MATH 801 — Braids

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DESCRIPTION

Everyone understands intuitively the concept of a braid as a collection of intertwined strands. However, there is a rigorous mathematical theory of braids, pioneered by Artin, which complements knot theory but is in many ways separate. Unlike knots, braids have a simple *algebraic* description in terms of a group, and can thus be manipulated more easily. This group structure also connects them to the *mapping class groups* of a surface: each element of this group describes a way of mapping a surface to itself. Braids thus provides insight into the structure of mappings of surfaces. We shall tie this to the powerful Thurston–Nielsen classification of surface diffeomorphisms.

This opens the door to many applications in fluid dynamics, since fluid motion can be regarded as a re-ordering of the fluid material. Braids are also being used in computational robotics, where the motion of robotic agents subject to constraints has to be optimized. On the algebraic side, braids are potentially useful in cryptography, since there are several computationally 'hard' problems associated with factoring braids — especially the *conjugacy problem*.

In this course I will present the basic theory of braids, assuming relatively few prerequisites. The approach will vary from semi-rigorous to rigorous, with some theorems proved and others merely motivated. The goal is to cover a lot of ground and not be slowed down by lengthy proofs. **The course is aimed at both applied and pure students**. A sample of topics to be covered, time permitting:

- Physical braids and the algebraic description of braids
- Configuration space
- Braids on other surfaces; The Dirac string problem
- Alexander's and Markov's theorem
- Garside normal form and the word and conjugacy problems
- The Burau representation and faithfulness
- Mapping class groups
- Thurston–Nielsen classification
- Topological entropy
- Robotics
- Stirring with braids

PREREQUISITES

Basic knowledge of group theory will help. Also a willingness to read widely!

TEXTBOOKS

We will use mainly handouts and papers from journal. See bibliography at the end.

BIBLIOGRAPHY

This bibliography is intended as a sample of papers we will refer to in the course. It is not meant to be exhaustive. To get a rough idea of the content of the course, check out *Braids: A Survey* by Joan Birman and Tara Brendle, `http://arxiv.org/abs/math/0409205`.

References

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[7] V. L. Hansen. *Braids and Coverings*. Number 18 in London Mathematical Society Student Texts. Cambridge University Press, Cambridge, U.K., 1989.

[8] J.-L. Thiffeault and M. D. Finn. Topology, braids, and mixing in fluids. *Phil. Trans. R. Soc. Lond. A*, 364:3251–3266, December 2006.

[9] W. P. Thurston. On the geometry and dynamics of diffeomorphisms of surfaces. *Bull. Am. Math. Soc.*, 19:417–431, 1988.