Identifying Convectively Coupled Equatorial Waves Using Theoretical Wave Eigenvectors

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ABSTRACT

Convectively coupled equatorial waves (CCEWs) are often identified by space-time filtering techniques that make use of the eigenvalues of linear shallow water theory. Here, instead, a method is presented for identifying CCEWs by projection onto the eigenvectors of the theory. This method does not use space-time filtering; instead, wave signals corresponding to the first baroclinic Kelvin, Rossby, and mixed Rossby-gravity (MRG) waves are constructed from reanalysis data by a series of projections onto (i) vertical and meridional modes and (ii) the wave eigenvectors. In accordance with the theory, only dry variables, that is, winds and geopotential height, are used; no proxy for convection is used. Using lag-lead regression, composites of the structures associated with each eigenvector signal during boreal summer are shown to contain all the features of the theory as well as some additional features seen in previous observational studies, such as vertical tilts. In addition, these composites exhibit propagation in good agreement with the theory in certain regions of the tropics: over the eastern Pacific ITCZ for the Kelvin and MRG composites and over the Pacific warm pool for the Rossby composite. In these respective regions, the Kelvin eigenvector signal is also in good agreement with space-time-filtered outgoing longwave radiation (OLR), and the Rossby and MRG eigenvector signals are in reasonable agreement with space-time-filtered OLR; it is shown that the eigenvector projections used here contribute to this agreement. Finally, a space-time-filtered version of the eigenvector projection is briefly discussed, as are potential applications of the method.

1. Introduction: Background and motivation

Convectively coupled equatorial waves (CCEWs) are a major source of tropical synoptic-scale variability (Kiladis et al. 2009). Identification and prediction of these waves is important because of their interaction with both tropical cyclogenesis (Bessafi and Wheeler 2006; Frank and Roundy 2006) and planetary-scale waves like the Madden–Julian oscillation (MJO; Zhang 2005; Dias et al. 2013).

Space-time spectral analysis of outgoing longwave radiation (OLR) and cloudiness data (Takayabu 1994; Wheeler and Kiladis 1999; Wheeler et al. 2000) has

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shown that many tropical convective anomalies propagate in a manner consistent with solutions to the shallow water equations developed by Matsuno (1966) for the tropical atmosphere. This shallow water theory appears to best describe the motion of convection when an equivalent depth between 15 and 50 m is used. These equivalent depths are smaller than those corresponding to dry equatorial waves, and several theories have been proposed to explain the need for this reduced depth. One such theory is based on quasi-equilibrium theory and reduced moist stability theory (Charney 1963; Arakawa and Schubert 1974; Gill 1982; Emanuel et al. 1994; Neelin and Zeng 2000; Frierson et al. 2004). This theory proposes that convection reduces the effective static stability of the tropical atmosphere, which then reduces the characteristic wave speed and equivalent depth of the atmosphere. This reduced effective static stability also affects the horizontal structure of waves,

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specifically resulting in waves with a smaller meridional length scale.

In this linear shallow water theory, each of the wave solutions has two components: an eigenvalue describing its propagation and an eigenvector describing the spatial structure of winds and pressure. Many methods for identifying CCEWs make use of this theory, but due perhaps to the early success of space–time spectral analysis, most of these methods utilize only the eigenvalues (e.g., Wheeler et al. 2000; Wheeler and Weickmann 2001). Studies that use this spatiotemporal approach then typically use statistical methods, for example, linear regression, to assess how closely the structures associated with this propagation resemble that of the theory.

Here, the opposite approach is pursued. Wave structures are identified through projection onto the eigenvectors only, so that no particular phase or group velocities are prescribed. This projection is achieved in several steps, including projection onto (i) the first baroclinic mode and the meridional modes and (ii) the wave eigenvectors; this method is similar to the techniques used in Stechmann and Majda (2015) and Ogrosky and Stechmann (2015b). The degree to which these structures propagate with the theoretical speed can then be assessed a posteriori using statistical methods and analyzing individual cases.

Two aspects of this method deserve immediate comment. First, since the theoretical solutions contain contributions from dry variables only, that is, winds and either pressure or geopotential height, no proxy for convection is used in identifying these wave structures. As a result, these eigenvector signals need not be correlated with convective anomalies. Assessing how well the eigenvector signals correspond with convection provides one simple test of the theory.

