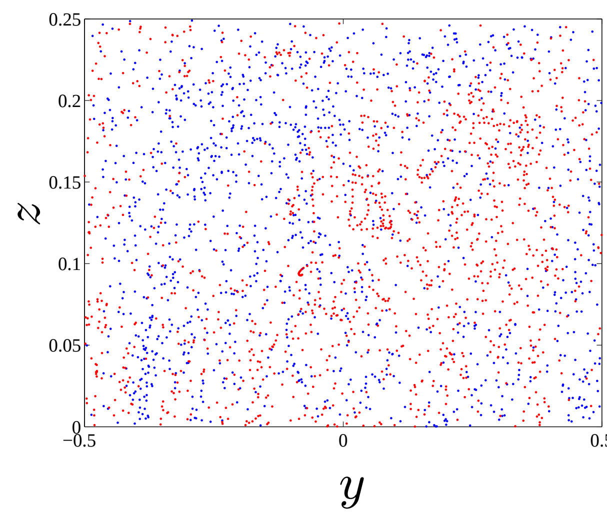
After 1 Period



After 5 Periods

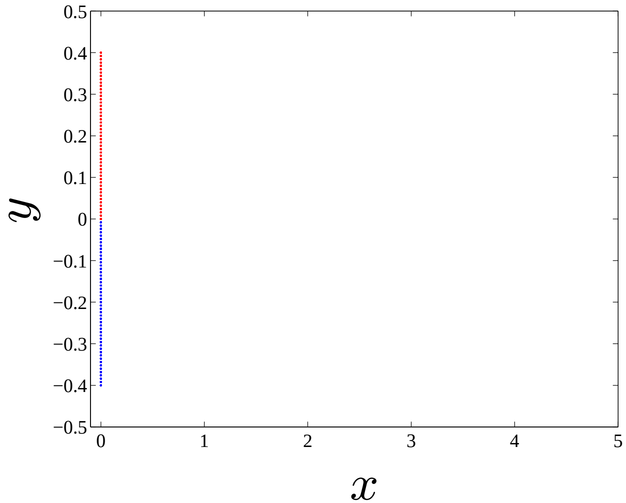


After 10 Periods

