Gravity Waves in Shear and Implications for Organized Convection

SAMUEL N. STECHMANN

Department of Mathematics, and Department of Atmospheric and Oceanic Sciences, University of California, Los Angeles, Los Angeles, California

ANDREW J. MAJDA

Department of Mathematics, and Center for Atmosphere Ocean Science, Courant Institute of Mathematical Sciences, New York University, New York, New York

(Manuscript received 10 October 2008, in final form 12 February 2009)

ABSTRACT

It is known that gravity waves in the troposphere, which are often excited by preexisting convection, can favor or suppress the formation of new convection. Here it is shown that in the presence of wind shear or barotropic wind, the gravity waves can create a more favorable environment on one side of preexisting convection than the other side.

Both the nonlinear and linear analytic models developed here show that the greatest difference in favorability between the two sides is created by jet shears, and little or no difference in favorability is created by wind profiles with shear at low levels and no shear in the upper troposphere. A nonzero barotropic wind (or, equivalently, a propagating heat source) is shown to also affect the favorability on each side of the preexisting convection. It is shown that these main features are captured by linear theory, and advection by the background wind is the main physical mechanism at work. These processes should play an important role in the organization of wave trains of convective systems (i.e., convectively coupled waves); if one side of preexisting convection is repeatedly more favorable in a particular background wind shear, then this should determine the preferred propagation direction of convectively coupled waves in this wind shear.

In addition, these processes are also relevant to individual convective systems: it is shown that a barotropic wind can lead to near-resonant forcing that amplifies the strength of upstream gravity waves, which are known to trigger new convective cells within a single convective system. The barotropic wind is also important in confining the upstream waves to the vicinity of the source, which can help ensure that any new convective cells triggered by the upstream waves are able to merge with the convective system.

All of these effects are captured in a two-dimensional model that is further simplified by including only the first two vertical baroclinic modes. Numerical results are shown with a nonlinear model, and linear theory results are in good agreement with the nonlinear model for most cases.

1. Introduction

Tropical convection can be organized across a wide range of space and time scales, from mesoscale convective systems (Houze 2004) to convectively coupled waves on equatorial synoptic scales (Kiladis et al. 2009) to the Madden–Julian oscillation on planetary/intraseasonal scales (Zhang 2005). These phenomena remain chal-

E-mail: stechmann@math.ucla.edu

DOI: 10.1175/2009JAS2976.1

© 2009 American Meteorological Society

lenging both theoretically and practically; not only are their physical mechanisms not adequately understood, but even numerical simulation of these multiscale phenomena is a challenging endeavor.

To better understand the physical mechanisms of organized convection, numerical simulations are often carried out using cloud-resolving models (CRMs). Many CRM studies have documented the role of gravity waves in organizing convection into wave trains (Oouchi 1999; Shige and Satomura 2001; Lac et al. 2002; Liu and Moncrieff 2004; Tulich et al. 2007; Tulich and Mapes 2008). Convective heating generates a variety of gravity waves that propagate away from the convection, and

Corresponding author address: Samuel N. Stechmann, UCLA Mathematics Department, 520 Portola Plaza, Los Angeles, CA 90095-1555.

these gravity waves can favor or suppress the formation of new convection. New convection is favored when gravity waves promote ascent of air in the free troposphere, which is also accompanied by adiabatic cooling and moistening, and new convection is suppressed in the opposite situation. In addition to gravity waves, density currents can play a similar role in triggering new convection (Moncrieff and Liu 1999; Tompkins 2001; Fovell 2005). Furthermore, in addition to their role in favoring or suppressing new convective systems, gravity waves and density currents can play a similar role in favoring or suppressing new convective cells within a single convective system (McAnelly et al. 1997; Lane and Reeder 2001; Fovell 2002; Fovell et al. 2006).

In addition to (and in some cases preceding) these CRM studies, several theoretical studies using simplified models have also been carried out to better understand the role of gravity waves and density currents in organizing convection (Bretherton and Smolarkiewicz 1989; Nicholls et al. 1991; Pandya et al. 1993; Mapes 1993; Liu and Moncrieff 2004). These studies use a simplified setup with linearized dynamics and imposed heat sources to represent cloud heating. The exact linear response to the heating can then be found analytically for any heating profile, usually chosen to represent deep convective and stratiform heating. Mapes (1993) used such a model with two vertical modes to demonstrate how the formation of new convection could be favored or triggered near preexisting convection by convectively generated gravity waves; such a mechanism was posited to be important for convective organization, and the observations and CRM simulations mentioned in the previous paragraph largely confirm this.

For the sake of simplicity, these simplified modeling studies mentioned above all assumed a motionless, shear-free background wind. The main purpose of the present paper is to investigate the effect of wind shear and barotropic wind on gravity waves by using the simplified nonlinear model of Stechmann et al. (2008, hereafter SMK08) and a linearized version of this model.

The main simplification used by SMK08, as in the other simplified model studies mentioned above, is a truncated vertical structure with only two baroclinic modes; however, unlike previous truncated models, SMK08 included nonlinear interactions between the vertical modes, which allows nonlinear interactions between gravity waves and wind shear. The truncated vertical structure of this model has the important advantage that it reduces the system dimension, which greatly reduces the computation time needed to solve the equations numerically. In addition to results with

this nonlinear model, an exact linear theory with background shear is developed here; these results are also shown below, and they are in good agreement with the nonlinear numerical results. The implications of these results for organized convection are discussed throughout the paper.

The paper is organized as follows: The simplified model of SMK08 is described in section 2. Results for various wind profiles are presented in section 3. The physical mechanisms at work are discussed throughout section 3 and in section 4, and some sensitivity studies including higher vertical resolution are shown in section 5. Finally, conclusions are summarized in section 6. To streamline the presentation, some analytic details of the model are relegated to the appendices.

2. The simplified model

In this section, the simplified model of SMK08 is reviewed. The representation of convective heating as an imposed heat source is also described; it will allow certain linear theory techniques to be used, which are also described below.

a. Description of the simplified nonlinear model

The setup considered here is a hydrostatic Boussinesq fluid with a background potential temperature $\theta_{bg}(z)$ that is linear with height. For simplicity, and following previous work mentioned in the introduction, a twodimensional setup with zonal coordinate x and vertical coordinate z will be used here. The fluid is bounded above and below by rigid lids where free slip boundary conditions are assumed: $W|_{z=0,H} = 0$. The upper boundary here is taken to be z = H = 16 km, which is roughly the height of the tropopause in the tropics, and the height z will be nondimensionalized so that $z = \pi$ is the upper boundary in nondimensional units. Standard equatorial scales are chosen as reference units so that the horizontal spatiotemporal scales are 1500 km and 8 h (Majda 2003), with a reference speed of 50 m s⁻¹ corresponding to the first baroclinic mode gravity wave speed. See Table 1 for the reference scales used to nondimensionalize the model variables.

In this situation there is an infinite set of horizontally propagating linear gravity waves with phase speeds c_j and vertical profiles $\sin(jz)$ for j = 1, 2, 3, ... (Majda 2003). The waves c_j are decoupled from all waves c_k ($k \neq j$) for the case of linear dynamics, but the waves become coupled in the case of nonlinear dynamics, as will be shown below. For the nonlinear model of this paper, only the first two baroclinic modes are kept so that the model variables are expanded as

Parameter	Derivation	Value	Description
β	_	$2.3 imes 10^{-11} { m m}^{-1} { m s}^{-1}$	Variation of Coriolis parameter with latitude
$\theta_{\rm ref}$		300 K	Reference potential temperature
g	_	9.8 m s^{-2}	Gravitational acceleration
H	_	16 km	Tropopause height
N^2	$(g/\theta_{\rm ref}) d\theta_{\rm bg}/dz$	$10^{-4} \mathrm{s}^{-2}$	Buoyancy frequency squared
С	\widetilde{NH}/π	50 m s^{-1}	Velocity scale
L	$\sqrt{c/\beta}$	1500 km	Equatorial length scale
Т	L/c	8 h	Equatorial time scale
$\overline{\alpha}$	$HN^2\theta_{\rm ref}/(\pi g)$	15 K	Potential temperature scale
_	H/π	5 km	Vertical length scale
_	$H/(\pi T)$	0.2 m s^{-1}	Vertical velocity scale
_	c^2	$2500 \text{ m}^2 \text{ s}^{-2}$	Pressure scale

TABLE 1. Model parameters and scales.

$$U(x, z, t) = u_0 + u_1(x, t)\sqrt{2} \cos(z) + u_2(x, t)\sqrt{2} \cos(2z),$$

$$\Theta(x, z, t) = \theta_1(x, t)\sqrt{2} \sin(z) + \theta_2(x, t)2\sqrt{2} \sin(2z),$$

$$P(x, z, t) = p_1(x, t)\sqrt{2} \cos(z) + p_2(x, t)\sqrt{2} \cos(2z), \text{ and}$$

$$W(x, z, t) = w_1(x, t)\sqrt{2} \sin(z) + w_2(x, t)\sqrt{2} \sin(2z).$$

(1)

Note that the convention here is to expand Θ on the basis of $j\sqrt{2} \sin(jz)$, not $\sqrt{2} \sin(jz)$. This vertical struc-

ture is illustrated in Fig. 1. The hydrostatic and continuity equations lead to the relationships

$$\theta_j = -p_j \quad \text{and} \quad w_j = -\frac{1}{j} \partial_x u_j.$$
(2)