Second, no temporal filtering is used in this method since no propagation information is prescribed in identifying these waves. As a result, these eigenvector signals need not propagate with the theoretical free wave speeds. Assessing the degree to which the wave structures propagate with the theoretical speeds provides another simple test of the theory. In addition, the absence of temporal filtering suggests that this method could potentially be a useful component of real-time CCEW prediction methods, at least in regions and during seasons where these eigenvector signals are strongly coupled with convection. While the emphasis here is on convectively coupled waves, this method could also potentially be used to identify dry wave activity after adjusting the equatorial Rossby radius as in Ogrosky and Stechmann (2015b).

We note that other studies have utilized a combination of space-time filtering and spatial information to identify CCEWs, such as Yang et al. (2003, 2007a,b,c). In their methodology, space-time filtering is used to distinguish eastward-moving and westward-moving information, and meridional basis functions from shallow water theory are used after estimating the equatorial Rossby radius through a best-fit approach. Several elements of the full theory are not imposed during the spatial projection, however, including vertical structure and the eigenvectors corresponding to each wave. Hendon and Wheeler (2008) also used space-time filtering and found that the space-time coherence between OLR and zonal winds was a useful metric for identifying CCEW activity. Other studies, including Stechmann and Ogrosky (2014), Ogrosky and Stechmann (2015b), and Castanheira and Marques (2015), have investigated the dry eigenvectors; the latter study used a combination of a spatial projection onto the three-dimensional normal mode functions (which extend well into the stratosphere) of the linearized primitive equations and space-time filtering. Other methods that do not rely at all on the shallow water theory are also frequently used; see, for example, Wheeler and Hendon (2004), Roundy and Schreck (2009), and Roundy (2012) for methods employing empirical orthogonal functions (EOFs) and extended EOFs (EEOFs) to identify CCEWs and the MJO.

With this background in mind, the goal of the current study is to use an eigenvector projection method to identify CCEWs. This is done by assessing how well these eigenvector structures (i) propagate with the appropriate characteristics and (ii) correspond with convective anomalies. Furthermore, this new method of CCEW identification is in some ways the opposite of the traditional method of Wheeler and Kiladis (1999) and Wheeler et al. (2000) in that only dry variables are used instead of only a convective proxy and only eigenvector information is used instead of only eigenvalue information.

Before proceeding, we note that this CCEW theory is linear and only describes first-baroclinic-mode dynamics and deep convection. In contrast, observed convectively coupled Rossby waves have a significant barotropic component as well. In addition, second baroclinic mode theories incorporate stratiform convection (Mapes 2000) and also congestus convection (Khouider and Majda 2006, 2007, 2008). They capture additional features of CCEWs such as vertical tilts, progression in cloud types from congestus to deep convection to stratiform, and nonlinear solutions with realistic amplitudes. We use first-baroclinic-mode theory here instead of a multicloud model for simplicity.

The rest of the paper is organized as follows. Section 2 provides a brief overview of linear shallow water and

quasi-equilibrium theory. Section 3 describes the data and methods used here. The evolution of the identified eigenvector signals and their correlation with convective activity is studied in section 4; results are presented for the Kelvin, Rossby, and (westward) mixed Rossby– gravity (MRG) waves. A comparison of these waves with those identified by convective signals is given in section 5, where it will be shown that the correlation between eigenvector signal and convection is strongest for the Kelvin mode. Potential modifications to the method used here are discussed in section 6, including a method that combines space–time filtering with the eigenvector projection. Section 7 contains additional discussion of the results, and conclusions are given in section 8.

2. Overview of the theory

Our starting point is the simple, linear, first-baroclinicmode model for CCEWs:

