

# Stochastic PDEs for Tropical Weather and Climate

Samuel N. Stechmann<sup>1</sup>  
University of Wisconsin–Madison

May 11, 2016

Tropical weather and climate is different than in the midlatitudes, where most of the United States is located. Yes, it's warmer in the tropics. But the cloud patterns and rainfall are also organized in different ways, as shown in Figure 1. Over the United States and Eurasia, elongated clouds form along fronts, familiar from weather maps. In contrast, near the equator, fronts are absent, and the clouds instead appear to be clustered more randomly.

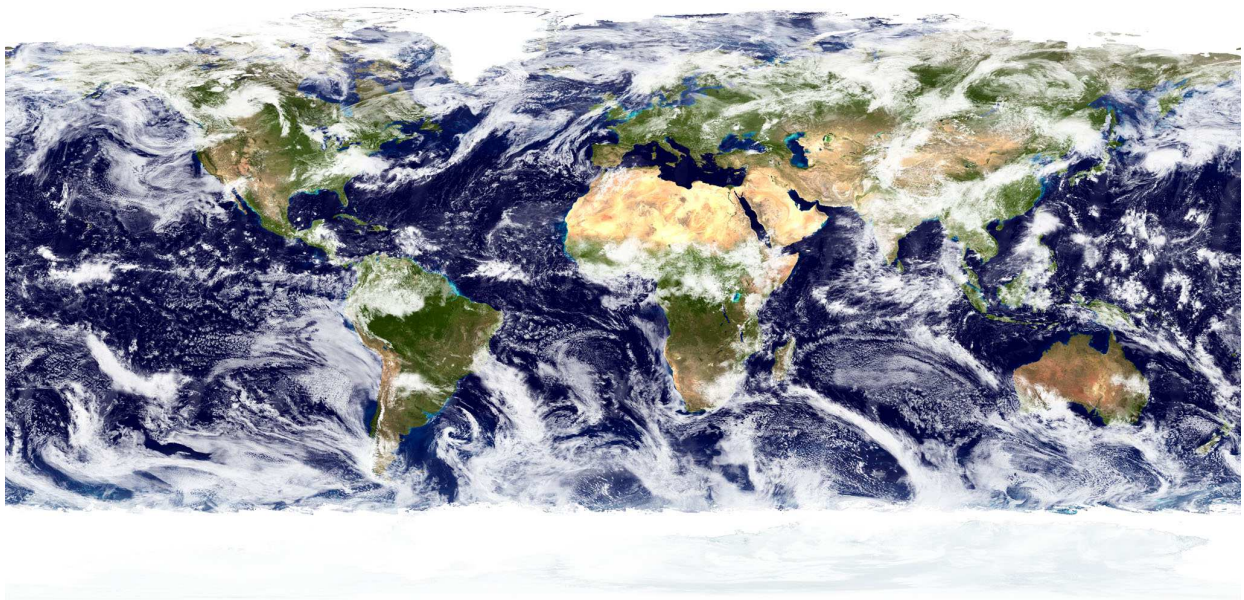


Figure 1: Near the equator, clouds appear to be clustered more randomly.

[ Image is from NASA, which says “These images are freely available to educators, scientists, museums, and the public.” ]

[ See source webpage: <http://visibleearth.nasa.gov/view.php?id=57735> ]

What causes the differences in cloud patterns between the midlatitudes and tropics? It is related to the Earth's rotation axis  $\vec{f}$  and the associated Coriolis force. In the midlatitudes, the axis  $\vec{f}$  and the Earth's inward gravitational acceleration vector  $\vec{g}$  are somewhat aligned ( $\vec{f} \cdot \vec{g} \neq 0$ ) and the rotation rate  $|\vec{f}|$  is relatively large. The rotation here plays a strong role in governing fronts, atmospheric motions, and the organization of clouds and rainfall. In

---

<sup>1</sup>Samuel N. Stechmann is associate professor of mathematics at the University of Wisconsin–Madison. His email address is [stechmann@wisc.edu](mailto:stechmann@wisc.edu)

contrast, at the equator, the vectors  $\vec{f}$  and  $\vec{g}$  are orthogonal ( $\vec{f} \cdot \vec{g} = 0$ ), and consequently tropical clouds are not as strongly influenced by the Earth's rotation. Cloud clusters that appear to be somewhat random are the dominant patterns.

Simplified partial differential equations (PDEs) for weather patterns in the midlatitudes have been known for over half a century. In contrast, simplified PDEs for the tropics are still actively proposed and investigated. Given the seemingly random appearance of cloud clusters in the tropics, one can imagine that tropical weather and climate could be modeled using stochastic PDEs. Other important aspects include the effects of dispersive equatorial waves and of nonlinearity from phase changes of water. Together, all of these aspects help form a complex dynamical system whose mathematical (and physical) understanding is still under development.

The societal impact is potentially enormous. Many of the most extreme weather phenomena are tropical, such as hurricanes and monsoons; and a large portion of the world's population lives under the influence of tropical weather. Moreover, weather in the United States is influenced by tropical phenomena, including El Niño and the increasingly discussed Madden–Julian oscillation. These examples have important impacts on the frontier of forecasting: the ability to make predictions not only a few days in advance but weeks or even months in advance. Such long-range forecasting stands as a great contemporary challenge, as foreseen by John von Neumann in his 1955 paper on “Some remarks on the problem of forecasting climate fluctuations.” It is an exciting area for mathematicians.

Photo of author:



[No photo credit needed. From personal photo collection.]