When the hydrostatic Boussinesq equations are projected onto the first two baroclinic modes, including the projections of the nonlinear advection terms, the result is (SMK08)

$$\begin{cases} \frac{\partial u_1}{\partial t} + u_0 \frac{\partial u_1}{\partial x} - \frac{\partial \theta_1}{\partial x} = -\frac{3}{\sqrt{2}} \left(u_2 \frac{\partial u_1}{\partial x} + \frac{1}{2} u_1 \frac{\partial u_2}{\partial x} \right), \\ \frac{\partial \theta_1}{\partial t} + u_0 \frac{\partial \theta_1}{\partial x} - \frac{\partial u_1}{\partial x} = -\frac{1}{\sqrt{2}} \left(2 u_1 \frac{\partial \theta_2}{\partial x} - u_2 \frac{\partial \theta_1}{\partial x} + 4 \theta_2 \frac{\partial u_1}{\partial x} - \frac{1}{2} \theta_1 \frac{\partial u_2}{\partial x} \right) + S_1; \\ \begin{cases} \frac{\partial u_2}{\partial t} + u_0 \frac{\partial u_2}{\partial x} - \frac{\partial \theta_2}{\partial x} = 0, \\ \frac{\partial \theta_2}{\partial t} + u_0 \frac{\partial \theta_2}{\partial x} - \frac{1}{4} \frac{\partial u_2}{\partial x} = -\frac{1}{2\sqrt{2}} \left(u_1 \frac{\partial \theta_1}{\partial x} - \theta_1 \frac{\partial u_1}{\partial x} \right) + S_2. \end{cases}$$
(3)

(In three dimensions, the u_2 equation includes terms proportional to $\mathbf{u}_1 \cdot \nabla \mathbf{u}_1 - \mathbf{u}_1 \nabla \cdot \mathbf{u}_1$, which are zero in this two-dimensional case.) Notice that the left-hand side shows the familiar linear dynamics for the baroclinic modes with phase speeds 1 and $\frac{1}{2}$ in the nondimensional units used here. The right-hand side shows how the baroclinic modes interact nonlinearly because of the projection of the nonlinear advection terms. These nonlinear terms also allow for nonlinear interactions between gravity waves and wind shear. Note that these equations have a conserved energy,

$$E = \frac{1}{2}(u_1^2 + u_2^2 + \theta_1^2 + 4\theta_2^2), \qquad (4)$$

and they can be written in matrix form as

$$\partial_t \mathbf{u} + \mathbf{A}(\mathbf{u})\partial_x \mathbf{u} = \mathbf{S},\tag{5}$$

where $\mathbf{u} = (u_1, \theta_1, u_2, \theta_2)^{\mathrm{T}}$, $\mathbf{S} = (0, S_1, 0, S_2)$, and the matrix $\mathbf{A}(\mathbf{u})$ is written out in appendix A. Because these equations cannot be written as a system of conservation laws, designing a numerical method for them is a challenge. The numerical methods used here were designed and extensively tested by SMK08; the reader is referred there for more details.

We also note here that a propagating source term $\mathbf{S}(x - st)$ is equivalent to a stationary source term with a barotropic wind of $u_0 = -s$. To see this, start with (5) and assume a source term of the form $\mathbf{S} = \mathbf{S}(x - st)$. By changing variables into a reference frame moving with the source term, with x' = x - st and t' = t, this equation becomes



FIG. 1. Vertical profiles for the first few modes of (a) velocity and (b) potential temperature, as described in (1).

$$\partial_{t'} \mathbf{u} - s \partial_{x'} \mathbf{u} + \mathbf{A}(\mathbf{u}) \partial_{x'} \mathbf{u} = \mathbf{S}(x'), \tag{6}$$

which is equivalent to a stationary source with a change of -s to the barotropic wind. Because of this equivalence, all of the results in this paper could be interpreted in many ways, since, for instance, even the case of zero barotropic wind and a stationary source is equivalent to any case with $u_0 = \overline{v}$ and $s = \overline{v}$ for any choice of \overline{v} .

b. Representation of convective heating

Convective heating will be represented as an imposed heat source with components from two vertical baroclinic modes, as it was represented in the previous simplified modeling studies mentioned in the introduction (Mapes 1993):

$$S(x, z) = S_1(x)\sqrt{2}\sin z + S_2(x)2\sqrt{2}\sin 2z.$$
 (7)

The first baroclinic heating S_1 represents deep convective heating, whereas the second baroclinic heating S_2 represents stratiform and congestus heating. The stan-



FIG. 2. Contour plot of the heat source $S_{\theta}(x, z)$ from (7) and (8) used to represent deep convective and stratiform heating. Contours are drawn from 50 to 300 K day⁻¹ with a contour interval of 50 K day⁻¹; an additional contour at 20 K day⁻¹ is also shown. The minimum heating is -23 K day⁻¹ near x = 50 km, z = 4 km, but no negative contours are shown. This is the standard heat source used unless otherwise specified.

dard top-heavy heating that will be used here unless otherwise stated represents a combination of deep convective and stratiform heating with

$$S_1(x) = a_1 e^{-x^2/2\sigma_1^2}$$
 and $S_2(x) = a_2 e^{-x^2/2\sigma_2^2}$. (8)

The standard values used here will be $a_1 = 200$ K day⁻¹, $2a_2 = -50$ K day⁻¹, $\sigma_1 = 20$ km, and $\sigma_2 = 40$ km, unless otherwise specified. These values are chosen to represent the mesoscale-filtered convective heating of a squall line or mesoscale convective system, and they are similar to the values used in previous studies (Nicholls et al. 1991; Mapes 1993; Pandya and Durran 1996; Mapes 1998); in fact, the ratio of total deep convective to stratiform heating is equivalent to that used by Mapes (1993), as explained below. This standard heating S(x, z), given by these standard parameter values and (7)–(8), is shown in Fig. 2. In the time-dependent numerical simulations shown below, this heating profile is turned on at the start of the simulation at time t = 0 and maintained thereafter.

A delta function approximation to the zonal heating profile (8) will be used for approximate analytical solutions as described later:

$$ae^{-x^{2}/2\sigma^{2}} = a\sigma\sqrt{2\pi}(2\pi\sigma^{2})^{-1/2}e^{-x^{2}/2\sigma^{2}}$$
$$\approx a\sigma\sqrt{2\pi}\delta(x). \tag{9}$$

Notice that the magnitude of the delta function, which will be denoted by S_j^* , is determined by the Gaussian profile's amplitude and standard deviation by

$$S_j(x) \approx a_j \sigma_j \sqrt{2\pi} \,\delta(x) = S_j^* \,\delta(x) \tag{10}$$

for j = 1, 2. Notice that σ_j , the standard deviation of the Gaussian profile, is just as important to the delta function magnitude as a_j , the amplitude of the Gaussian heating. When this is taken into account, the delta function form of the standard heating used here is equivalent to that of Mapes (1993) because the larger width of the stratiform heating compensates for its weaker amplitude.

c. Exact linear solutions

When linearized about the sheared background state $\overline{\mathbf{u}} = (\overline{u}_1, 0, \overline{u}_2, 0)^{\mathrm{T}}$, the simplified model in (5) becomes

$$\partial_t \mathbf{u} + \mathbf{A}(\overline{\mathbf{u}})\partial_x \mathbf{u} = \mathbf{S}^* \delta(x),$$
 (11)

where a delta function source term is also included. When this linear system is forced by a delta function source term, four discontinuous waves are excited and propagate away from the source. These propagating fronts are what Mapes (1993) referred to as "pulses" or "buoyancy bores." The jumps associated with these propagating fronts can be found using the so-called Rankine–Hugoniot jump conditions (Evans 1998; LeVeque 2002). In addition, there can also be discontinuities in the variables at the location of the source, x = 0, and the jumps associated with this discontinuity can also be found by using a form of the Rankine–Hugoniot jump conditions (LeVeque 2002):

$$\mathbf{A}[\mathbf{u}] = \mathbf{S}^*,\tag{12}$$

where $\mathbf{A} = \mathbf{A}(\overline{\mathbf{u}})$ is the background advection matrix from (11) and $[\mathbf{u}] = \mathbf{u}_+ - \mathbf{u}_-$ is the jump in \mathbf{u} across the location of the source, where \mathbf{u}_+ and \mathbf{u}_- are the values of \mathbf{u} just to the right and left of the source, respectively. This is a simple linear system of equations. Therefore, given the background wind shear \overline{u} , the barotropic wind u_0 , the source propagation speed *s*, and the magnitude \mathbf{S}^* of the convective heating, one can use the Rankine– Hugoniot jump conditions in (12) to easily determine $[\mathbf{u}]$, which measures the difference in the variables on the east and west sides of the heat source. (Exact solutions to the complete linear problem with shear can also be readily found as shown in appendix A.)

In particular, if there is a jump in θ_1 and/or θ_2 across the location of the source, this represents a difference in the thermodynamic state (and therefore a difference in favorability for convection) between the western and eastern sides of the source. Determining how this difference in the thermodynamic state depends on $\overline{\mathbf{u}}$, u_0 , and s is the main purpose of the present paper. In addition to numerical simulations of the equations in (3), analytical solutions to the jump conditions in (12) will also provide insight. Comparisons are shown below to demonstrate that the linear approximation in (11) is accurate in most cases.

One way to think of the jump conditions in (12) is to recall the relationship $w_j = -\partial_x u_j/j$ from the continuity equation in (2). If the jump $[u_j]$ is interpreted as a derivative, then $[u_j]$ can be thought of as being proportional to the vertical velocity at the location of the source. The source initially excites four waves that subsequently propagate away from the source, and the state [**u**] is the steady balanced state left behind after the adjustment process, with the source term partially balanced by the "vertical velocity" $[u_j]$. For an interpretation in terms of the physical vertical coordinate *z* instead of vertical modes, see section 4.

