These same conditions could hold at $x = L$, noting that the change of sign ($-H$ becoming H) is necessary for Newton's law of cooling. One boundary condition occurs at each boundary. It is not necessary that both boundaries satisfy the same kind of boundary condition. For example, it is possible for $x = 0$ to have a prescribed oscillating temperature

$$u(0,t) = 100 - 25\cos t,$$

and for the right end, $x = L$, to be insulated,

$$\frac{\partial u}{\partial x}(L,t) = 0.$$

EXERCISES 1.3

1.3.1. Consider a one-dimensional rod, $0 \leq x \leq L$. Assume that the heat energy flowing out of the rod at $x = L$ is proportional to the temperature difference between the end temperature of the bar and the known external temperature. Derive (1.3.5) (briefly, physically explain why $H > 0$).

*1.3.2. Two one-dimensional rods of different materials joined at $x = x_0$ are said to be in **perfect thermal contact** if the temperature is continuous at $x = x_0$:

$$u(x_0-,t) = u(x_0+,t)$$

and no heat energy is lost at $x = x_0$ (i.e., the heat energy flowing out of one flows into the other). What mathematical equation represents the latter condition at $x = x_0$? Under what special condition is $\partial u/\partial x$ continuous at $x = x_0$?

*1.3.3. Consider a bath containing a fluid of specific heat c_f and mass density ρ_f that surrounds the end $x = L$ of a one-dimensional rod. Suppose that the bath is rapidly stirred in a manner such that the bath temperature is approximately uniform throughout, equaling the temperature at $x = L, u(L,t)$. Assume that the bath is thermally insulated except at its perfect thermal contact with the rod, where the bath may be heated or cooled by the rod. Determine an equation for the temperature in the bath. (This will be a boundary condition at the end $x = L$.) (*Hint*: See Exercise 1.3.2.)

1.4 Equilibrium Temperature Distribution

1.4.1 Prescribed Temperature

Let us now formulate a simple, but typical, problem of heat flow. If the thermal coefficients are constant and there are no sources of thermal energy, then the temperature $u(x,t)$ in a one-dimensional rod $0 \leq x \leq L$ satisfies

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}. \tag{1.4.1}$$