$$u_t - yv - \theta_x = S_u, \tag{1a}$$

$$v_t + yu - \theta_v = S_v, \tag{1b}$$

$$\theta_t - u_x - v_y = \frac{1}{\tau}q + S_\theta, \qquad (1c)$$

$$q_t + \tilde{Q}(u_x + v_y) = -\frac{1}{\tau}q + S_q, \qquad (1d)$$

where (u, v) are the horizontal velocities in the (x, y)directions, θ is potential temperature, q is lower tropospheric moisture, Q is the dimensionless background vertical moisture gradient, and τ is a convective relaxation time scale parameter. The terms S_u , S_v , S_{θ} , and S_q represent forcing due to, for example, heating, cooling, and dissipation. Equations (1) have been made dimensionless by use of typical dry scales for u, v, and θ (Ogrosky and Stechmann 2015b); moisture q has been made dimensionless by $L_v/c_p\tilde{\alpha}$, where L_v is the latent heat of vaporization, c_p is the specific heat of dry air at constant pressure, and $\tilde{\alpha}$ is the reference potential temperature scale (Ogrosky and Stechmann 2015a). These equations can also be derived from the quasi-equilibrium theory introduced in Emanuel et al. (1994) and Neelin and Zeng (2000) and also presented in Frierson et al. (2004). For example, the system (1) with $S_u = S_v = S_\theta = S_q = 0$ is identical to Eqs. (3.16)–(3.19) in Frierson et al. (2004)if their P is parameterized by q/τ [i.e., their Eq. (2.55)– (2.56) with $\hat{q} = 0$].

In the case of short relaxation time, that is, $\tau \ll 1$ (the strict quasi-equilibrium limit), each of the variables and forcing terms in (1) can be expanded in τ , for example,

$$= u^{(0)} + \tau u^{(1)} + O(\tau^2), \qquad (2a)$$

$$v = v^{(0)} + \tau v^{(1)} + O(\tau^2),$$
 (2b)

$$\theta = \theta^{(0)} + \tau \theta^{(1)} + O(\tau^2),$$
 (2c)

$$q = q^{(0)} + \tau q^{(1)} + O(\tau^2).$$
(2d)

Substituting (2) into (1) and collecting terms of $O(\tau^{-1})$ results in

$$q^{(0)} = 0. (3)$$

Collecting all O(1) terms results in

u

$$u_t^{(0)} - yv^{(0)} - \theta_x^{(0)} = S_u^{(0)}, \qquad (4a)$$

$$v_t^{(0)} + yu^{(0)} - \theta_y^{(0)} = S_v^{(0)}, \qquad (4b)$$

$$\theta_t^{(0)} - u_x^{(0)} - v_y^{(0)} = q^{(1)} + S_{\theta}^{(0)}, \qquad (4c)$$

$$\tilde{Q}[u_x^{(0)} + v_y^{(0)}] = -q^{(1)} + S_q^{(0)}.$$
 (4d)

Combining (4c) and (4d) results in the usual linearized equatorial shallow water system [dropping superscripts of (0) hereafter],

$$u_{t} - yv - \theta_{y} = S_{u}, \qquad (5a)$$

$$v_t + yu - \theta_v = S_v, \tag{5b}$$

$$\theta_t - (1 - \tilde{Q})(u_x + v_y) = \tilde{S}_{\theta},$$
 (5c)

where $\bar{S}_{\theta} = S_{\theta} + S_q$. System (5) has a modified wave speed $c_m = \sqrt{1 - \tilde{Q}c_d}$, where $c_d \approx 50 \text{ m s}^{-1}$ is the dry atmospheric wave speed. The corresponding modified equivalent depth is $H_m = c_m^2/g$, where g is acceleration due to gravity. The modified wave speed c_m can also simply be absorbed into the variables by a rescaling, that is,

$$u^* = u(1 - \tilde{Q})^{-1/2}, \quad v^* = v(1 - \tilde{Q})^{-1/2},$$

$$\theta^* = \theta(1 - \tilde{Q})^{-1}, \quad (6a)$$

$$x^* = x(1 - \tilde{Q})^{-1/4}, \quad y^* = y(1 - \tilde{Q})^{-1/4},$$

$$t^* = t(1 - \tilde{Q})^{1/4}.$$
 (6b)

Substitution of (6) into (5) results in the usual system:

$$u_{t^*}^* - y^* v^* - \theta_{x^*}^* = S_u^*, \tag{7a}$$

$$v_{t^*}^* + y^* u^* - \theta_{v^*}^* = S_v^*, \tag{7b}$$

$$\theta_{t^*}^* - (u_{x^*}^* + v_{y^*}^*) = \tilde{S}_{\theta}^*, \tag{7c}$$

where the forcing terms have also been rescaled accordingly. We will reference (7) and drop stars hereafter, though the moist scales are understood to be used for generating all results presented here.