3. Results

The results in this section will be of two types: (i) numerical solutions to the nonlinear equations in (3) and (ii) analytical solutions to the jump conditions in (12). First the simplest case with initially motionless wind is described, after which the effect of wind shear is considered, then barotropic wind, and then both effects combined.

a. Simplest case with initially motionless wind

As an initial illustration of the gravity wave response to diabatic forcing, consider the simplest case of initial conditions with motionless, shear-free winds and a localized heat source that is turned on at time t = 0 and maintained thereafter. The heat source used here is the standard source shown in Fig. 2 except with the deep convective and stratiform heatings widened by factors of 2 and 4, respectively, in order to broaden the propagating fronts for illustration purposes. The amplitudes a_j of the heatings were reduced by factors of 2 and 4 to keep the total heating magnitudes S_j^* fixed at their standard values.

The resulting gravity wave response to this heating is shown in Fig. 3 at time t = 4 h. The first baroclinic mode response can be seen near $x = \pm 720$ km. Its signature is subsidence and adiabatic warming through the depth of the troposphere, which tend to suppress the formation of new convection. This component of the response propagates away from the source at 50 m s⁻¹, whereas the second baroclinic mode response propagates at only 25 m s⁻¹ and reaches only $x = \pm 360$. This slower response has low-level ascent and adiabatic cooling as its signature, and these effects tend to favor the formation of new convection (Mapes 1993; Tulich and Mapes 2008). In a more realistic situation with water vapor, the



FIG. 3. Numerical solution of the simplified nonlinear model (3) with motionless initial conditions. Snapshots are shown at time t = 4 h for (a) potential temperature $\Theta(x, z)$ and (b) vertical velocity W(x, z). Solid (dashed) contours denote positive (negative) values, and the zero contour is not shown. The contour intervals for Θ and W are 0.25 K and 10 cm s⁻¹, respectively.

subsidence (ascent) would be accompanied by drying (moistening), which accentuates the dry effects on the favorability for new convection. In short, the effect of the fast wave is to initially suppress the formation of new convection, and the effect of the slow wave is to create a nearby neighborhood of the preexisting convection that is favorable for the formation of new convection. These basic physical mechanisms of the gravity wave response have been described in earlier work (Bretherton and Smolarkiewicz 1989; Nicholls et al. 1991; Pandya et al. 1993; Mapes 1993), and the role of the slower, shallow second baroclinic mode response in triggering convection has been documented in several numerical simulations (Lac et al. 2002; Tulich and Mapes 2008). This mechanism, whereby the second baroclinic mode preconditions the lower troposphere for convection, has also been included in models of convectively coupled waves that agree well with observations (Mapes 2000; Majda and Shefter 2001; Khouider and Majda 2006).

Note that in addition to the important effects of subsidence or ascent at the location of the front on the triggering of new convection, the resulting changes in the thermodynamic state after the passage of the front should also be important because they set up an environment that is favorable or unfavorable for the formation of new convection. In terms of elementary parcel stability theory (Emanuel 1994), regions with $\Theta(x, z) < 0$ (>0) should be favorable (unfavorable) environments for rising parcels and convection. In addition, the vertical velocity is a less reliable variable in this model because it must be computed numerically as the derivative of horizontal velocity through (2). For these reasons, the thermodynamic state $\Theta(x, z)$ will be used throughout this paper as a measure of the favorability for the formation of new convection.

As demonstrated below, the presence of wind shear or barotropic wind can substantially alter the east–west symmetry of this picture, with important implications for the organization of convection. One way in which the east–west symmetry can be broken is through differences in the strength of the subsidence (or the low-level ascent) to the east and west of the source. Other mechanisms can also cause east–west asymmetries, as described below.

b. Effect of different wind shears

The numerical response of the nonlinear model (3) to the standard heat source (7) is shown in Fig. 4 for four



FIG. 4. Numerical simulation of waves generated by the localized heat source in Fig. 2 using the simplified nonlinear model (3). (left) Shear profile used as the initial conditions. (middle) Vertical profile of potential temperature $\Theta(z)$ to the west (east) of the heat source at x = -100 km (x = +100 km) shown with a dashed (solid) line at time t = 4 h. (right) First baroclinic mode potential temperature θ_1 at time t = 4 h.

different initial shear profiles $\overline{U}(z)$. All of these cases have zero barotropic wind, and the potential temperature plots in the middle and right columns are snapshots at time t = 4 h. Figures 4a–c show results with a shearless initial condition, which can be identified with the contour plots in Fig. 3. Figure 4b shows the potential temperature $\Theta(z)$ just to the west and east of the source. Each side of

the source has the same $\Theta(z)$, with $\Theta(z) < 0$ in the lower troposphere, representing favorable conditions for the formation of new convection (cf. Fig. 3). This east–west symmetry can also be seen clearly in Fig. 4c, which shows the first baroclinic mode potential temperature θ_1 at time t = 4 h.

Figures 4d-f show results with a jet shear initial condition that roughly resembles the shear profiles of the westerly wind burst phase of the Madden-Julian oscillation (Lin and Johnson 1996; Biello and Majda 2005). In terms of vertical baroclinic modes, this case uses initial conditions of $u_1 = 10 \text{ m s}^{-1}$ and $u_2 = -10 \text{ m s}^{-1}$. For this case there is a large difference in the gravity wave response to the west and east of the source. As shown in Fig. 4e, $\Theta(z)$ is more negative to the west of the source by roughly 1.5 K through most of the troposphere. This asymmetry is also clear in the snapshot of θ_1 in Fig. 4f, which shows that the west (east) of the source has become more (less) favorable for the formation of new convection in comparison to the shearless case in Figs. 4a-c. This is in agreement with actual observations of the formation of new convection in this type of shear profile (Wu and LeMone 1999).

These results, when considered in terms of organized convection, provide predictions for the preferred propagation direction of wave trains of convective systems (i.e., convectively coupled waves). In a particular background wind shear, if new convection is repeatedly formed on the more favorable side of preexisting convection, then a wave train of convective systems will take form. For the westerly wind burst-like shear in Figs. 4d–f, such a convectively coupled wave should then propagate westward because the west side of preexisting convection is more favorable. Notice that squall lines in this wind shear would be expected to propagate eastward (LeMone et al. 1998; Wu and LeMone 1999), which is in the opposite direction of the expected propagation of the convectively coupled wave. In fact, this arrangement, with the envelope propagating in the opposite direction of the convective systems within it, is what tends to be seen in observations and simulations (Nakazawa 1988; Grabowski and Moncrieff 2001; Tulich et al. 2007); the results shown here suggest that jet shears provide an ideal environment for this type of behavior. This preferred propagation direction of explicitly resolved convectively coupled waves in different wind shears was also captured in the recent model of Majda and Stechmann (2009), which uses a convective parameterization and does not resolve the squall lines in detail.

Figures 4g-i show results with a jet shear initial condition that roughly resembles the shear profiles of the westerly onset phase of the Madden–Julian oscillation (Lin and Johnson 1996; Biello and Majda 2005). In

TABLE 2. Numerical (num.) and analytical (anal.) predictions of the jump $[\theta_1]$ for different wind shears and different amplitudes of stratiform heating.

$(\mathrm{m} \mathrm{s}^{-1})$	$(m s^{-1})$	$-2a_2$ (K day ⁻¹)	$\begin{bmatrix} \theta_1 \end{bmatrix}$ (num.) (K)	$\begin{bmatrix} \theta_1 \end{bmatrix}$ (anal.) (K)
0	0	50	0	0
10	-10	50	1.29	1.42
-5	10	50	-1.03	-1.14
-9	-3	50	-0.16	-0.15
0	0	50	0	0
5	-5	50	0.65	0.71
10	-10	50	1.29	1.42
15	-15	50	1.93	2.14
20	-20	50	2.59	2.85
25	-25	50	3.29	3.56
10	-10	200	2.05	2.85
10	-10	100	1.59	1.90
10	-10	50	1.29	1.42
10	-10	25	1.13	1.19
10	-10	12	1.04	1.07

terms of vertical baroclinic modes, this case uses initial conditions of $u_1 = -5$ m s⁻¹ and $u_2 = 10$ m s⁻¹. The results in this case are qualitatively similar to the case in Figs. 4d–f, except the east side of the source is now more favorable for convection because an easterly jet is used for this case.

Figures 4j-l show results with an initial shear that roughly resembles those associated with midlatitude squall lines (Pandya and Durran 1996; Fovell 2002). In terms of vertical baroclinic modes, this case uses initial conditions of $u_1 = -9 \text{ m s}^{-1}$ and $u_2 = -3 \text{ m s}^{-1}$. Figure 4k shows that $\Theta(z)$ is nearly identical to the immediate west and east of the source, signifying that each side is nearly equally favorable for the formation of new convection, although Fig. 4l shows that the entire gravity wave response is not identical to the west and east of the source. In such a situation with equal favorability on each side, one would not expect wave trains of convective systems to develop because convection should form sometimes to the east and sometimes to the west of preexisting convection. This is consistent with the absence of wave trains of convective systems in the midlatitudes where shears like this are common, although rotational effects also suppress wave trains of squall lines in midlatitudes (Liu and Moncrieff 2004).

