## EXERCISES 3.5

3.5.1. Consider

$$x^2 \sim \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{L}. \tag{3.5.12}$$

(a) Determine $b_n$ from (3.3.11), (3.3.12), and (3.5.6).

(b) For what values of $x$ is (3.5.12) an equality?

*(c) Derive the Fourier cosine series for $x^3$ from (3.5.12). For what values of $x$ will this be an equality?

3.5.2. (a) Using (3.3.11) and (3.3.12), obtain the Fourier cosine series of $x^2$.

(b) From part (a), determine the Fourier sine series of $x^3$.

3.5.3. Generalize Exercise 3.5.2, in order to derive the Fourier sine series of $x^m$, $m$ odd.

*3.5.4. Suppose that $\cosh x \sim \sum_{n=1}^{\infty} b_n \sin n\pi x/L$.

(a) Determine $b_n$ by correctly differentiating this series twice.

(b) Determine $b_n$ by integrating this series twice.

3.5.5. Show that $B_n$ in (3.5.9) satisfies $B_n = a_n/(n\pi/L)$, where $a_n$ is defined by (3.5.1).

3.5.6. Evaluate

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} + \cdots$$

by evaluating (3.5.5) at $x = 0$.

*3.5.7. Evaluate

$$1 - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \cdots$$

using (3.5.6).

# 3.6 Complex Form of Fourier Series

With periodic boundary conditions, we have found the theory of Fourier series to be quite useful:

$$f(x) \sim a_0 + \sum_{n=1}^{\infty} \left( a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L} \right), \tag{3.6.1}$$