EXERCISES 9.2

9.2.1. Consider

$$\begin{aligned} \frac{\partial u}{\partial t} &= k\frac{\partial^2 u}{\partial x^2} + Q(x,t) \\ u(x,0) &= g(x). \end{aligned}$$

In all cases obtain formulas similar to (9.2.20) by introducing a Green's function.

(a) Use Green's formula instead of term-by-term spatial differentiation if

$$u(0,t) = 0 \qquad \text{and} \qquad u(L,t) = 0.$$

(b) Modify part (a) if

$$u(0,t) = A(t) \qquad \text{and} \qquad u(L,t) = B(t).$$

Do not reduce to a problem with homogeneous boundary conditions.

(c) Solve using any method if

$$\frac{\partial u}{\partial x}(0,t) = 0 \qquad \text{and} \qquad \frac{\partial u}{\partial x}(L,t) = 0.$$

*(d) Use Green's formula instead of term-by-term differentiation if

$$\frac{\partial u}{\partial x}(0,t) = A(t) \qquad \text{and} \qquad \frac{\partial u}{\partial x}(L,t) = B(t).$$

9.2.2. Solve by the method of eigenfunction expansion

$$c\rho\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(K_0\frac{\partial u}{\partial x}\right) + Q(x,t)$$

subject to $u(0,t) = 0$, $u(L,t) = 0$, and $u(x,0) = g(x)$, if $c\rho$ and K_0 are functions of x. Assume that the eigenfunctions are known. Obtain a formula similar to (9.2.20) by introducing a Green's function.

*9.2.3. Solve by the method of eigenfunction expansion

$$\begin{aligned} \frac{\partial^2 u}{\partial t^2} &= c^2\frac{\partial^2 u}{\partial x^2} + Q(x,t) \\ u(0,t) = 0 \qquad & u(x,0) = f(x) \\ u(L,t) = 0 \qquad & \frac{\partial u}{\partial t}(x,0) = g(x). \end{aligned}$$

Define functions (in the simplest possible way) such that a relationship similar to (9.2.20) exists. It must be somewhat different due to the two initial conditions. (*Hint*: See Exercise 8.5.1.)

9.2.4. Modify Exercise 9.2.3 (using Green's formula if necessary) if instead

(a) $\frac{\partial u}{\partial x}(0,t) = 0$ and $\frac{\partial u}{\partial x}(L,t) = 0$

(b) $u(0,t) = A(t)$ and $u(L,t) = 0$

(c) $\frac{\partial u}{\partial x}(0,t) = 0$ and $\frac{\partial u}{\partial x}(L,t) = B(t)$