In summary, the results of Fig. 4 demonstrate that wind shear can (but does not necessarily) cause the east and west of the source to be unequally favorable for the formation of new convection. Notice that this asymmetry comes from two parts. In Fig. 4f, for example, there is greater warming by the +50 m s⁻¹ wave than by the -50 m s⁻¹ wave, and there is warming (cooling) in θ_1 due to the +25 m s⁻¹ (-25 m s⁻¹) wave. Both of these

effects involve interactions between the vertical modes and are not seen in the motionless case in Figs. 4a–c.

A comparison of these nonlinear numerical results with linear theory results using (12) is shown in Table 2. The numerical and theoretical values of $[\theta_1]$ are within 10% of each other for each of the cases from Fig. 4, indicating that linear theory provides a good estimate of the nonlinear results. The reason for this is that the wave and source amplitudes are weak in a suitable sense. Although a jump in θ_1 of 1–2 K is large in the sense that its impact on the formation of new convection is very important, it is small in a nondimensional sense with respect to the reference temperature $\overline{\alpha} = 15$ K. This response is ultimately determined by the strength of the source terms; and while the amplitude of the heating is a typically large value of $O(300 \text{ K day}^{-1})$, this heating is actually weak in terms of the nondimensional form of $S_i^* = a_i \sigma_i \sqrt{2\pi}$. This heating magnitude also includes the effect of the heating width σ_j , which is small in comparison to the longer far-field length scales of interest here.

Also shown in Table 2 are results of further nonlinear simulations with varying shear strength and varying stratiform heating strength (and corresponding results using linear theory). The jump $[\theta_1]$ varies approximately linearly with the strength of the wind shear for the cases like the westerly jet of Figs. 4d–f, with $[\theta_1]$ reaching values larger than 3 K for the strongest winds considered, and the linear theory values are again within 10% of the numerical values. As the strength of the stratiform heating decreases, with fixed deep convective heating, the jump $[\theta_1]$ decreases and linear theory agrees better with the nonlinear results.

Many of the results from Fig. 4 and Table 2 (and other results in this paper) can be seen directly from the exact linear formulas for the jumps, which are found by solving the system of equations in (12):

$$[u_1] = -\frac{2 - 3\overline{u}_1^2}{\text{denom}} S_1^* + \frac{6\overline{u}_1\overline{u}_2}{\text{denom}} S_2^*, \quad [u_2] = \frac{3\sqrt{2}\overline{u}_2}{\text{denom}} S_1^* - \frac{6\sqrt{2}\overline{u}_1}{\text{denom}} S_2^*, \quad \text{and}$$

$$[u_{2}] = -\frac{6\overline{u}_{1}\overline{u}_{2}}{\text{denom}}S_{1}^{*} - \frac{8 + 12\overline{u}_{2}^{2}}{\text{denom}}S_{2}^{*},$$
and $[\theta_{2}] = 0,$
(13)

where the denominator (denom) for these fractions is $2 - 3\overline{u}_1^2 + 3\overline{u}_2^2$. These formulas show that $[\theta_1] = 0$ for the shearless case of $\overline{u}_1 = \overline{u}_2 = 0$, which represents balance between the vertical velocity $[u_i]$ and the heating S_i^* . Also notice that $[\theta_2] = 0$ for any shear (although $[\theta_2] \neq 0$ for nonzero barotropic winds or for shears with higher vertical resolution, as shown below).

c. Effects of barotropic wind and propagating source

Several interesting effects emerge with a nonzero barotropic wind or a propagating heat source [which are equivalent, as discussed in (6)]. Figure 5 shows results of nonlinear simulations with $u_1 = u_2 = 0$ initially and with varying barotropic winds of $u_0 = 0, -5, -10,$ and -15 m s^{-1} . (Cases with $u_0 > 0$ are mirror images about x = 0 for this case with $u_1 = u_2 = 0$ initially.) The left column shows $\Theta(z)$ just to the west and east of the source, and the middle and right columns show snapshots of θ_1 and θ_2 , respectively, at time t = 4 h. Notice that the headwind confines the eastward-moving waves to the neighborhood of the source, and it helps the westward-moving waves travel farther from the source. Thus, the barotropic wind may affect whether the gravity waves are able to propagate far enough away from a preexisting convective system to excite a new convective system, or whether the gravity waves are held close enough to an individual convective system to excite new convective cells. Also notice that the barotropic wind creates a large jump in θ_2 . The large value of $[\theta_2]$ corresponds to a state to the east of the source that is very favorable for convection in the lower troposphere, as shown in the left column of Fig. 5 at x = +100 km by the solid lines. If the easterly barotropic wind is interpreted as an eastward-propagating heat source, then this represents a preconditioning of the upstream environment, which has been observed and studied in numerical simulations of squall lines (Fovell 2002; Fovell et al. 2006).

The large increase in $[\theta_2]$ as the barotropic wind becomes stronger is due to near-resonant forcing of the second baroclinic mode, which has a propagation speed of 25 m s⁻¹. This can be seen in the exact linear theory solutions to (12) for this case of zero baroclinic background winds and varying barotropic wind:

$$[u_{1}] = -\frac{1}{1 - u_{0}^{2}} S_{1}^{*}, \quad [u_{2}] = -\frac{1}{\frac{1}{4} - u_{0}^{2}} S_{2}^{*},$$

$$[\theta_{1}] = -\frac{u_{0}}{1 - u_{0}^{2}} S_{1}^{*}, \quad \text{and} \quad [\theta_{2}] = -\frac{u_{0}}{\frac{1}{4} - u_{0}^{2}} S_{2}^{*}.$$
(14)

The jumps $[\theta_j]$ are zero for $u_0 = 0$, but $[\theta_2]$ becomes large as u_0 approaches 25 m s⁻¹ (which is $\frac{1}{2}$ in nondimensional



FIG. 5. As in Fig. 4, but with shearless initial conditions with different barotropic wind $u_0 = 0, -5, -10$, and -15 m s^{-1} from top to bottom. (left) Vertical profile of potential temperature $\Theta(z)$ to the west (east) of the heat source at x = -100 km (x = +100 km) shown with a dashed (solid) line at time t = 4 h; (middle), (right) θ_1 and θ_2 , respectively, at time t = 4 h.

units). Notice that this effect is also significant in the θ_1 response, with Fig. 5 showing clear differences in the warming of θ_1 to the east and west of the source.

One nonlinear feature in Fig. 5k is the overshooting front that appears in θ_1 near x = 500 km as the barotropic

wind becomes stronger. At the same time, the downstream-propagating front in the region -1000 km < x < -500 km is depressed as the barotropic wind becomes stronger. These same trends have also been observed in numerical simulations of density currents (Liu and Moncrieff 1996). If there is a resultant change in the vertical velocity at the front, then this could change the triggering or suppressing effect of the front.

A comparison of the nonlinear numerical results with linear theory is shown in Table 3. The jumps in θ_2 are in agreement to within 10%, and the jumps in θ_1 show the same trends as u_0 varies but differ quantitatively, probably because of the nonlinear overshooting front discussed in the previous paragraph.

d. Effects of both wind shear and barotropic wind

The effect of wind shear in combination with a barotropic wind is shown in Fig. 6. The westerly wind burstlike shear from Figs. 4d–f, with $u_1 = 10$ and $u_2 =$ -10 m s^{-1} , is shown with three values of the barotropic wind: $u_0 = -10, 0, \text{ and } 10 \text{ m s}^{-1}$. These cases can also be interpreted in terms of different propagation speeds of the heat source. For instance, if a squall line formed in the wind shear in Fig. 6e and propagated eastward at the jet max speed of roughly 10 m s⁻¹, then the wind $\overline{U}(z)$ in Fig. 6a would be that seen in a reference frame moving with the squall line. This case displays a combination of the results from Figs. 4d-f and Figs. 5g-i, which showed the separate effects of a jet shear and an easterly barotropic wind, respectively. The upstream environment (x > 0) is much less favorable than the downstream environment (x < 0) for the formation of deep convection (because $[\theta_1] = 1.77$ K), but the upstream environment is favorable for low-level convection because $\Theta(z) <$ -1 K in the lower troposphere (Fig. 6b), which is a phenomena studied by Fovell (2002) and Fovell et al. (2006).

Another important effect of the barotropic wind in Fig. 6c is that it helps the downstream waves more quickly leave the environment of the preexisting convective system and reach locations where new convective systems could form. At the same time, it prevents the upstream waves from leaving the vicinity of the preexisting convection, which could prevent any new convection from forming a new convective system that is distinct from the preexisting system. These effects are likely important in the formation of wave trains of convective systems, which typically show a succession of distinct convective systems propagating transverse to the envelope of the wave train (Oouchi 1999; Shige and Satomura 2001; Tulich et al. 2007).

The case in Figs. 6i–l, with $u_0 = 10 \text{ m s}^{-1}$, is possibly a more realistic approximation to a westerly wind burst than the case in Fig. 6e because it has nonzero westerly winds at the lowest levels of the free troposphere (Lin and Johnson 1996; Biello and Majda 2005). The western side of the source in this case is much more favorable for convection than the eastern side (Figs. 6j–l), which is in agreement

TABLE 3. Numerical and analytical predictions of the jumps $[\theta_1]$ and $[\theta_2]$ across the location of the source. Initial conditions are shearless with varying values of barotropic wind u_0 .

