Lecture 13: Complex variable methods

[Wally]

Summary: • $f(z)$ is really $f(z) = u(x,y) + i s(x,y)$

• For $\frac{df}{dz}$ to be well-defined, require

$$\frac{\partial u}{\partial x} = \frac{\partial s}{\partial y}, \quad \frac{\partial u}{\partial y} = -\frac{\partial s}{\partial x}$$

Cauchy–Riemann equations

• This implies $\nabla^2 u = \nabla^2 s = 0$ u, s harmonic

• We say that $f(z)$ is analytic is it satisfies C-R + convergent power series (in some domain).

• Thus, solving the steady heat equation in 2D is the same as finding an analytic function whose real or imaginary part satisfies the appropriate boundary condition.

• Conformal maps $w = f(z)$ map the complex plane z to another, w. Preserves angles except at critical points ($f'(z) = 0$). An analytic function of z maps to an analytic function of w.

For example: wedges, strips, polygons... can be mapped to half-plane.