1. a). Show $\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$

b) Evaluate. $\int \frac{x^2+2x-1}{x^2+9}\,dx$

2. a) $y = \ln(\ln(x))$

b) $\ln y = e^y \sin x$

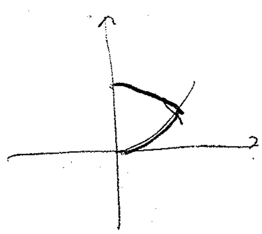
c) $y = 2^{\sin(3x)}$

3. $\frac{dV}{dt} = -\frac{1}{40}V$. $V(0) = V_0$

a) Solve for $V(t)$

b). Show how long will it take for V to reach 10% of its initial value

4. $y = 2 - x^2$. $y = x^2$ $x = 0$ about y axis



5. Find total area between

$y = x\sqrt{a^2 - x^2}$ $a > 0$ and $y = 0$

for $-a \le x \le a$.

6. Sketch. (min max intercepts. asymptote.) $f(x) = x^{2/3}\left(\frac{5}{2} - x\right)$

7. The sum of two nonnegative number is 20. Find the number if.

a) the product of one and square of the other max

b) one + square of other is max

8. 1) $\int \frac{\sin(2t+1)}{\cos^2(2t+1)}\,dt$

2) $\int \frac{1}{x^2}\sqrt{2 - \frac{1}{x}}\,dx$

3) $\int_0^{\frac{\pi}{2}} \frac{\sin x}{(3+2\cos x)^2}\,dx$

1. a) $\tan(\tan^{-1}(x)) = x$

$\Rightarrow \quad \sec^2(\tan^{-1}(x)) \cdot \dfrac{d\tan^{-1}(x)}{dx} = 1$

$\Rightarrow \left[\tan(\tan^{-1}x)^2 + 1\right] \dfrac{d\tan^{-1}x}{dx} = 1$

$\Rightarrow \quad \dfrac{d\tan^{-1}x}{dx} = \dfrac{1}{[\tan(\tan^{-1}x)]^2+1} = \dfrac{1}{1+x^2}$

b) $\displaystyle\int \frac{x^2+2x-1}{x^2+9}\,dx = \int \frac{x^2+9+2x-10}{x^2+9}\,dx$

$\displaystyle = \int 1 + \frac{2x}{x^2+9} - \frac{10}{x^2+9}\,dx$

$= x + \ln(x^2+9) - \frac{10}{9}\cdot 3 \cdot \arctan(\frac{x}{3}) + C$

$= x + \ln(x^2+9) - \frac{10}{3}\arctan\frac{x}{3} + C$

2. a) $\dfrac{d\ln(\ln x)}{dx} = \dfrac{1}{\ln x}\cdot\dfrac{1}{x}$

b) $\dfrac{1}{y}\cdot\dfrac{dy}{dx} = e^y \cdot \sin x \dfrac{dy}{dx} - e^y \cos x$

$\Rightarrow \quad \dfrac{dy}{dx} = \dfrac{e^y\cos x}{e^y \sin x - \frac{1}{y}}$

c) $\dfrac{dy}{dx} = \dfrac{d2^{\sin(3x)}}{dx} = 2^{\sin 3x}\cdot \ln 2 \cdot \cos 3x \cdot 3$

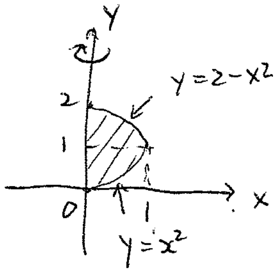
$= 3\cdot 2^{\sin 3x}\cdot \ln 2 \cdot \cos 3x$

3. a) $V(t) = V(0)\cdot e^{-\frac{1}{40}t}$

b) $V(0)\, e^{-\frac{1}{40}t} = 0.1\,V(0) \quad \Rightarrow \quad e^{-\frac{1}{40}t} = 0.1$

$\Rightarrow \quad t = -40\ln(0.1) = 40\ln 10.$

4.



$$V = \int_0^1 2\pi \cdot x \cdot [2-x^2-x^2]\,dx$$

$$= 2\pi \int_0^1 2x\,(1-x^2)\,dx$$

$$= 4\pi \int_0^1 (x-x^3)\,dx = 4\pi \left[\tfrac{1}{2}x^2 - \tfrac{1}{4}x^4\right]\Big|_0^1$$

$$= \pi$$

5. $$A = \int_{-a}^{0} -x\sqrt{a^2-x^2}\,dx + \int_0^a x\sqrt{a^2-x^2}\,dx$$

$$\overset{u=a^2-x^2}{=} \int_{x=-a}^{0} -x\cdot\sqrt{u}\cdot -\frac{1}{2x}\,du + \int_{x=0}^{a} x\sqrt{u}\cdot -\frac{1}{2x}\,du.$$