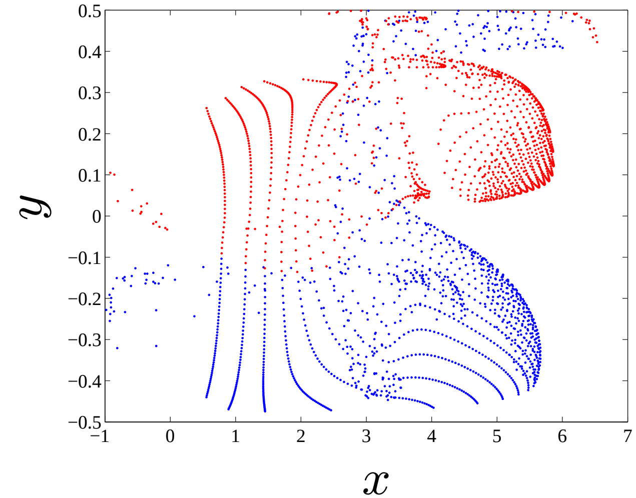


Dispersion of Particles: Top Down View

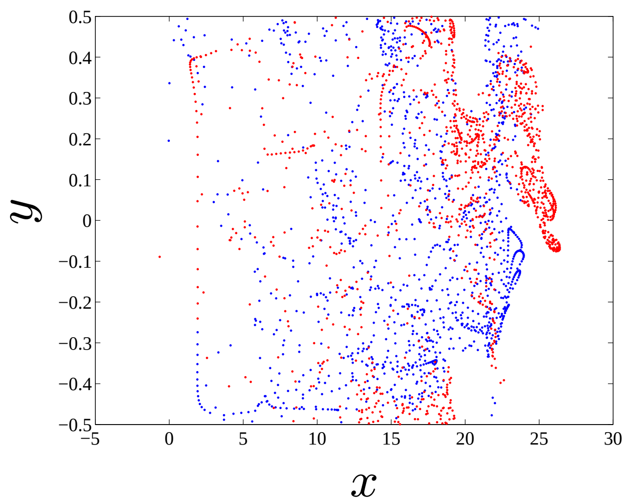
Initially