FIG. 1. Meridional basis functions associated with the moist wave speed $c_m \approx 16 \text{ m s}^{-1}$ for m = 0, 1, 2, 3, and 4.

An estimate of \tilde{Q} is now needed in order to determine an approximate modified wave speed. The value $\tilde{Q} = 0.9$ has been used successfully in other models of the tropical atmosphere (e.g., Neelin and Zeng 2000; Frierson et al. 2004; Majda and Stechmann 2009; Thual et al. 2014) and will be used here as well. This estimate results in a characteristic wave speed of $c_m \approx 16 \text{ m s}^{-1}$. The corresponding equivalent depth of $H_m \approx 26 \text{ m}$ lies well within the range 12–50 m that has been found to provide the best agreement with observations (Wheeler and Kiladis 1999).

This choice of Q corresponds to setting the meridional trapping scale to be approximately 8°. The first five parabolic cylinder functions, which form a natural set of meridional basis functions, are shown in Fig. 1 for this reduced trapping scale. For reference, these dimensionless functions are

$$\phi_0(y) = \frac{1}{\pi^{1/4}} e^{-y^2/2}, \quad \phi_1(y) = \frac{1}{\pi^{1/4}} \frac{1}{\sqrt{2}} y e^{-y^2/2},$$
 (8a)

$$\phi_2(y) = \frac{1}{\pi^{1/4}} \frac{1}{2\sqrt{2}} (4y^2 - 2)e^{-y^2/2},$$

$$\phi_1(y) = \frac{1}{2} \frac{1}{\sqrt{2}} (8y^3 - 12y)e^{-y^2/2}$$
(8b)

$$\phi_3(y) = \frac{1}{\pi^{1/4}} \frac{1}{2\sqrt{12}} (8y^3 - 12y)e^{-y^2/2}, \tag{8b}$$

$$\phi_4(y) = \frac{1}{\pi^{1/4}} \frac{1}{4\sqrt{24}} (16y^4 - 48y^2 + 12)e^{-y^2/2}.$$
 (8c)

The system (7) can be solved by defining characteristic variables

$$r^* = \frac{1}{\sqrt{2}}(u^* - \theta^*)$$
 and $l^* = \frac{1}{\sqrt{2}}(u^* + \theta^*).$ (9)

Substitution of (9) into (7) and projection of the result onto the meridional basis functions ϕ_m results in the usual decoupled systems whose solutions are eigenvectors representing the familiar Kelvin, equatorial Rossby, westward MRG, and inertio-gravity waves (see, e.g., Matsuno 1966; Majda 2003). For example, the MRG wave is composed of r_1 and v_0 , and the equatorial Rossby wave (shortened to simply Rossby wave in what follows) is composed of r_2 , v_1 , and l_0 , where subscripts denote meridional mode number. The propagation of each of these waves is governed by its corresponding eigenvalue $\omega(k)$, where k is the zonal wavenumber.

The projection of first baroclinic data onto the (i) parabolic cylinder functions and then (ii) the eigenvectors gives a unique way to express first baroclinic variables in terms of the shallow water eigenvectors. Moreover, since the linear operator of the (r, l, v) shallow water system is a skew-symmetric matrix, these eigenvectors are all orthogonal to one another (Majda 2003). Thus, one can also recover a given first baroclinic perturbation from a linear combination of eigenvectors. Last, each primitive variable at a single pressure level (850 or 200 hPa) may be recovered from the first baroclinic mode if the corresponding barotropic mode is known as well.

3. Data and methods

National Centers for Environmental Prediction-National Center for Atmospheric Research (NCEP-NCAR) reanalysis daily zonal winds, meridional winds, and geopotential height are used to estimate u, v, and θ , respectively (Kalnay et al. 1996). These datasets have a horizontal spatial resolution of $2.5^{\circ} \times 2.5^{\circ}$ and are recorded at 17 pressure levels ranging from 1000 to 10 hPa. NCEP-NCAR reanalysis vertical velocity is also used for the regression plots and has the same horizontal resolution as the other dry variables, but is only available at the 12 pressure levels ranging from 1000 to 100 hPa. The time period used in this study is the 33-yr period from 1 January 1980 to 31 December 2012. As a proxy for convective activity, NOAA interpolated daily OLR is used (Liebmann and Smith 1996), which also has a horizontal spatial resolution of $2.5^{\circ} \times 2.5^{\circ}$. The dry variable data are made dimensionless by use of the moist scales corresponding to a choice of $c_m = 16 \text{ m s}^{-1}$; that is, winds are made dimensionless by c_m , and geopotential height is made dimensionless by c_m^2/g .

