Homework 4: Questions on conformal mappings

1. (a) Show that $w = f(z) = z + \frac{1}{z}$ maps the region $\{(x,y) \mid y \geq 0 \text{ & } x^2+y^2 \geq 1\}$ to the upper-half w-plane.

(b) Where are the critical points and by how much does the angle change at these points?

(c) Find the complex Green's function satisfying homogeneous Dirichlet boundary conditions.

(d) Where are the images?

2. (a) Show that the map $w = f(z) = q\left(pz + \frac{1}{pz}\right)$ with $q>0$, $p>1$, maps $|z|=1$ to an ellipse. What are the semimajor and semiminor radii?

(b) Show that $f(z)$ maps $|z|>1$ to outside the ellipse.

(c) Find the Green's function for a point charge outside an elliptical region of semimajor radius a, semiminor radius b, with homogeneous Dirichlet conditions.