$\frac{u_0}{(m \ s^{-1})}$	$\begin{bmatrix} \theta_1 \end{bmatrix}$ (num.) (K)	$\begin{bmatrix} \theta_1 \end{bmatrix}$ (anal.) (K)	$\begin{bmatrix} \theta_2 \end{bmatrix}$ (num.) (K)	$\begin{bmatrix} \theta_2 \end{bmatrix}$ (anal.) (K)
0	0	0	0	0
-5	0.12	0.23	-0.25	-0.23
-10	0.23	0.47	-0.57	-0.53
-15	0.27	0.73	-1.09	-1.03

with observations of the actual formation of new convection in similar wind shears (Wu and LeMone 1999).

These nonlinear results with the jet shear and varying barotropic wind are compared with linear theory solutions of (12) in Table 4. The results agree to within about 10% except for the cases with strong easterly barotropic wind, which appear to exhibit nonlinear overshooting fronts (Fig. 6c near x = 500 km). Interestingly, the cases with easterly (westerly) barotropic wind are more (less) nonlinear than the $u_0 = 0$ case with this jet shear, possibly due to the difference in the strength of the fast θ_1 fronts near x = -900 and +700 km in this jet shear in Fig. 6g.

Cases with the midlatitude squall line–like shear with different barotropic winds are shown in Fig. 7. The case that most resembles the reference frame moving with the squall line would be the left column of Fig. 7. In this case, the upstream side of the source is much more favorable at low levels (because of near-resonant forcing) and less favorable at upper levels. This type of thermo-dynamic environment is consistent with the formation of new low-level convective cells in the upstream environment as studied by Fovell (2002) and Fovell et al. (2006); because of the reduced speed of the upstream-propagating waves, these effects are confined to the neighborhood of the preexisting convection, which allows the new convective cells to become part of the preexisting convective system.

e. Optimal shears

Given that different wind shears lead to different values of $[\theta_1]$, as shown in Fig. 4, two questions come to mind: Which wind shears give the largest jump $[\theta_1]$? Which wind shears give $[\theta_1] = 0$? To answer these questions, the jump $[\theta_1]$ was computed using the exact formula from (13) for the family of two baroclinic mode wind shears satisfying $\overline{u}_1^2 + \overline{u}_2^2 = (10 \text{ m s}^{-1})^2$. Results are shown in Fig. 8 for three cases of the stratiform heating with the deep convective heating held fixed at its standard value. The jump $[\theta_1]$ is maximized for a jet shear (Fig. 8b) and is zero for midlatitude squall line–like shears (Fig. 8c). These results can also be seen from



FIG. 6. As in Fig. 4, but using initial jet shears with $u_0 = -10$, 0, and 10 m s⁻¹ from left to right. (top)–(bottom) Wind profile used as the initial conditions; vertical profile of potential temperature $\Theta(z)$ to the west (east) of the heat source at x = -100 km (x = +100 km) shown with a dashed (solid) line at time t = 4 h; θ_1 at time t = 4 h; and θ_2 at time t = 4 h.

the linear theory formulas in (13). Assuming $S_j^* = -4S_2^*$ as for the standard heating profile used here, and assuming that the \overline{u}_j^2 terms in the denominator are negligible, the formula in (13) for $[\theta_1]$ is approximately

$$[\theta_1] \approx (2\overline{u}_2 - \overline{u}_1) 6\sqrt{2S_1^*}.$$
(15)

This will be largest when \overline{u}_1 and \overline{u}_2 have different signs (as in the jet shears in Fig. 4) and it will be smallest in magnitude when \overline{u}_1 and \overline{u}_2 have the same sign (as in the midlatitude squall line–like shear). In the next section, these results are discussed in terms of the physical vertical coordinate z rather than vertical modes.

TABLE 4. As in Table 3, but initial conditions include a jet shear with $\overline{u}_1 = 10$ and $\overline{u}_2 = -10$ m s⁻¹.

$\frac{u_0}{(m s^{-1})}$	$\begin{bmatrix} \theta_1 \end{bmatrix}$ (num.) (K)	$\begin{bmatrix} \theta_1 \end{bmatrix}$ (anal.) (K)	$\begin{bmatrix} \theta_2 \end{bmatrix}$ (num.) (K)	$\begin{bmatrix} \theta_2 \end{bmatrix}$ (anal.) (K)
-15	2.02	3.29	-1.15	-1.44
-10	1.77	2.31	-0.69	-0.68
-5	1.50	1.78	-0.30	-0.28
0	1.29	1.42	0	0
5	1.14	1.18	0.28	0.27
10	1.06	1.02	0.62	0.59
15	1.03	1.00	0.98	1.15

4. Physical mechanisms

In this section the physical mechanisms of the eastwest asymmetries are summarized in terms of vertical baroclinic mode behavior. Then these mechanisms are discussed in terms of the physical vertical coordinate zrather than vertical modes, and the link with advection by the background wind can be seen more clearly.

The results in the previous section with two baroclinic modes clearly demonstrate the effects of wind shear and barotropic wind on diabatically forced gravity waves. The effect of wind shear is mainly to create east-west asymmetries in θ_1 . This asymmetry comes from two effects: (i) differences in the deep warming of the $50 \,\mathrm{m \, s^{-1}}$ waves to the east and west of the source and (ii) differences in the θ_1 contributions to the 25 m s⁻¹ waves. The effects of barotropic wind are mainly (i) to increase the low-level cooling on the upstream side due to nearresonant forcing, (ii) to create east-west asymmetries in the θ_1 response, and (iii) to slow down (speed up) the propagation of upstream (downstream) waves. These effects have important implications for the formation of organized convection (both new convective cells and new convective systems), and these effects were highlighted throughout the previous section. Because these effects are captured well by linear theory, the underlying mechanisms are presumably due to advection by the background wind, which is described in the following discussion.

To see more clearly how the background wind creates east–west asymmetries in diabatically forced gravity waves, consider the steady-state linear setup of (11)–(12)in terms of the physical vertical coordinate *z* instead of vertical modes. The horizontal momentum and potential temperature equations take the form

$$-\overline{U}(z)\frac{dW^*}{dz} + W^*\frac{d\overline{U}}{dz} + [P] = 0, \quad \text{and} \qquad (16)$$

$$\overline{U}(z) \frac{d[P]}{dz} + W^* = S^*(z), \qquad (17)$$

after applying the hydrostatic and continuity equations, $[\Theta] = d[P]/dz$ and $[U] = -dW^*/dz$. This setup is described in more detail in appendix B. These two equations can be combined to give a single second-order ordinary differential equation for [P](z):

$$\overline{U}^2 \frac{d^2[P]}{dz^2} + [P] = \overline{U} \frac{dS^*}{dz} - S^* \frac{d\overline{U}}{dz}.$$
 (18)

This can be thought of as the physical space analog of the jump conditions in (12) for the vertical modes. In fact, if one assumes vertical structures with only two baroclinic modes, the right-hand side of (18) is equivalent to the leading-order terms (ignoring the \overline{u}_j^2 terms) of the formulas for $[\theta_1]$ and $[\theta_2]$ in (13). This suggests that the leading-order part of (18) is the simple formula

$$[P] \approx \overline{U} \, \frac{dS^*}{dz} - S^* \, \frac{d\overline{U}}{dz},\tag{19}$$

which is a concise description of the east-west asymmetry of diabatically forced gravity waves due to background wind shear.

There is an equivalent, alternative way to arrive at (19) that draws the connection with advection. If the dominant balance in (17) is $W^* = S^*(z)$, then (19) is simply the horizontal momentum Eq. (16). Thus, $-\overline{U}S_z^*$ and $S^*\overline{U}_z$ are simply the terms for advection of diabatically forced horizontal momentum by the background wind $\overline{U}(z)$.

While the simplified Eq. (19) appears to capture the leading-order effects of the solutions with truncated vertical structures, one must be careful about taking the limit of small \overline{U}^2 in (18); this is a singular limit, and the limiting equations could thus have qualitatively different solutions than the original equation. In fact, this appears to be the case in this situation: the equation in (18) allows for the possibility of critical layers when $\overline{U}(z) = 0$ at some height, but the simplified formula in (19) does not. The absence of critical layer effects also seems to be a property of the jump conditions (12) for the vertically truncated system. This is likely due to the model's low vertical resolution, since critical layer effects typically occur on small vertical length scales and are associated with high-frequency gravity waves with significant vertical energy propagation (Booker and Bretherton 1967; Lindzen and Tung 1976; Lin 1987; Shige and Satomura 2001).

In short, the simplified Eq. (19) and the truncated model (11)–(12) effectively filter out critical layer effects by considering only low-frequency, horizontally propagating



FIG. 7. As in Fig. 6, but using initial shear similar to the environment of a midlatitude squall line with $u_0 = (\text{left}) - 10$, (middle) 0, and (right) 10 m s⁻¹.

waves. These are useful approximations for the purposes of the present paper, since the main focus is on the farfield response to diabatic forcing. Critical layer effects could be studied in the future using (18) and the associated forced Taylor–Goldstein equation shown in appendix B. Diabatically forced Taylor–Goldstein equations, both nonhydrostatic and hydrostatic, were also recently derived by Majda and Xing (2009). In terms of implications for organized convection, it was found by Shige and Satomura (2001) that critical layers play an important role by creating a wave duct, beneath which high-frequency gravity waves can propagate without losing too much energy to the stratosphere. This is not inconsistent with the results of the present paper; here the focus is on the far-field effects of low-frequency gravity waves that are forced by the



FIG. 8. Linear theory calculations of $[\theta_1]$ for the family of wind shears with $u_0 = 0$ and $\overline{u}_1^2 + \overline{u}_2^2 = (10 \text{ m s}^{-1})^2$; (a) $[\theta_1]$ as a function of \overline{u}_1 ; (b) wind shear $\overline{U}(z)$, which maximizes $[\theta_1]$; (c) wind shear $\overline{U}(z)$ for which $[\theta_1] = 0$. Plots are shown for three different values of the stratiform heating amplitude: $2a_2 = -25$ (dashed–dotted), -50 (dashed), and -75 K day⁻¹ (solid).

mesoscale filtered heating of a convective system rather than on the high-frequency waves forced by fluctuations of convective cells. Both high- and low-frequency gravity waves are important for triggering or favoring the formation of new convection, with high- (low) frequency waves probably most important for triggering (favoring). Lowfrequency gravity waves, unlike high-frequency waves, can propagate long distances horizontally without losing too much energy to the stratosphere (Shige and Satomura 2001). Several studies have shown that the tropopause itself is enough to trap low-frequency waves in the troposphere (Mapes 1993, 1998; Xue 2002; Liu and Moncrieff 2004), and the use of an upper rigid lid in this paper is meant to be an approximation of this effect, but it could also be a stand-in for a wave duct. In summary, both high- and low-frequency waves are important for convective organization, but the main focus of the present paper is on low-frequency waves, in which case critical layers are not an essential effect.

