Math 322: Applied Mathematical Analysis (001) SP22

General Course Information

Generic Math 322 Course Description

Sturm-Liouville theory; Fourier series, including mean convergence; boundary value problems for linear second order partial differential equations, including separation of variables and eigenfunction expansions. Enroll Info: None

Prerequisites: MATH 321 or 376 or graduate/professional standing

Course website: https://canvas.wisc.edu/courses/284829

Course Designations and Attributes

Breadth – Natural Science
Level – Advanced
L&S Credit – Counts as Liberal Arts and Science credit in L&S

Credit Hours

This *3-credit class* meets for *three 50-minute class periods* each week over the spring semester and carries the expectation that students will work on course learning activities (reading, writing, problem sets, studying, etc.) for about *2 hours* out of the classroom *for every class period*. This syllabus includes additional information about meeting times and expectations for student work.

Instructor

Dr Thomas Chandler (he/him), tgchandler@wisc.edu, https://people.math.wisc.edu/~tgchandler/

Availability: The instructor will be available at the following:

- Office Hours: M 11:00 PM – 12:00 PM and W 1:00 PM – 2:00 PM via Zoom (virtual, link found on canvas: https://canvas.wisc.edu/courses/284829/external_tools/14065).
- Extra (in-person or virtual) instructor office hours can be arranged on request. Please contact the instructor via canvas or email.
- Contributing to Q&A on Piazza predominantly on Monday and Wednesday evenings.

Class Instructional Mode, Time, and Location

Class	Date & Time	Location
322-001	MWF 8:50 AM – 9:40 AM	Van Vleck B115 (in person)

There will be a total of 41 classes (40 lectures + 1 midterm) with no classes during spring recess (03/12 – 03/20).

Other Course Information

Required Textbook, Software, & Other Course Materials

Campus provides students with technology guidelines and recommendations for instruction. Students should consult these resources prior to the start of the semester.

Textbook: This course will predominately follow a draft copy of Sam Stechmann's upcoming book, *Applied Partial Differential Equations: Deterministic and Stochastic*, this book can be found on the canvas page (https://canvas.wisc.edu/courses/284829/pages/stechmann-book-and-more). Along with the Stechmann Book, you may like a published textbook for reference, I'd recommend *Applied Partial Differential Equations with Fourier Series and Boundary Value Problems*, by Richard Habermann, other recommendations are available upon request.

Other: Handwritten class notes, as well as homework and midterm solutions, will be posted on canvas.

Course Website, Learning Management System & Digital Instructional Tools

- All course material, homework, announcements, and grades will be uploaded onto Canvas (https://canvas.wisc.edu/courses/284829)
- All virtual contact (e.g. virtual office hours) shall be via Zoom, with link found on the Canvas website (https://canvas.wisc.edu/courses/284829/external_tools/14065)
- Questions of general interest related to the course should be posted and discussed on the course Piazza site (https://piazza.com/class/kxys4xcaog6hh) or brought to the office hours. The course instructor will be regularly contributing to the Q&A section son Piazza.
- All other questions/problems should be sent to the instructor directly via Canvas or email.
- Solutions to the homework sheets should be submitted on canvas in a PDF format. There is a lot of support available on the web for doing this; for example, see scanning on iPhone/iPad and scanning on Android.

Grading

This course will be assessed via weekly problem sheets (with the two lowest scores being dropped), one midterm exam, and a final exam. The scales given below may be decreased to allow for grade curving.

Assessment Weighting: 30% Assessed Homework, 30% Midterm, 40% Final.

Tentative Grading Scales: 0-59 F, 60-69 D, 70-78 C, 79-81 BC, 82-88 B, 89-91 AB, 92-100 A

Exams

The exams shall be in person and closed book. No communication with anyone (in or out of the class) is allowed during the exams. The exams are cumulative with no make-up exams. By signing up to this course, you are agreeing to take the midterm and final exams at the following (Madison, WI) times:

Exam	**Date & Time**	**Location**
Midterm	11 March 8:50 AM – 9:40 AM	Van Vleck B115 (in person)
Final Exam	09 May 10:05 AM – 12:05 PM	To be announced

Please inform the instructor of any problems sitting these exams as soon as possible.

Exam Proctoring: Exam proctoring will be required for this course. Failure to use the proctoring service assigned will results in a zero on the exam. Further details will be given before each exam on canvas and in class.

Assessed Homework: Problem Sheets

A problem sheet will be posted weekly on canvas, these will be assessed and count towards your final grade (see grading above). The problem sheets shall focus on the content presented in the corresponding week's classes, as will be indicated on the canvas course site and in class. This assessed homework is cumulative, but the two lowest scores will be dropped.

Solutions to the problem sheets should be submitted on canvas as PDFs before the posted deadline. *A grace period of 6 hours shall be given for submissions, however any late submission during this period (without prior permission) will be docked 10%.* Tentatively, the problem sheets shall be made available on Friday and due the following Thursday. The instructor should be informed of any problems with completing the assessed homework.