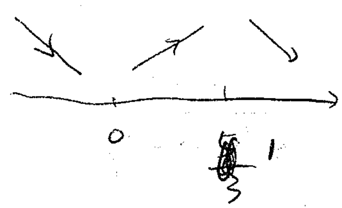
$$= \frac{1}{2}\left[\int_0^{a^2} \sqrt{u}\,du - \int_{a^2}^{0} \sqrt{u}\,du\right] = \int_0^{a^2} \sqrt{u}\,du.$$

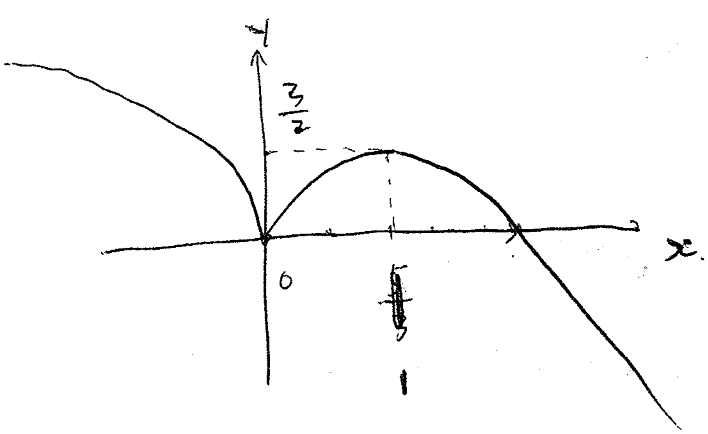
$$= \frac{2}{3}u^{\frac{3}{2}}\Big|_0^{a^2} = \frac{2}{3}a^3$$

6. $$y' = \frac{2}{3}x^{-\frac{1}{3}}\left(\frac{5}{2}-x\right) - x^{\frac{2}{3}}$$

$$= \frac{5}{3}x^{-\frac{1}{3}} - \frac{5}{3}x^{\frac{2}{3}}$$

$$= \frac{5-5x}{3x^{\frac{1}{3}}}$$





7. a) $x+y=20 \quad x\geq 0 \quad y\geq 0$.

maximize. $f(x,y)=xy^2$

$$= x(20-x)^2$$

$$\frac{df}{dx} = (20-x)^2 - x2(20-x).$$

$$= (20-x)(20-3x)$$

$$\Rightarrow x=20. \text{ or. } \frac{20}{3}=x.$$

$$\Rightarrow x=\frac{20}{3}. \quad \Rightarrow y=\frac{40}{3}$$

$$\Rightarrow \max_{\substack{x\geq 0,\, y\geq 0 \\ x+y=20}} f(x,y) = \frac{20}{3}\left(\frac{40}{3}\right)^2 = \frac{32000}{27}$$

b) maximize. $f(x,y)= x+y^2$

$$= x+(20-x)^2.$$

$$\frac{df}{dx} = 1 - 2(20-x) \Rightarrow x=\frac{39}{2}$$

$$\Rightarrow y=\frac{1}{2}$$

8). a) $\displaystyle\int \frac{\sin(2t+1)}{\cos^2(2t+1)}\,dt \quad u=\cos(2t+1)$

$du = -2\sin(2t+1)\,dt.$

$$= \int \frac{\cancel{\sin(2t+1)}}{u^2}\cdot -\frac{1}{2\cancel{\sin(2t+1)}}\,dt.$$

$$= \int -\frac{1}{2u^2}\,du = \frac{1}{2u}+c = \frac{1}{2\cos(2t+1)}+C$$

b) $\int \frac{1}{x^2}\sqrt{2-\frac{1}{x}}\,dx$ $\qquad u = 2-\frac{1}{x}$

$du = \frac{1}{x^2}\,dx$

$= \int \cancel{\frac{1}{x^2}}\sqrt{u}\cdot \cancel{x^2}\,du$

$= \frac{2}{3}u^{\frac{3}{2}} + C = \cancel{\frac{2}{3}\frac{1}{x^3} + C} \qquad \frac{2}{3}\left(2-\frac{1}{x}\right)^{\frac{3}{2}} + C$

$\boxed{\frac{2}{3}\cdot\frac{1}{\left(2-\frac{1}{x}\right)^{\frac{3}{2}}}}$

wrong

c) $\int_0^{\frac{\pi}{2}} \frac{\sin x}{(3+2\cos x)^2}\,dx$ $\qquad u = 3+2\cos x$

$du = -2\sin x\,dx$

$= \int_{x=0}^{\frac{\pi}{2}} \frac{\sin x}{u^2}\cdot\frac{1}{-2\sin x}\,du,$

$= -\int_5^3 \frac{1}{2u^2}\,du = \frac{1}{2}\int_3^5 \frac{1}{u^2}\,du = \frac{1}{2}\left[-\frac{1}{u}\right]_3^5$

$= \frac{1}{2}\left[-\frac{1}{5}+\frac{1}{3}\right] = \frac{1}{15}$