This choice of c_m (or equivalently \tilde{Q}) corresponds to a choice of meridional length scale. In nature, however, convectively coupled waves occur over a range of meridional length scales. Thus, waves with a different meridional length scale than the one specified in the current method can be expected to project onto not only the appropriate eigenvector of corresponding wave type but also partly onto other eigenvectors. To briefly test the sensitivity of the results to this choice of \tilde{Q} , the main

results of the paper were reproduced with alternate values of Q = 0.975 and Q = 0.6 (not shown), corresponding to characteristic wave speeds of 8 and $32 \,\mathrm{m \, s}^{-1}$ respectively; furthermore, similar results for $c_m = 50 \text{ m s}^{-1}$ are given in Ogrosky and Stechmann (2015b). Use of $\tilde{Q} = 0.6$ corresponding to $32 \,\mathrm{m \, s^{-1}}$ resulted in composites with all the same primary features as those seen with the standard choice Q = 0.9; use of Q = 0.975 also resulted in composites that had similar features to those found using the standard choice, but the correlation between the dry eigenvector signal and convection was weaker (not shown). The relatively minor changes to the results using such large changes in \tilde{Q} is consistent with previous studies that have also reported robustness to choice of equivalent depth (e.g., Yang et al. 2003).

The variables (u, v, θ) in (5) are then isolated in the dimensionless data through the following series of spectral projections. First, to estimate the first baroclinic mode of each variable, the top, z = H, and bottom, z = 0, of the troposphere are associated with the 200- and 850-hPa pressure levels, respectively. When each velocity component is expressed as the sum of the barotropic mode and a first baroclinic mode, the first baroclinic component can be estimated by

$$u_{\rm BC}(x, y, t) = \frac{u(850 \,\mathrm{hPa}) - u(200 \,\mathrm{hPa})}{2\sqrt{2}},$$

$$v_{\rm BC}(x, y, t) = \frac{v(850 \,\mathrm{hPa}) - v(200 \,\mathrm{hPa})}{2\sqrt{2}}.$$
 (10)

Potential temperature θ can be estimated by geopotential height Z using hydrostatic balance as in Ogrosky and Stechmann (2015b), that is

$$\theta_{\rm BC}(x, y, t) = -\frac{Z(850\,{\rm hPa}) - Z(200\,{\rm hPa})}{2\sqrt{2}}.$$
 (11)

Isolation of the first baroclinic mode through this simple vertical projection reduces a 3D (x, y, z) dataset to a 2D (x, y) dataset. The specification of a vertical structure in the data projection is one distinction between the current method and that of Yang et al. (2003).

An alternate projection method was tested that makes use of each pressure level between 850 and 200 hPa for which reanalysis data are reported; in this alternate method, the first baroclinic basis function is assumed to be a cosine function of pressure. The resulting first baroclinic variables \tilde{u}_m , \tilde{v}_m , and $\tilde{\theta}_m$ all have a pattern correlation with their two-level counterparts u_m , v_m , and θ_m of 0.8 or higher at every longitude for m = 0, 1, 2. At most longitudes this pattern correlation was 0.9 or higher, suggesting that the two-level projection method is able to adequately capture the primary features of the first baroclinic mode. Additional support for the crude vertical structures in (10) and (11) is provided by Stechmann and Ogrosky (2014), who show that steady, diabatically forced shallow water theory is in excellent agreement with observational data based on (10) and (11). We also note that one could alternatively use vertical basis functions $\Phi(z)$ that incorporate contributions from all heights z or pressure levels p (e.g., Fulton and Schubert 1985). Such an approach would also include contributions from stratospheric variations. While CCEWs are correlated with stratospheric variations [such as "boomerang" structures (Kiladis et al. 2009)], many other physical mechanisms unrelated to CCEWs can create stratospheric variability. To avoid stratospheric variations, and to focus on deep, first baroclinic variability in the troposphere, the simple formulation (10) and (11) is used.