Non-Assessed Homework: Suggested Reading and Further Work

On top of the assessed homework, suggested reading and further work shall be posted regularly on canvas and given in class, but will not be collected or graded. This suggested work is intended to help you learn how to solve problems on your own, consolidate your knowledge, and prepare for the exams. It is highly recommended that you carefully study the assigned sections of the book. Feel free to post any questions about the reading on Piazza or bring them to the office hours/classes.

Course Outline:

Detailed Course Description: This is an introductory course to Partial Differential Equations (PDEs). The material is focused on the theoretical foundations and mathematical techniques for solving the three canonical second order PDEs: the heat equation, Laplace's equation, and the wave equation. Two techniques considered in this course are Green's functions and separation of variables. The latter involves the construction of eigensolutions of homogeneous problems, Fourier series, and solutions to Sturm–Liouville problems.

Tentative Schedule: Math 322 covers a lot of material and is not an easy course. The material will not be understood upon a first read but should be studied in multiple iterations. In general, the course will follow the Stechmann book in order, however it may deviate at times. Before taking this course, you should be comfortable with manipulating and solving Ordinary Differential Equations (ODEs); a review of prerequisite courses, such as Math 321 or Math 376, is highly recommended. This course will follow the tentative schedule:

Week	Material	Stechmann
1	Introduction to PDEs and overview of course. Heat equation (parabolic): derivation, fundamental solution first look.	§1.1, §2.1, §2.2
2	Heat equation (parabolic): fundamental solution derivation, initial value and forced problems.	§2.2–§2.4
3	Laplace's equation (elliptic): applications, fundamental solution, Poisson's equation, examples problems.	§3.1–§3.4
4	Laplace's equation (elliptic): mean value formula, maximum principle, Dirichlet's principle of energy minimization, calculus of variations. Wave equation (hyperbolic): derivations, energy, applications.	§3.5–§3.7, §4.1, §4.2
5	Wave equation (hyperbolic): initial value problems, forced problems	§4.3
6	Wave equation (hyperbolic): fundamental problem, solution in 1D and 3D. Boundaries/Green functions: heat equation: method of images and homogenizing transformations.	§4.3, §5.1
7	Boundaries/Green functions: heat equation: superposition of boundary and initial conditions and forcing. Material review and midterm exam.	§5.1, midterm
8	Boundaries/Green functions: Laplace/Poisson's equation: Green's formula, Method of images in 2D and 3D. Wave equation: reflection, radiation and more applications.	§5.2, §5.3
9	Separation of variables: heat equation, Dirichlet boundary conditions, Fourier series, computing Fourier coefficients.	§6.1–§6.4, §7.1–§7.4
10	Separation of variables: Neumann and periodic boundary conditions, initial value problems, forced problems, nonhomogeneous boundary conditions.	§7.3, §7.4, §8.2
11	Separation of variables: heat equation in 2D and 3D, Laplace's equation, Poisson's equation, wave equation. Fourier series: term-by-term differentiation and integration.	§8.2, §8.3, §9.1, §9.2
12	Fourier series: convergence, Gibbs phenomenon. Separation of variables: cylindrical coordinates.	§8.2
13	Separation of variables: spherical coordinates, Sturm–Liouville theory.	§8.2, §9.3
14	Fourier and Laplace Transform for solving PDEs, dispersive waves. Material review.	§10.1, §10.2

Generic Course Learning Outcomes

Students will be able to:

- Check the premises of theorems used in complex and vector calculus in order to apply their conclusions (e.g. that a finite series solution minimizes error).
- Verify if a mathematical object has a given property used in elementary partial differential equations, PDEs, (e.g. homogenous and symmetric).
- Recall and state the formal definitions, properties, techniques, and theorems associated with elementary PDEs (e.g. derive series solutions to a given PDE and apply the finite element method to approximate a PDE solution).

- Express informal mathematical arguments in English using appropriate mathematical terminology and notation.
- Use separation of variables to find formal solutions to homogenous boundary value problems involving the heat, Laplace's, and the wave equations.
- Establish orthogonality of sines and cosines using direct computation.
- Construct Fourier, sine, and cosine series expansions for given piecewise smooth functions and state/explain the conditions necessary for convergence and term-by-term differentiation.
- State the definition and main theorems of a second-order regular Sturm–Liouville boundary value problem.
- Be able to apply the method of characteristics to derive D'Alembert's solution to the wave equation on the real line.
- Use the fundamental solution to linear PDEs to create more general solutions.

How to Succeed in This Course

The students should strive to attend all classes, whilst the office hours and discussion boards are highly recommended. The classes are instructional lectures ran by the instructor; however, they are informal and, thus, any interruption for questions or observations are welcomed and recommended — there are no 'silly' questions in this course! The office hours are for any questions (either about the course or the bigger picture) but are also a chance to just say 'hi' and chat with fellow mathematicians. You do not have to have a specific question to come to the office hours but can instead just come to listen. Finally, any course related questions should be sent to me, the instructor, via email (tgchandler@wisc.edu) or canvas (https://canvas.wisc.edu/courses/284829).