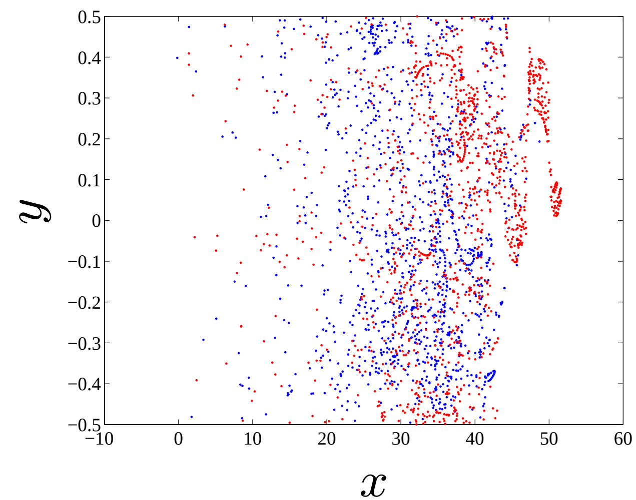
After 1 Period



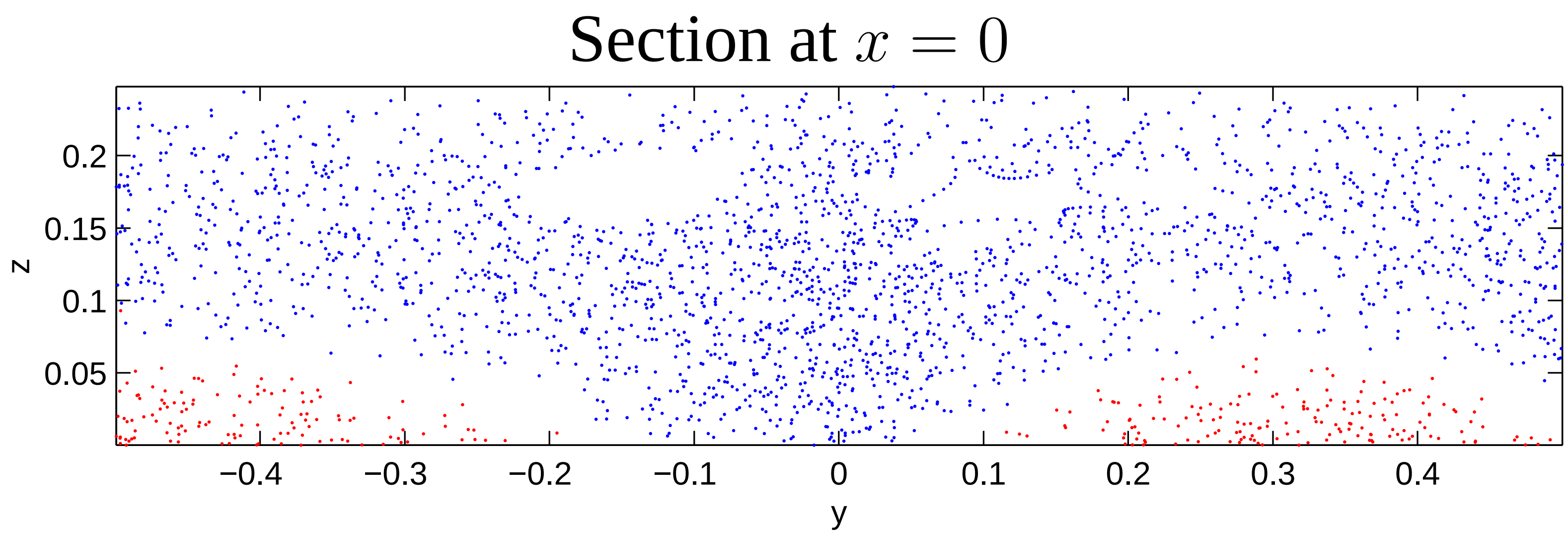
After 5 Periods

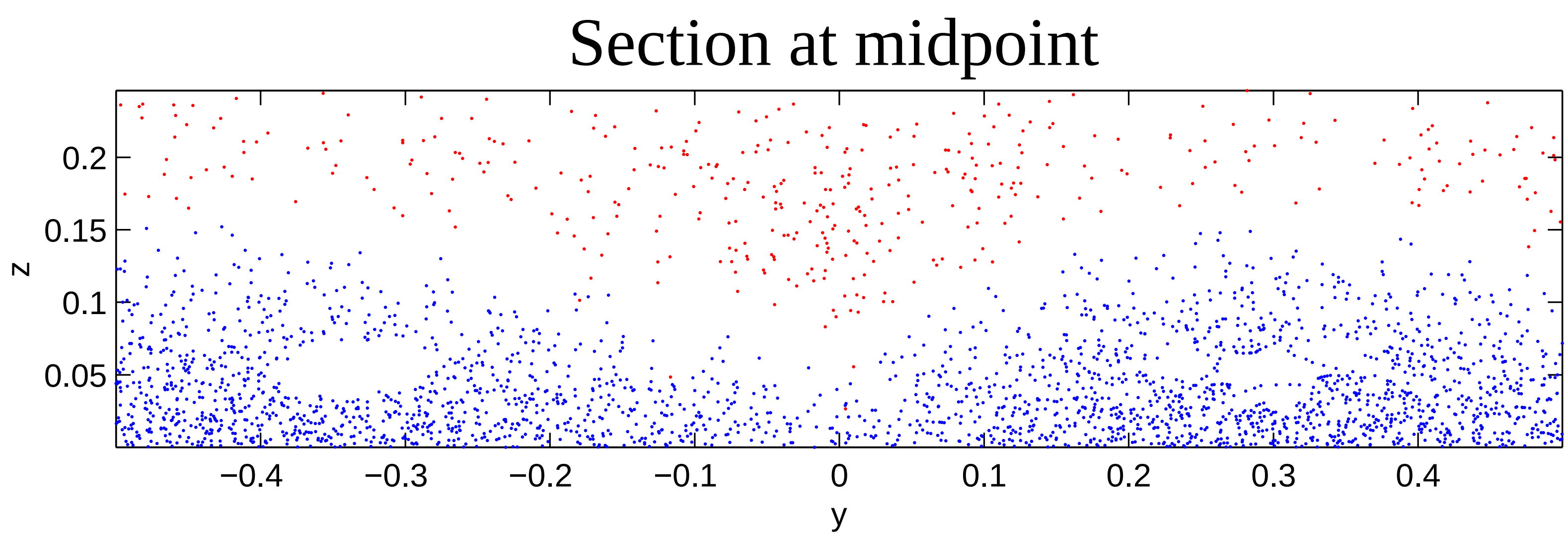