Next, the dimensionless first baroclinic data are decomposed into their meridional mode components, utilizing the meridional basis functions that take the form of parabolic cylinder functions. Each first baroclinic variable can be expressed as a linear combination of parabolic cylinder functions, for example,

$$u_{\rm BC}(x, y, t) = \sum_{m=0}^{\infty} u_m(x, t)\phi_m(y),$$
 (12)

where $\phi_m(y)$ are the basis functions, the first five of which are given in (8). The spectral coefficients $u_m(x, t)$ can be approximated by evaluating the integral

$$u_m(x,t) = \int_{-\infty}^{\infty} u_{\rm BC}(x,y,t)\phi_m(y)\,dy.$$
 (13)

Similar formulas apply for v and θ . This meridional projection reduces the 2D (x, y) dataset to a 1D (x) dataset. More details of these projection steps can be found in Stechmann and Majda (2015); similar techniques have been used in other studies as well (e.g., Yang et al. 2003; Gehne and Kleeman 2012; Stechmann and Ogrosky 2014).

As discussed above, the effects of projecting waves in observational data that have a different equivalent depth than the one specified in the current method onto eigenvectors can be expected to at least somewhat hamper identification of CCEWs with the current method. However, we note that other methods in the literature are subject to similar difficulties. For example, many methods of CCEW identification use data lying only within a tropical strip along the equator; that is, data are averaged over a range of latitudes, such as 10°S–10°N, with all other data neglected in the analysis. An abrupt latitudinal truncation of the data may be expected to suffer from the same sort of difficulties, and one might view the smooth tapering of the parabolic cylinder functions to zero with increasing distance from the equator as a positive feature in defining the edges of the tropics.

For each spectral coefficient u_m , v_m , and θ_m , a seasonal cycle is identified at each longitude x by the mean and first three annual harmonics. This cycle is then removed at each longitude. To remove low-frequency variability, a 120-day running time average is also subtracted on each day at each longitude.

Finally, reanalysis data are projected onto the wave eigenvectors. The eigenvectors of these first baroclinic waves are well documented and can be found in, for example, Matsuno (1966) and Majda (2003), and they are different for each zonal wavenumber. For each wave type, a corresponding eigenvector signal can be constructed, such as K(x, t) for the Kelvin wave. The eigenvector signals K, R_1 , and MRG are constructed exactly as in Ogrosky and Stechmann (2015b), but using the variables r^* , l^* , and v^* , which were scaled by moist reference scales instead of dry reference scales, and using the corresponding eigenvectors defined in terms of moist reference scales instead of dry reference scales. The eigenvector definition also includes an arbitrary complex phase factor that we have chosen so that the eigenvector signal is in phase with low-level divergence so that the wave amplitude signal is in phase with OLR. The eastward inertio-gravity waves EIG₀ and EIG₁ and westward inertio-gravity wave WIG1 structures were also studied by the authors, but we leave a detailed study of their properties to future work.

We note that this projection method results in wave amplitude signals, for example, K(x, t) and $R_1(x, t)$, that are not necessarily orthogonal to one another in xand t; that is, it is not necessarily the case that $\int_{0}^{T} \int_{0}^{P_{E}} K(x, t) R_{1}(x, t) dx dt = 0$, where T is the temporal extent of the dataset being projected and P_E is the circumference of the earth. This type of orthogonality is a feature of space-time-filtered OLR signals, provided that each wavenumber-frequency combination (k, ω) is ascribed to at most one wave type during the filtering. Here, instead, the wave signals are orthogonal to each other in the sense that $\langle \mathbf{K}, \mathbf{R}_1 \rangle = \int_{-\infty}^{\infty} \int_0^{P_E} \mathbf{K}^{\mathrm{T}} \mathbf{R}_1 \, dx \, dy =$ $\int_{-\infty}^{\infty} \int_{0}^{P_E} u_K u_{R_1} + v_K v_{R_1} + \theta_K \theta_{R_1} \, dx \, dy = 0, \quad \text{where} \quad \mathbf{K}^{\mathrm{T}} =$ (u_K, v_K, θ_K) and where, for example, u_K are the zonal winds ascribed to the Kelvin wave structure. This integral is the natural inner product based on the conserved shallow water energy, and one can obtain each of the u_K, v_K , and θ_K from the wave amplitude signal K(x, t)by reversing the steps in the data projection method

outlined above and discussed further in Ogrosky and Stechmann (2015b).