The following campus services might also be useful:

- University Health Services
- Undergraduate Academic Advising and Career Services
- Office of the Registrar
- Office of Student Financial Aid
- Dean of Students Office

Covid-19 Guidelines

Students should continually monitor themselves for COVID-19 symptoms and get tested for the virus if they have symptoms or have been in close contact with someone with COVID-19. Students should reach out to the instructor as soon as possible if they become ill or need to isolate or quarantine, in order to make alternate plans for how to proceed with the course. Students are expected to comply with the University's current COVID rules and policies that are maintained here: https://covidresponse.wisc.edu (see in particular https://covidresponse.wisc.edu/faq/).

Students who do not comply with these rules can be asked to leave the classroom, and students who repeatedly fail to comply will be referred to the Office of Student Conduct and Community Standards. Any student who requires an exemption to current policies must contact the McBurney Office, as instructors do not have the authority to grant such exceptions.

Course Evaluation

Students will be provided with an opportunity to evaluate this course and your learning experience. Student participation is an integral component of this course, and your confidential feedback is important to me. I strongly encourage you to participate in the course evaluation.

Privacy of Student Records & the Use of Audio Recorded Lectures Statement

Lecture materials and recordings for this course are protected intellectual property at UW-Madison. Students in this course may use the materials and recordings for their personal use related to participation in this class. Students may also take notes solely for their personal use. If a lecture is not already recorded, you are not authorized to record my lectures without my permission unless you are considered by the university to be a qualified student with a disability requiring accommodation. [Regent Policy Document 4-1] Students may not copy or have lecture materials and recordings outside of class, including posting on internet sites or selling to commercial entities. Students are also prohibited from providing or selling their personal notes to anyone else or being paid for taking notes by any person or commercial firm without the instructor's express written permission. Unauthorized use of these copyrighted lecture materials and recordings constitutes copyright infringement and may be addressed under the university's policies, UWS Chapters 14 and 17, governing student academic and non-academic misconduct.

Students' Rules, Rights & Responsibilities

To see the Undergraduate Guide's Rules, Rights, and Responsibilities information, please refer to https://guide.wisc.edu/undergraduate/#rulesrightsandresponsibilitiestext

Diversity & Inclusion Statement

Diversity is a source of strength, creativity, and innovation for UW-Madison. We value the contributions of each person and respect the profound ways their identity, culture, background, experience, status, abilities, and opinion enrich the university community. We commit ourselves to the pursuit of excellence in teaching, research, outreach, and diversity as inextricably linked goals. The University of Wisconsin-Madison fulfills its public mission by creating a welcoming and inclusive community for people from every background – people who as students, faculty, and staff serve Wisconsin and the world.

Academic Integrity Statement

By virtue of enrollment, each student agrees to uphold the high academic standards of the University of Wisconsin-Madison; academic misconduct is behavior that negatively impacts the integrity of the institution. Cheating, fabrication, plagiarism, unauthorized collaboration, and helping others commit these previously listed acts are examples of misconduct which may result in disciplinary action. Examples of disciplinary action include, but is not limited to, failure on the assignment/course, written reprimand, disciplinary probation, suspension, or expulsion.

Accommodations for Students with Disabilities Statement

The University of Wisconsin-Madison supports the right of all enrolled students to a full and equal educational opportunity. The Americans with Disabilities Act (ADA), Wisconsin State Statute (36.12), and UW-Madison policy (UW-855) require the university to provide reasonable accommodations to students with disabilities to access and participate in its academic programs and educational services. Faculty and students share responsibility in the accommodation process. Students are expected to inform faculty [me] of their need for instructional accommodations during the beginning of the semester, or as soon as possible after being approved for accommodations. Faculty [I], will work either directly with the student [you] or in coordination with the McBurney Center to provide reasonable instructional and course-related accommodations. Disability information, including instructional accommodations as part of a student's educational record, is confidential and protected under FERPA. (See: McBurney Disability Resource Center.)

Academic Calendar & Religious Observances

The 2022 academic calendar and religious observances can be found at https://secfac.wisc.edu/academic-calendar/. Pursuant to university policy UW-880 (https://policy.wisc.edu/library/UW-880), students are required to inform their instructors during the first two weeks of class about any religious conflicts with quizzes and exams taking place during the semester. Students who will miss quizzes and/or exams during the semester because of religious holidays/observances must email the instructor Dr Thomas G J Chandler at tgchandler@wisconsin.edu to inform of possible conflicts. He will work with the individual student to find suitable alternatives that adhere to university and departmental guidelines. Note that if a conflict is not raised during the initial two-week period, then we cannot guarantee that suitable accommodations will be provided. Because of this, it is vital that students with religious conflicts contact their instructors in a timely manner during the first two weeks of class.