Many other assumptions were made in the models here that should be kept in mind when interpreting the results, such as hydrostatic balance, two-dimensional flow, and the simple imposed form of the convective heating. These assumptions have been commonly used in the earlier studies mentioned in the introduction as well; the reader is referred to these references for further discussion. The effects of higher vertical modes are also not studied in detail here (but some results are shown in the next section). Other studies have shown that the third baroclinic mode may play an important role in convective organization (Lane and Reeder 2001; Tulich and Mapes 2008). This might be explained by convective forcing of the third baroclinic mode due to convection with lower cloud-top heights. It is also possible that the effect of the higher baroclinic modes, in the context of wave trains of convective systems, is limited by their slow propagation speeds, which confine them to the close neighborhood of the source.

In any case, the simplified model used here is not meant to capture every detail of this problem; it is meant to isolate a few important effects and demonstrate the basic effects of wind shear and barotropic wind in a simplified setting. Indeed, one important strength of this model is its simplicity, which allows clear, inexpensive computation of a wide range of possibilities as well as analytical understanding. This simplified model and the results shown here should help guide further studies that include additional physical effects.

5. Sensitivity studies with higher vertical resolution

In this section results are shown using the linear theory from (12) extended to include the effects of the third and fourth baroclinic modes.

a. Sensitivity to number of vertical modes

Earlier results in this paper emphasize that wind shear and/or nonlinearities lead to interactions between vertical modes. One might then expect that higher vertical modes beyond the first two modes would be excited even if the source term includes contributions from only the first two modes. For instance, the 25 and 50 m s⁻¹ waves might include contributions from higher vertical modes, or wind shear could cause the 12 and 17 m s⁻¹ waves to be excited in the presence of wind shear even if the source term includes contributions from only the first two modes.

To check that the results of section 3 do not change much when the gravity wave response is allowed to include more vertical modes, several cases are repeated here with the same wind shears and source terms with only two vertical modes, but with a wave response that can include contributions from the third and fourth vertical modes. This is done by computing the jumps $[\theta_j]$ using the linear theory (12) with an advection matrix $\mathbf{A}(\mathbf{\overline{u}})$ that is 6×6 and 8×8 for the cases including

TABLE 5. Linear theory calculations with two, three, and four baroclinic modes for the case of a jet shear with $\overline{u}_1 = 10$ and $\overline{u}_2 = -10 \text{ m s}^{-1}$. All values are in K.

modes	$[heta_1]$	$[\theta_2]$	$[\theta_3]$	$[heta_4]$
2	1.42	0	_	_
3	1.58	-0.14	-0.10	_
4	1.69	-0.22	-0.07	-0.10

responses in modes up to the third and fourth, respectively. These larger advection matrices are given in appendix A. Table 5 shows a comparison of linear theory results with two, three, and four baroclinic modes for the westerly wind burst-like wind shear with $\overline{u}_1 = 10$ and $\overline{u}_2 = -10$ m s⁻¹. Although the jumps $[\theta_j]$ change depending on the number of modes, the changes are not drastic. Similar results for the wind shear that resembles conditions for midlatitude squall lines (with $u_1 = -9$ and $u_1 = -3$ m s⁻¹) are shown in Table 6. Again, although the jumps $[\theta_j]$ change depending on the number of modes, the changes are not drastic. These results suggest that the simple case with two modes provides a reasonable approximation to $[\Theta](z)$ when the wind shear and source term are composed of two modes.

b. Sensitivity to jet height

Up to this point, the wind shear and source term included contributions from only the first and second vertical modes. However, the wind shear profiles associated with squall lines typically have lower tropospheric shear that cannot be adequately represented by the first two baroclinic modes (Pandya and Durran 1996; Fovell 2002; Majda and Xing 2009). In this section, background wind shears will be used with contributions from the third and fourth baroclinic modes, so wind shear profiles with jets in the lower troposphere can be used. The heat source, however, will still be the standard case from Fig. 2 with contributions from only two vertical modes. As in the previous subsection, the linear theory results (12) will use the advection matrix $A(\overline{u})$ with four vertical modes, which means that the gravity wave response includes contributions from the third and fourth vertical modes.

The top row of Fig. 9 illustrates three families of background shears. These families are the projections onto four baroclinic modes of wind profiles with constant shear of 2.5 m s⁻¹ km⁻¹ in the lower troposphere up to heights of 8, 6, or 4 km. The mid- and upper-tropospheric winds above this height also have constant shear, and results are computed for a range of upper-tropospheric shears that includes jet shears, midlatitude squall line–like shears, and constant shears. These shear profiles are

TABLE 6. As in Table 5, but for the case of a midlatitude squall line–like shear with $\overline{u}_1 = -9$ and $\overline{u}_2 = -3$ m s⁻¹.

		1	2	
Number of modes	$[heta_1]$	$[\theta_2]$	$[\theta_3]$	$[heta_4]$
2	-0.15	0	_	_
3	-0.19	-0.01	-0.25	
4	-0.21	-0.04	-0.34	-0.01

similar to those used by Liu and Moncrieff (2001). Whereas the top row of Fig. 9 shows $\overline{U}(z)$ including a barotropic component, the calculations of $[\theta_j]$ use zero barotropic wind.

The middle row of Fig. 9 shows the jumps $j[\theta_i]$ for these different wind shears. [The factor *j* is included in $i[\theta_i]$ to give each mode's vertical structure function an amplitude of 1; see the expansion in (1).] The circles represent the values of $j[\theta_i]$ corresponding to the shear profiles in the top row of Fig. 9. For each shear family, $[\theta_1] > 0$ for the jet shear, $[\theta_1] \approx 0$ for the midlatitude squall line–like shear, and $[\theta_1] < 0$ for the constant shear. These results are consistent with the two-mode results with similar shears in Fig. 4. The jumps $2[\theta_2]$ and $4[\theta_4]$ are relatively small for all of these shears, and they do not vary much for the different shears considered here. The jump $3[\theta_3]$ also changes little except for the different jet profiles; it is negative for the midtropospheric jet in Fig. 9b and positive for the lowertropospheric jet in Fig. 9h. Although there are some changes as the jet height changes, the jumps $[\Theta](z)$ are generally determined by $[\theta_1]$, as shown in the middle row of Fig. 9. No matter what the jet height is, it is approximately true that $[\Theta](z) > 0$ for the jet profiles, $[\Theta](z) \approx 0$ for the midlatitude squall line–like profiles, and $[\Theta](z) < 0$ for the linear profiles, which is the trend shown in $[\theta_1]$.

6. Conclusions

The effects of wind shear and barotropic wind on gravity waves were studied with a focus on the implications for organized convection. It was shown that because of advection by a background wind shear or barotropic wind, eastward- and westward-propagating gravity waves can have different properties, and this asymmetry can create differences in the environments to the east and west of a heat source. Therefore, because of wind shear or barotropic wind, one side of preexisting convection can be more favorable for the formation of new convection than the other side. These effects were discussed in the context of two settings for convective organization: the formation of new convective systems near a



FIG. 9. Linear theory calculations of $j[\theta_i]$ for three different families of wind shears with jet heights of 8, 6, and 4 km from the left column to the right column. (top) Sample wind shears from each family; (middle) $j[\theta_i]$ for a range of upper tropospheric wind shears: $[\theta_1]$ (solid), $2[\theta_2]$ (dashed), $3[\theta_3]$ (dashed–dotted), and $4[\theta_4]$ (dotted); (bottom) $[\Theta](z)$ for the cases of a jet profile (solid) and linear $\overline{U}(z)$ (dashed).

preexisting convective system and the formation of new convective cells of a single convective system.

Nonlinear and linear models showed that the greatest difference in favorability between the two sides is created by jet shears, and little or no difference in favorability is created by wind profiles with shear at low levels and no shear in the upper troposphere. These results did not appear to be sensitive to the height of the jet, whether it is in the lower or middle troposphere. The cases with jet shear similar to the westerly wind burst phase of the Madden-Julian oscillation were in agreement with actual observations of the formation of new convection during that period. These processes should play an important role in the organization of wave trains of convective systems. For instance, they provide predictions for the preferred propagation direction of wave trains of convective systems in different background wind shears. Also, midlatitude squall lines typically propagate in an environment with shear at low levels and no shear in the upper troposphere (Pandya and Durran 1996; Fovell 2002) and are often isolated. The results here are broadly consistent with these facts, although the effect of rotation also suppresses wave trains of squall lines in midlatitudes (Liu and Moncrieff 2004).

