The Role of Walls in Chaotic Mixing

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University of Adelaide, 22 August 2008

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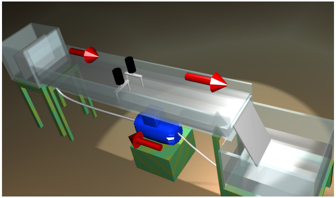
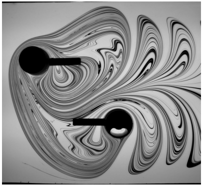
Stirring and Mixing of Viscous Fluids

- Viscous flows ⇒ no turbulence! (laminar)
- Open and closed systems
- Active (rods) and passive



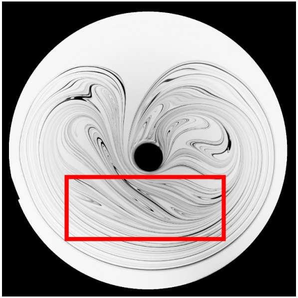


Understand the mechanisms involved.
Characterise and optimise the efficiency of mixing.

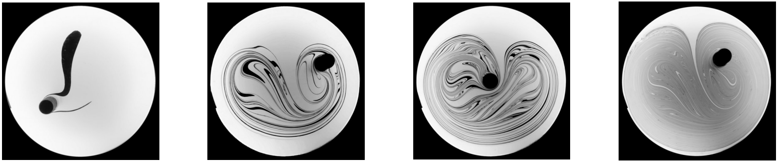
Stirring and Mixing: What's the Difference?

- Stirring is the mechanical motion of the fluid (cause);
- Mixing is the homogenisation of a substance (effect, or goal);
- Two extreme limits: Turbulent and laminar mixing, both relevant in applications;
- Even if turbulence is feasible, still care about energetic cost;
- For very viscous flows, use simple time-dependent flows to create chaotic mixing.
- Here we look at the impact of the vessel walls on mixing rates.

The Figure-Eight Stirring Protocol



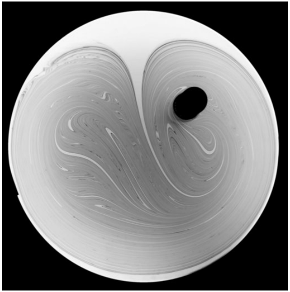
- Circular container of viscous fluid (sugar syrup);
- A rod is moved slowly in a 'figure-eight' pattern;
- Gradients are created by stretching and folding, the signature of chaos.

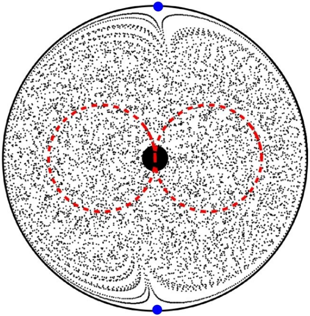


[movie 1] Experiments by E. Gouillart and O. Dauchot (CEA Saclay).

The Mixing Pattern

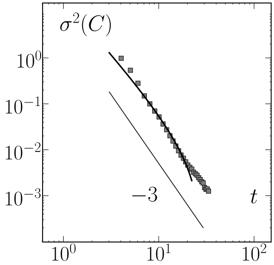
- Kidney-shaped mixed region extends to wall;
- Two parabolic points on the wall, one associated with injection of material;
- Asymptotically self-similar, so expect an exponential decay of the concentration ('strange eigenmode' regime).
  (Pierrehumbert, 1994; Rothstein et al., 1999; Voth et al., 2003)





Mixing is Slower Than Expected

Concentration field in a well-mixed central region



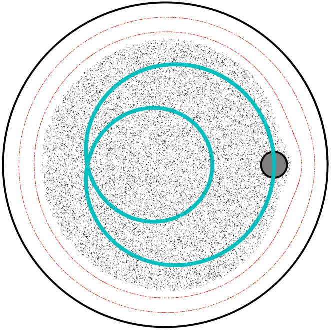
Variance $= \int |\theta|^2 dV$

$\Rightarrow$ Algebraic decay of variance $\neq$ Exponential

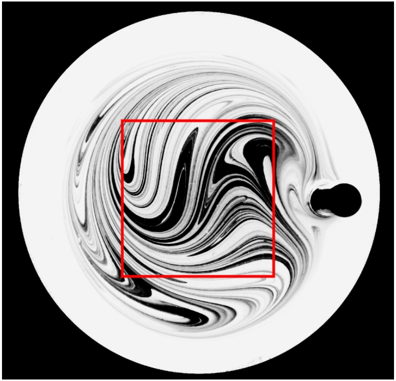
The 'stretching and folding' action induced by the rod is an exponentially rapid process (chaos!), so why aren't we seeing exponential decay?

A Second Scenario

How do we mimic a slip boundary condition?

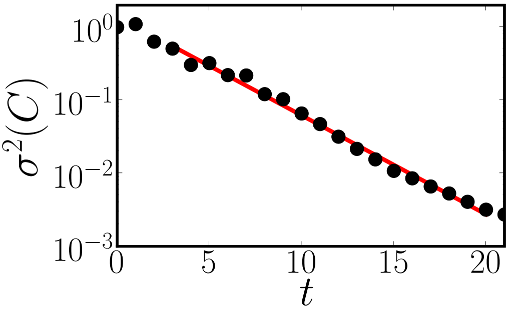


"Epitrochoid" protocol

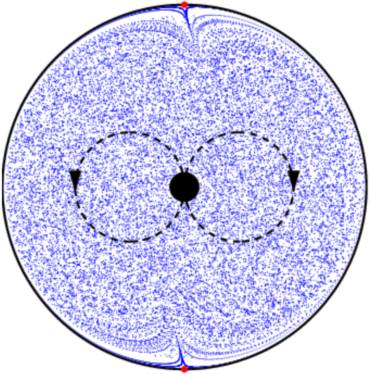


Central chaotic region + regular region near the walls.

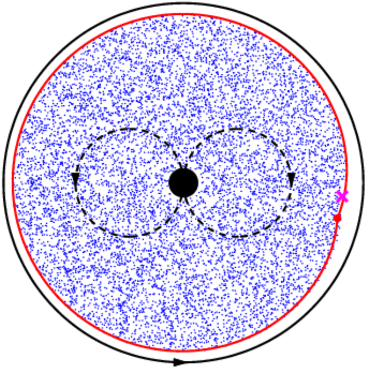
Recover Exponential Decay



Rotating the Wall



Fixed wall: parabolic separation point (algebraic)



Moving wall: hyperbolic fixed point (exponential)

Conclusions

- If the chaotic region extends to the walls, then the decay of concentration is algebraic (typically $(\log t)/t^{-2}$ for variance).
- The no-slip boundary condition at the walls is to blame.
- Would recover a strange eigenmode for very long times, once the mixing pattern is within a Batchelor length from the edge (not very useful in practice!).
- We can shield the mixing region from the walls by wrapping it in a regular island — rotate the wall!
- We then recover exponential decay.
- How to control this in practice? Is it really advantageous? Is scraping the walls better?
- See **[Gouillart et al., PRL 99, 114501 (2007); PRE (2008)]**
- Thanks to Matt for use of his code!

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