The power contained in the anomalies of primitive variables u_m , v_m , and θ_m or eigenvector signals K_m , etc., can then be found through space-time spectral analysis where the following standard steps are taken here. First, a spatial Fourier transform is taken, and the data for each wavenumber are partitioned into shorter overlapping time segments. For each segment, the mean is removed and the first and last 10% of each segment is tapered to zero by a cosine tapering function. Here segments are taken to be 366 days long, with an overlap of 246 days. A temporal Fourier transform of each segment is taken next, and the power in each frequency is estimated by the square of the amplitude of these Fourier coefficients. This power is then averaged over all segments and is denoted throughout the paper by $|\hat{K}_m|^2$, etc.; the logarithm of these quantities can then be displayed in a wavenumber-frequency diagram (e.g., Wheeler and Kiladis 1999).

Composite structures are created for each wave type using standard lagged regression techniques, as described in detail in Hendon and Salby (1994) and Wheeler et al. (2000). Unfiltered winds, geopotential height, and OLR anomalies from a seasonal cycle are regressed against a reference time series given by a wave signal at a fixed longitude. A 120-day running average is removed from each dynamical field prior to the regression in order to remove the effects of low-frequency variability due to, for example, ENSO. A separate regression equation is solved for each variable at each longitude, latitude, pressure level, and time lag. The resulting linear regression coefficients are then used to produce a composite picture of the evolution of each wave type. In these composites, the winds are plotted only at locations where they are deemed to be significant at the 99% confidence level using a two-sided significance test and a decorrelation time scale as estimated by Eq. (1) in Livezey and Chen (1983). Finally, radiosonde zonal wind observations at Majuro, obtained from NOAA's Integrated Global Radiosonde Archive, were also regressed against the reference time series (Durre et al. 2006).

4. Wave structures, convection, and their propagation

We next present results for the Kelvin, Rossby, and MRG eigenvector signals. As each of these signals was created without temporal filtering, it is unclear a priori whether each will propagate with the expected phase and group velocities of the corresponding free wave. It is also unclear how well correlated these signals will be



FIG. 2. Hovmöller diagram of the (a) Kelvin, (b) first Rossby, and (c) MRG eigenvector signals from 1 Jul 2009 through 30 Jun 2010. Units are nondimensional (see section 3 for a description).

with convection. In this section, we address these two questions.

Before presenting a detailed analysis of each wave type individually, the evolution of each of these three wave signals is shown in Fig. 2 for the 1-yr period 1 July 2009 through 30 June 2010; space-time-filtered versions of each of these signals are shown and discussed later in section 6. Each signal has been defined so that negative anomalies correspond to low-level convergence and positive anomalies correspond to low-level divergence. The Kelvin eigenvector signal propagates primarily eastward at fast speeds, with some slower eastward events between 60°E and 150°W. The two slower events in November-February correspond with periods of documented high MJO activity during the Year of Tropical Convection (YOTC; Moncrieff et al. 2012; Waliser et al. 2012). The appearance of MJO events in the Kelvin wave signal will be discussed further in section 7, but we note here that this is consistent with Roundy (2012), whose analysis suggests that the MJO and Kelvin waves lie along a continuum in wavenumber-frequency space. Roundy (2012) also shows that for zonal wavenumber-4 Kelvin waves the phase relationship between the zonal winds and convection depends on the frequency considered; no such phase shift was observed in the analysis presented here. Note that there are rapidly eastward-moving signals within this slower-moving component. From 150° to 90°W, almost all information is moving rapidly eastward.

The first Rossby eigenvector signal (denoted hereafter as simply the Rossby eigenvector signal) contains discernible signals over a large range of wavenumbers (both eastward and westward) and frequencies. There does, however, appear to be more westward propagation of information, and this propagation appears to occur at a slower phase velocity than that of, say, the Kelvin signal. Like the Kelvin signal, there are slow-moving eastward events from October through February from 60°E through 150°W corresponding to high MJO activity, and we note that while the projection method used here creates orthogonal modes, a signal in raw data can contribute to more than one mode. There are, however, instances of smaller westward-moving waves within this larger envelope of eastward propagation.