After 10 Periods



Cross-Sections

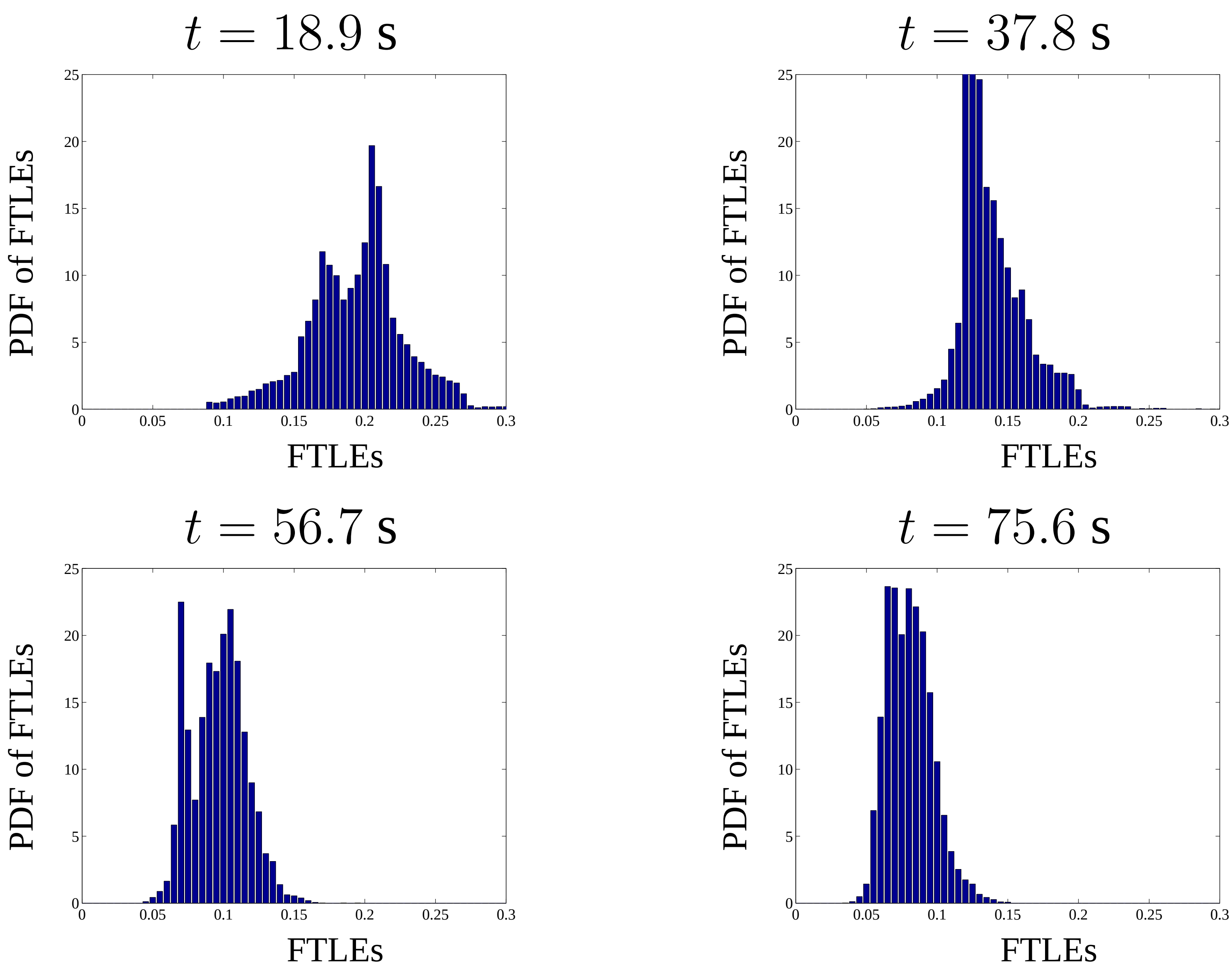




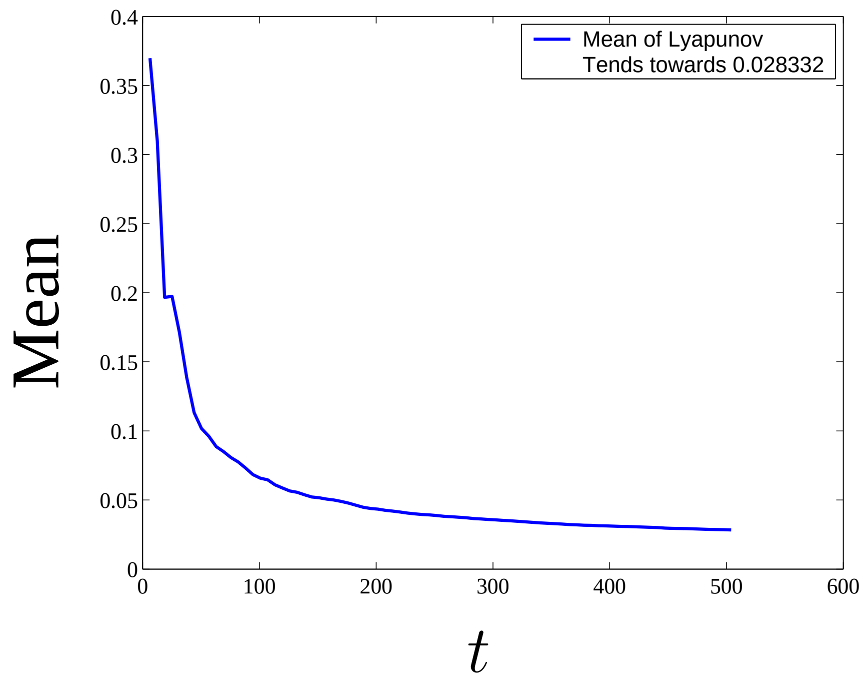