A nonzero barotropic wind (or, equivalently, a propagating heat source) was shown to also affect the favorability on each side of the preexisting convection. The barotropic wind was shown to amplify the strength of upstream gravity waves that are known to trigger new convective cells within a single convective system. The strong amplification of these waves seemed to be the result of near-resonant forcing as the source propagation speed approached the speed of second baroclinic mode gravity waves. The barotropic wind also had the effect of advecting downstream waves farther away from the source, where they have a greater chance of triggering or favoring a new convective system, and confining upstream waves to the vicinity of the source, where they have a greater chance of triggering a new convective cell that will merge with the preexisting convective system. In addition, the nonlinear simulations revealed overshooting fronts in the presence of a barotropic headwind, which has also been seen in two-dimensional simulations of density currents.

All of these effects are captured in a two-dimensional model that is further simplified by including only the first two vertical baroclinic modes. Numerical results were shown with a nonlinear model, and linear theory results were in good agreement with the nonlinear model for most cases. One important advantage of the vertically truncated model used here is that it is much less expensive computationally than models with full vertical resolution, and, in some cases, conceptually simpler as well.

Acknowledgments. The authors thank two anonymous reviewers for helpful suggestions. S. S. is supported by a NSF Mathematical Sciences Postdoctoral Research Fellowship. The research of A. M. is partially supported by Grants NSF DMS-0456713 and ONR N0014-08-0284.

APPENDIX A

Model Details

a. Model in matrix form

For the nonlinear model in matrix form (5), the advection matrix is

$$\mathbf{A}(\mathbf{u}) = \begin{pmatrix} u_0 + \frac{3}{\sqrt{2}}u_2 & -1 & \frac{3}{2\sqrt{2}}u_1 & 0\\ 1 + 2\sqrt{2}\theta_2 & u_0 - \frac{1}{\sqrt{2}}u_2 & -\frac{1}{2\sqrt{2}}\theta_1 & \sqrt{2}u_1\\ 0 & 0 & u_0 & -1\\ -\frac{1}{2\sqrt{2}}\theta_1 & \frac{1}{2\sqrt{2}}u_1 & -\frac{1}{4} & u_0 \end{pmatrix}.$$
(A1)

Similarly, for the case with three baroclinic modes in section 5, the advection matrix is

$$\mathbf{A}(\mathbf{u}) = \begin{pmatrix} u_0 + \frac{3}{\sqrt{2}}u_2 & -1 & \frac{3}{2\sqrt{2}}u_1 + \frac{5}{2\sqrt{2}}u_3 & 0 & \frac{5}{3\sqrt{2}}u_2 & 0 \\ -1 + 2\sqrt{2}\theta_2 & u_0 - \frac{1}{\sqrt{2}}u_2 & -\frac{1}{2\sqrt{2}}\theta_1 + \frac{9}{2\sqrt{2}}\theta_3 & \sqrt{2}(u_1 - u_3) & -\frac{2\sqrt{2}}{3}\theta_2 & \frac{3}{\sqrt{2}}u_2 \\ 2\sqrt{2}u_3 & 0 & u_0 & -1 & \frac{2\sqrt{2}}{3}u_1 & 0 \\ -\frac{1}{2\sqrt{2}}\theta_1 + \frac{9}{2\sqrt{2}}\theta_3 & \frac{1}{2\sqrt{2}}(u_1 - u_3) & -\frac{1}{4} & u_0 & -\frac{1}{6\sqrt{2}}\theta_1 & \frac{3}{2\sqrt{2}}u_1 \\ -\frac{1}{\sqrt{2}}u_2 & 0 & \frac{1}{2\sqrt{2}}u_1 & 0 & u_0 & -1 \\ -\frac{2\sqrt{2}}{3}\theta_2 & \frac{1}{3\sqrt{2}}u_2 & -\frac{1}{6\sqrt{2}}\theta_1 & \frac{\sqrt{2}}{3}u_1 & -\frac{1}{9} & u_0 \end{pmatrix}.$$
(A2)

These matrices are derived by projecting the equations of motion onto the different baroclinic modes, as explained by SMK08. The 8×8 advection matrix for the case with four baroclinic modes can be derived in the same way; its explicit form is available from the authors upon request.

b. Representation of convective heating

The use of a delta function heating in (9) is essentially a far-field approximation; that is, we are interested in the state of the model variables far away from the source, relative to certain length and time scales as discussed below. Quantitatively, this approximation can be illustrated by considering the model in (5) with a source term $\mathbf{S}(x)$ that varies with the length scale x. The state of the system far from the source is described in terms of larger distances and longer times $x' = \varepsilon x$ and $t' = \varepsilon t$, where $0 < \varepsilon \ll 1$. Rewriting (5) with source term $\mathbf{S}(x)$ in terms of these longer scales gives

$$\partial_{t'} \mathbf{u} + \mathbf{A}(\mathbf{u}) \partial_{x'} \mathbf{u} = \varepsilon^{-1} \mathbf{S}(\varepsilon^{-1} x').$$
 (A3)

In the limit $\varepsilon \to 0$ for far-field length and time scales, this source term has the appearance of a delta function:

$$\varepsilon^{-1}\mathbf{S}(\varepsilon^{-1}x') \approx \mathbf{S}^*\delta(x'),$$
 (A4)

where $\mathbf{S}^* = \int \mathbf{S}(x) dx$, as in (9).

The relevance of this approximation to the model used here can be seen by considering the model's space and time scales. The largest length scale of the source term in (8) is the standard deviation σ_2 , and the important time scale is σ_2/c_2 , where c_2 is the second baroclinic mode gravity wave speed (the slowest gravity wave speed of the model). These scales characterize how long it takes gravity waves to propagate sufficiently far from the source, leaving behind the far-field state of interest. As illustrated in all of the figures here, the length and time scales of interest are always sufficiently large relative to σ_2 and σ_2/c_2 (with the possible exception of cases when u_0 approaches c_2). If higher baroclinic modes were included, this approximation could become questionable because of the slower propagation speed of these modes.

Note that the far-field approximation also clusters nearby heat sources into one heat source at a single location. If the heat source is the sum of two phase-lagged heat sources, $\mathbf{S} = \mathbf{S}(x) + \mathbf{S}(x + x_0)$, then these heat sources would be located at the same location in the limit $\varepsilon \to 0$ if x_0 is a small phase shift (i.e., if $x_0 \ll \sigma$). Therefore, the deep convective and stratiform parts of the convective heating are at the same location in the far-field approximation. This ignores the circulations produced by variations in the heat sources as studied by Pandya and Durran (1996) and only reveals the jumps in the variables across the location of the entire heat source.

The far-field approximation allows simple computation of the jumps [**u**] from (12), which is an exact formula for the far-field jumps for linear dynamics in the limit $\varepsilon \rightarrow 0$. For nonlinear dynamics, however, the approximation is less exact because of breaking waves and the associated energy dissipation, which is illustrated in Tables 3 and 4.

c. Exact linear solutions

In section 2c the linearized system with a delta function source term in (11) was considered, and the Rankine– Hugoniot jump conditions for the jumps across the location of the source were given in (12). For this system, it is also possible to compute the entire exact solution including the speeds and amplitudes of the propagating discontinuities (Evans 1998; LeVeque 2002). One way to find the solution is to decompose the matrix problem in (11) into scalar problems associated with each of the eigenvectors \mathbf{e}_p of \mathbf{A} , where p = 1, 2, 3, 4. The solution $\mathbf{u}(x, t)$ can be written as $\mathbf{u}(x, t) = \sum_{p=1}^{4} \alpha_p(x, t) \mathbf{e}_p$, where $\alpha_p(x, t)$ satisfies the scalar problem $\partial_t \alpha_p + \lambda_p \partial_x \alpha_p =$ $S_p^* \delta(x)$, where λ_p is the *p*th eigenvalue of \mathbf{A} and S_p^* is the projection of \mathbf{S}^* onto \mathbf{e}_p . The solution to this scalar problem is

$$\alpha_p(x,t) = \begin{cases} 0 & \text{if } x < 0, \\ \frac{1}{\lambda_p} S_p^* & \text{if } 0 < x < \lambda_p t, \\ 0 & \text{if } x > \lambda_p t, \end{cases}$$
(A5)

if $\lambda_p > 0$ and similarly if $\lambda_p < 0$. Therefore, the state $\mathbf{u}(x, t)$ at a particular space–time location (x, t) is the sum of $\alpha_p \mathbf{e}_p$ for each wave that has passed location x before time t.

APPENDIX B

Arbitrary Vertical Structure

Throughout this paper, truncated vertical structures were used with only a few vertical modes. Linear theory for these cases had the form of Rankine–Hugoniot jump conditions in matrix form, as shown in (12). In this appendix, instead of using a finite number of vertical modes, we derive the jump conditions for arbitrary vertical structures as functions of the physical coordinate z. Instead of matrix equations, the jump condition will be an ordinary differential equation for $[\Theta](z)$.

