**One-dimensional wave equation**. If the only body force per unit mass is gravity, then $Q(x,t) = -g$ in (4.2.7). In many such situations, this force is small (relative to the tensile force $\rho_0 g \ll |T_0 \partial^2 u/\partial x^2|$) and can be neglected. Alternatively, gravity sags the string, and we can calculate the vibrations with respect to the sagged equilibrium position. In either way we are often led to investigate (4.2.7) in the case in which $Q(x,t) = 0$,

$$\rho_0(x)\frac{\partial^2 u}{\partial t^2} = T_0\frac{\partial^2 u}{\partial x^2} \tag{4.2.8}$$

or

$$\boxed{\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2},} \tag{4.2.9}$$

where $c^2 = T_0/\rho_0(x)$. Equation (4.2.9) is called the **one-dimensional wave equation**. The notation $c^2$ is introduced because $T_0/\rho_0(x)$ has the dimensions of velocity squared. We will show that c is a very important velocity. For a uniform string, c is constant.

## EXERCISES 4.2

4.2.1. (a) Using Equation (4.2.7), compute the sagged equilibrium position $u_E(x)$ if $Q(x,t) = -g$. The boundary conditions are $u(O) = 0$ and $u(L) = 0$.

(b) Show that $v(x,t) = u(x,t) - u_E(x)$ satisfies (4.2.9).

4.2.2. Show that $c^2$ has the dimensions of velocity squared.

4.2.3. Consider a particle whose $x$-coordinate (in horizontal equilibrium) is designated by $\alpha$. If its vertical and horizontal displacements are $u$ and $v$, respectively, determine its position $x$ and $y$. Then show that

$$\frac{dy}{dx} = \frac{\partial u/\partial \alpha}{1 + \partial v/\partial \alpha}.$$

4.2.4. Derive equations for horizontal and vertical displacements without ignoring $v$. Assume that the string is perfectly flexible and that the tension is determined by an experimental law.

4.2.5. Derive the partial differential equation for a vibrating string in the simplest possible manner. You may assume the string has constant mass density $\rho_0$, you may assume the tension $T_0$ is constant, and you may assume small displacements (with small slopes).