Cross-Sections: Animation

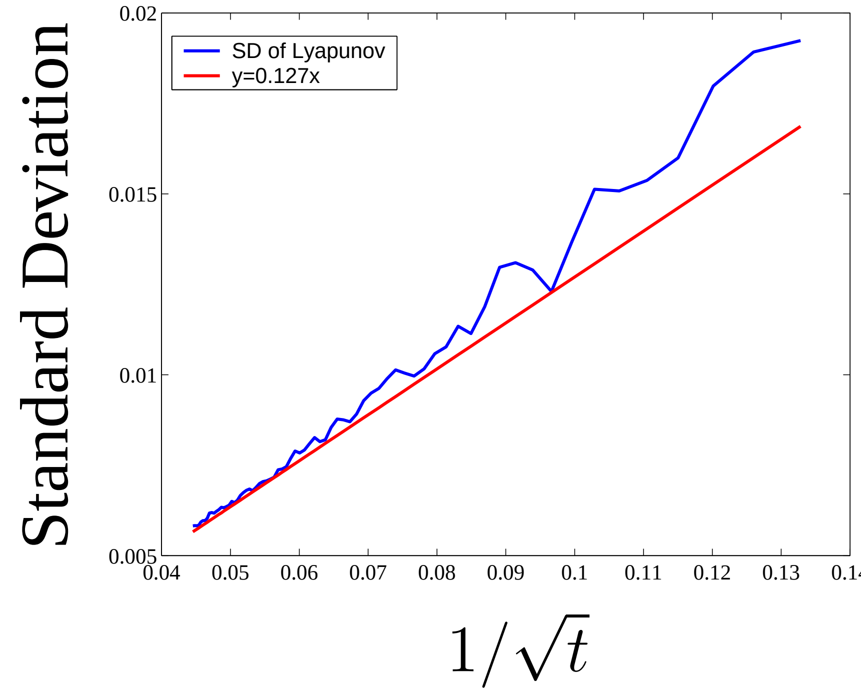
Animation of cross sections for $\alpha = 1$, $\beta = 2$. (8 Megs)