Our starting point is the steady-state linearized hydrostatic Boussinesq equations with a source term whose xdependence takes the form of a delta function:

$$\overline{U}(z)U_{x} + W \frac{d\overline{U}}{dz} + P_{x} = 0$$

$$\overline{U}(z)\Theta_{x} + W = S^{*}(z)\delta(x) \qquad (B1)$$

$$P_{z} = \Theta$$

$$U_{x} + W_{z} = 0.$$

Assume a solution to these equations of the form

$$U(x, z) = -\frac{dW^*}{dz} \mathcal{H}(x), \qquad W(x, z) = W^*(z)\delta(x),$$

$$\Theta(x, z) = \frac{d[P]}{dz} \mathcal{H}(x), \quad \text{and} \quad P(x, z) = [P](z)\mathcal{H}(x),$$
(B2)

where $\mathcal{H}(x)$ is the Heaviside function and the hydrostatic and incompressibility equations were used to phrase these solutions in terms of $W^*(z)$ and [P](z) alone.

With this solution form, and dropping the common $\delta(x)$ from each term, the $W^*(z)$ and [P](z) equations become

$$-\overline{U}(z) \frac{dW^*}{dz} + W^* \frac{d\overline{U}}{dz} + [P] = 0$$

$$\overline{U}(z) \frac{d[P]}{dz} + W^* = S^*(z).$$
(B3)

These two equations can be combined to give a single second-order ordinary differential equation for [P](z):

$$\overline{U}^2 \frac{d^2[P]}{dz^2} + [P] = \overline{U} \frac{dS^*}{dz} - S^* \frac{d\overline{U}}{dz}.$$
 (B4)

From this equation, $[\Theta](z)$ could then be computed using (B2). Alternatively, one could write a single differential equation for $W^*(z)$, which takes the form of a forced Taylor–Goldstein equation (Hazel 1972):

$$\overline{U}^2 \, \frac{d^2 W^*}{dz^2} + \left(1 - \overline{U} \, \frac{d^2 \overline{U}}{dz^2}\right) W^* = S^*. \tag{B5}$$

Other versions of forced nonhydrostatic and hydrostatic Taylor–Goldstein equations were also recently studied by Majda and Xing (2009). Although some discussion of physical mechanisms is provided in section 4, deeper examination of these equations is beyond the scope of this paper, but they should be useful for further studies.

REFERENCES

- Biello, J. A., and A. J. Majda, 2005: A new multiscale model for the Madden–Julian oscillation. J. Atmos. Sci., 62, 1694–1721.
- Booker, J. R., and F. P. Bretherton, 1967: The critical layer for internal gravity waves in a shear flow. J. Fluid Mech., 27, 513–539.
- Bretherton, C. S., and P. K. Smolarkiewicz, 1989: Gravity waves, compensating subsidence, and detrainment around cumulus clouds. J. Atmos. Sci., 46, 740–759.
- Emanuel, K. A., 1994: Atmospheric Convection. Oxford University Press, 580 pp.
- Evans, L., 1998: Partial Differential Equations. American Mathematical Society, 662 pp.
- Fovell, R., 2002: Upstream influence of numerically simulated squall-line storms. Quart. J. Roy. Meteor. Soc., 128, 893–912.
- —, 2005: Convective initiation ahead of the sea-breeze front. Mon. Wea. Rev., 133, 264–278.
- —, G. Mullendore, and S.-H. Kim, 2006: Discrete propagation in numerically simulated nocturnal squall lines. *Mon. Wea. Rev.*, 134, 3735–3752.
- Grabowski, W. W., and M. W. Moncrieff, 2001: Large-scale organization of tropical convection in two-dimensional explicit numerical simulations. *Quart. J. Roy. Meteor. Soc.*, **127**, 445–468.
- Hazel, P., 1972: Numerical studies of the stability of inviscid stratified shear flows. J. Fluid Mech., 51, 39–62.
- Houze, R. A., 2004: Mesoscale convective systems. *Rev. Geophys.*, 42, RG4003, doi:10.1029/2004RG000150.
- Khouider, B., and A. J. Majda, 2006: A simple multicloud parameterization for convectively coupled tropical waves. Part I: Linear analysis. J. Atmos. Sci., 63, 1308–1323.
- Kiladis, G. N., M. C. Wheeler, P. T. Haertel, K. H. Straub, and P. E. Roundy, 2009: Convectively coupled equatorial waves. *Rev. Geophys.*, 47, RG2003, doi:10.1029/2008RG000266.
- Lac, C., J. Lafore, and J. Redelsperger, 2002: Role of gravity waves in triggering deep convection during TOGA COARE. J. Atmos. Sci., 59, 1293–1316.

- Lane, T., and M. Reeder, 2001: Convectively generated gravity waves and their effect on the cloud environment. J. Atmos. Sci., 58, 2427–2440.
- LeMone, M., E. Zipser, and S. Trier, 1998: The role of environmental shear and thermodynamic conditions in determining the structure and evolution of mesoscale convective systems during TOGA COARE. J. Atmos. Sci., **55**, 3493–3518.
- LeVeque, R. J., 2002: *Finite Volume Methods for Hyperbolic Problems*. Cambridge University Press, 558 pp.
- Lin, X., and R. H. Johnson, 1996: Kinematic and thermodynamic characteristics of the flow over the western Pacific warm pool during TOGA COARE. J. Atmos. Sci., 53, 695–715.
- Lin, Y.-L., 1987: Two-dimensional response of a stably stratified shear flow to diabatic heating. J. Atmos. Sci., 44, 1375–1393.
- Lindzen, R. S., and K.-K. Tung, 1976: Banded convective activity and ducted gravity waves. *Mon. Wea. Rev.*, **104**, 1602–1617.
- Liu, C., and M. Moncrieff, 1996: A numerical study of the effects of ambient flow and shear on density currents. *Mon. Wea. Rev.*, 124, 2282–2303.
- —, and —, 2001: Cumulus ensembles in shear: Implications for parameterization. J. Atmos. Sci., 58, 2832–2842.
- —, and —, 2004: Effects of convectively generated gravity waves and rotation on the organization of convection. J. Atmos. Sci., 61, 2218–2227.
- Majda, A. J., 2003: Introduction to PDEs and Waves for the Atmosphere and Ocean. Courant Lecture Notes in Mathematics, Vol. 9, American Mathematical Society, 234 pp.
- —, and M. G. Shefter, 2001: Models for stratiform instability and convectively coupled waves. J. Atmos. Sci., 58, 1567–1584.
- —, and S. N. Stechmann, 2009: A simple dynamical model with features of convective momentum transport. J. Atmos. Sci., 66, 373–392.
- —, and Y. Xing, 2009: New multi-scale models on mesoscales and squall lines. *Comm. Math. Sci.*, in press.
- Mapes, B. E., 1993: Gregarious tropical convection. J. Atmos. Sci., 50, 2026–2037.
- —, 1998: The large-scale part of tropical mesoscale convective system circulations: A linear vertical spectral band model. J. Meteor. Soc. Japan, 76, 29–55.
- —, 2000: Convective inhibition, subgrid-scale triggering energy, and stratiform instability in a toy tropical wave model. J. Atmos. Sci., 57, 1515–1535.
- McAnelly, R., J. Nachamkin, W. Cotton, and M. Nicholls, 1997: Upscale evolution of MCSs: Doppler radar analysis and analytical investigation. *Mon. Wea. Rev.*, **125**, 1083–1110.
- Moncrieff, M., and C. Liu, 1999: Convection initiation by density currents: Role of convergence, shear, and dynamical organization. *Mon. Wea. Rev.*, **127**, 2455–2464.
- Nakazawa, T., 1988: Tropical super clusters within intraseasonal variations over the western Pacific. J. Meteor. Soc. Japan, 66, 823–839.
- Nicholls, M., R. Pielke, and W. Cotton, 1991: Thermally forced gravity waves in an atmosphere at rest. J. Atmos. Sci., 48, 1869– 1884.
- Oouchi, K., 1999: Hierarchical organization of super cloud cluster caused by WISHE. Convectively induced gravity waves and cold pool. J. Meteor. Soc. Japan, 77, 907–927.
- Pandya, R. E., and D. R. Durran, 1996: The influence of convectively generated thermal forcing on the mesoscale circulation around squall lines. J. Atmos. Sci., 53, 2924–2951.
- —, —, and C. Bretherton, 1993: Comments on "Thermally forced gravity waves in an atmosphere at rest." *J. Atmos. Sci.*, **50**, 4097–4101.

- Shige, S., and T. Satomura, 2001: Westward generation of eastwardmoving tropical convective bands in TOGA COARE. J. Atmos. Sci., 58, 3724–3740.
- Stechmann, S. N., A. J. Majda, and B. Khouider, 2008: Nonlinear dynamics of hydrostatic internal gravity waves. *Theor. Comput. Fluid Dyn.*, 22, 407–432.
- Tompkins, A., 2001: Organization of tropical convection in low vertical wind shears: The role of cold pools. J. Atmos. Sci., 58, 1650–1672.
- Tulich, S. N., and B. Mapes, 2008: Multiscale convective wave disturbances in the tropics: Insights from a two-dimensional cloud-resolving model. J. Atmos. Sci., 65, 140–155.
- —, D. Randall, and B. Mapes, 2007: Vertical-mode and cloud decomposition of large-scale convectively coupled gravity waves in a two-dimensional cloud-resolving model. *J. Atmos. Sci.*, 64, 1210–1229.
- Wu, X., and M. LeMone, 1999: Fine structure of cloud patterns within the intraseasonal oscillation during TOGA COARE. *Mon. Wea. Rev.*, **127**, 2503–2513.
- Xue, M., 2002: Density currents in shear flows: Effects of rigid lid and cold-pool internal circulation, and application to squall line dynamics. *Quart. J. Roy. Meteor. Soc.*, **128**, 47–73.
- Zhang, C., 2005: Madden–Julian oscillation. *Rev. Geophys.*, **43**, RG2003, doi:10.1029/2004RG000158.