

Math 221 Practice Test - December 16

1. a.) Show that $\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$
b.) Evaluate $\int \frac{x^2+2x-1}{x^2+9} dx$

2. Compute the derivatives $\frac{dy}{dx}$
 - a.) $y = \ln(\ln(x^2))$
 - b.) $\ln y = e^y \sin x$
 - c.) $y = 2^{\sin(3x)}$

3. $\frac{dV}{dt} = -\frac{1}{40}V$, $V(0) = V_0$
 - a.) Solve for $V(t)$
 - b.) Show how long it will take for V to reach ten percent of its initial value.

4. Find the volume of the figure obtained by rotating the region between $y = 2 - x^2$, $y = x^2$, and $x = 0$ about the y -axis. (Hint: the shell method will be easier here, but use whatever method you wish)

5. Find the total area between $y = x\sqrt{a^2 - x^2}$ and $y = 0$ for $-a \leq x \leq a$ ($a > 0$)

6. Sketch (including minima, maxima, intercepts, asymptotes and any other relevant data you know how to compute) $f(x) = x^{\frac{2}{3}}(\frac{5}{2} - x)$

7. The sum of two non-negative numbers is 20. Find the numbers if:
 - a.) the product of one number with the square root of the other is to be as large as possible.
 - b.) one number plus the square root of the other number is to be as

large as possible.

8. Evaluate

a.) $\int \frac{\sin(2t+1)}{\cos^2(2t+1)} dt$

b.) $\int \frac{1}{x^2} \sqrt{2 - \frac{1}{x}} dx$

c.) $\int_0^{\frac{\pi}{2}} \frac{\sin x}{(3+2 \cos x)^2} dx$