Distribution of Finite-time Lyapunov Exponents



FTLEs: Evolution of Mean and STD

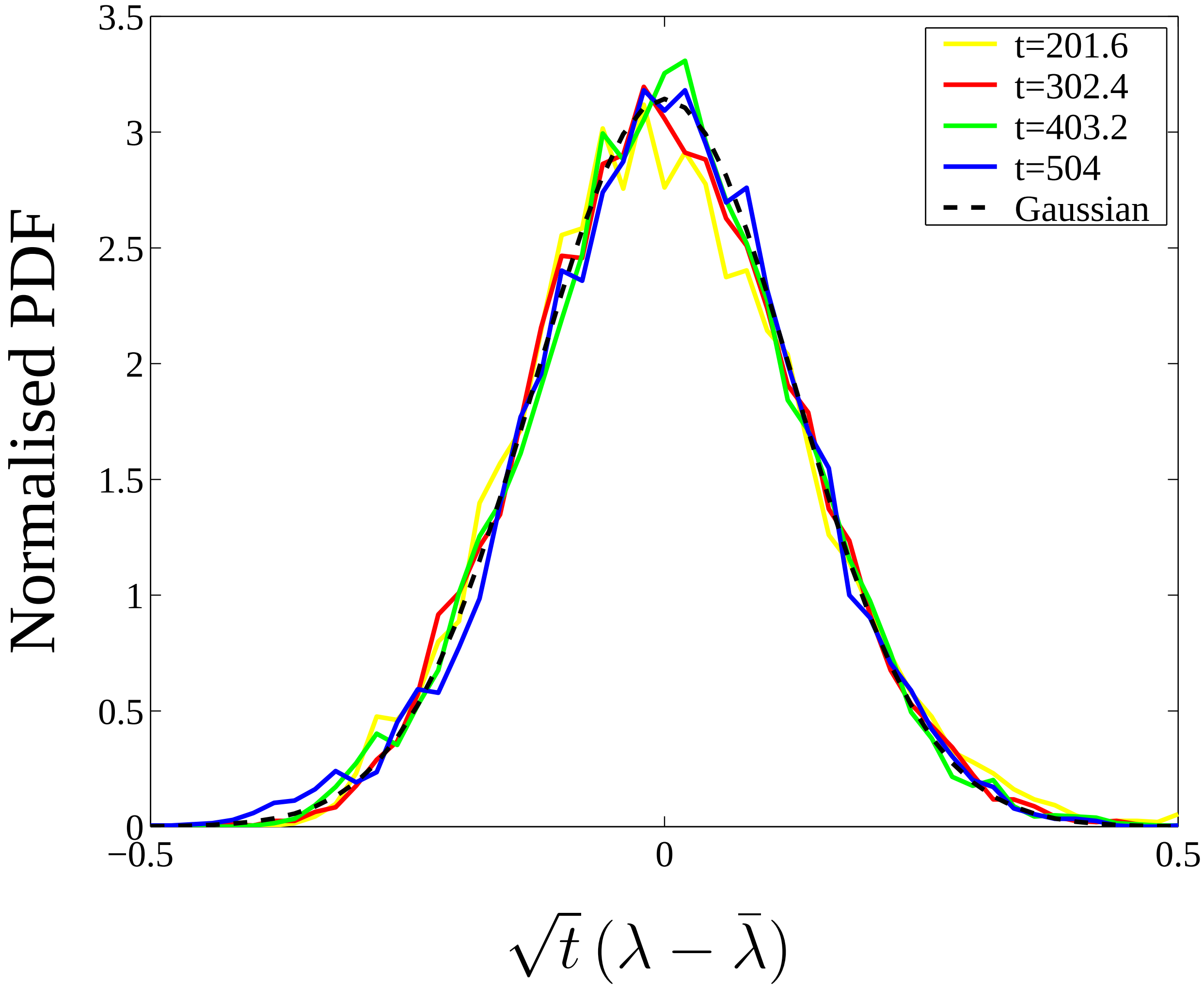




$$\bar{\lambda} \simeq 0.0283\ \mathrm{s}^{-1} \qquad \sigma = \sqrt{\frac{\nu}{t}} \qquad \nu \simeq 0.0161\ \mathrm{s}^{-1}$$

$$P(\lambda, t) \simeq \sqrt{\frac{t}{2\pi\,\nu^2}}\ \exp\left\{-\frac{t\,(\lambda-\bar{\lambda})^2}{2\nu}\right\}$$

Rescaled Distribution



Decay Rate of Variance

$$\langle \theta^2 \rangle \sim \int_{-\infty}^{\infty} \mathrm{e}^{-\lambda\, t}\, P(\lambda, t)\, \mathrm{d}t = \mathrm{e}^{-\left(\bar{\lambda} - \frac{1}{2}\,\nu\right)t} = \mathrm{e}^{-\gamma_2\, t}$$

$$\begin{aligned} \gamma_2 &= \bar{\lambda} - \tfrac{1}{2}\,\nu \\ &\simeq 0.0283 - \tfrac{1}{2}\,0.0161 = 0.0202\ \mathrm{s}^{-1} \end{aligned}$$

- The "mixing time" is $\gamma_2^{-1} \simeq 50$ seconds.
- $\bar{\lambda}$ is the mean stretching rate.
- ν reflects the "bias" of the fluctuations: they do more harm than good.
- Fluctuations decrease rate by 25%.

Conclusions

- The decay rate for the passive scalar depends on the distribution of finite-time Lyapunov exponents.
- The fluctuations in the Lyapunov exponents tend to work against good mixing.
- There may be regions of poor mixing (regular regions).
- Plenty of room for optimisation (staggered, etc.).
- This regime breaks down eventually: must also understand the role of strange eigenfunctions.
- Range of validity is poorly understood.
- Comparison to direct solution is needed.