FIG. 3. Power spectrum of the (a) Kelvin, (b) first Rossby, and (c) MRG eigenvector signals from 1980 to 2012. The bold black lines denote the dispersion relation for the appropriate wave type using an equivalent depth of 25 m.

The MRG eigenvector signal shows primarily smaller wavenumber and higher-frequency activity than that of the Kelvin signal and is most active in the western hemisphere, as also described in other investigations of CCEWs (e.g., Roundy and Frank 2004a). The signal appears to propagate with primarily westward phase velocity. There are some instances where there appears to be an eastward group velocity, perhaps most clearly seen in September or late October between 180° and 90°W. This signal attains its largest amplitude at the end of the MJO event in February, when it appears to have slower phase velocity than at other times.

The power spectrum of each eigenvector signal is shown in Fig. 3, where the entire time period 1 January 1980 through 31 December 2012 has been used. These plots confirm many of the spatiotemporal characteristics of each signal shown in Fig. 2, including (i) eastwarddominant propagation for the Kelvin signal and westward-dominant propagation for the MRG and Rossby signals and (ii) low-wavenumber information projecting almost exclusively onto the Kelvin and Rossby eigenvectors.

One aspect of Fig. 3 that deserves comment is that the spectral peaks do not necessarily lie along the theoretical dispersion curves associated with each eigenvector. We note that while *free* waves may be expected to propagate in a manner determined by the eigenvalues of the linear theory, the picture is more complicated for *forced* waves. In nature, waves may be forced by heating and cooling, dissipation, extratropical forcing, etc., and these forces contain contributions from many wavenumbers k and frequencies ω that do not lie along the dispersion curves for free waves. In such cases, a *forced* wave can arise where the structure of a given wave type propagates in a manner inconsistent with the *free* wave dispersion curves. While the degree to which various forces contribute to the signals seen in Fig. 3 is not known, extratropical Rossby waves have been identified as a likely cause of spectral peaks at zonal wavenumbers in the range 5 < |k| < 10 in low-level meridional winds (Gehne and Kleeman 2012); we speculate that they likewise contribute to the MRG spectral peaks near wavenumber |k| = 5 in Fig. 3. In addition, the lack of power at smaller wavenumbers |k| < 4 in Fig. 3c is consistent with longwave balanced equatorial dynamics, which predicts that MRG waves are essentially "filtered out," and has been noted in Ogrosky and Stechmann (2015b).

Figure 2 also suggests that the variability in these eigenvector signals is a function of region, consistent with other studies (e.g., Roundy and Frank 2004a; Dias and Kiladis 2014). For example, Roundy and Frank (2004a) show that MRG waves have highest activity over the ITCZ during boreal summer and fall, consistent with the MRG eigenvector signal studied here. We note that the Rossby eigenvector signal has significant variability at all longitudes, including over the eastern Pacific ITCZ (where it has been shown that convectively coupled Rossby waves have a predominantly barotropic vertical structure; e.g., Kiladis and Wheeler 1995; Wang and Xie 1996). The Kelvin eigenvector variability does not vary as strongly with region as the other two wave types, also consistent with other studies.

We next study each of the three wave types in detail, examining their structure, propagation, and correlation with convection. We concentrate primarily on the May– October season, when impacts of the MJO on each wave signal are minimal; these impacts during the boreal winter are considered briefly in section 7.

a. Kelvin wave

As convectively coupled Kelvin waves (CCKWs) have been shown to have highest variance over the Pacific ITCZ during boreal summer and fall (Roundy and



FIG. 4. Longitude–time diagram of OLR anomalies (averaged from 0° to 15°N) regressed onto the May–October Kelvin eigenvector signal at 125°W (red \times). Negative (positive) anomalies are depicted by shading (contours). The three levels of shading represent OLR anomalies < -2.5, -3.5, and -4.5 W m⁻²; the three contours represent OLR anomalies of 2.5, 3.5, and 4.5 W m⁻². The solid dashed lines depict propagation at 16 and 6 m s⁻¹.

Frank 2004a), the composite wave structure associated with the Kelvin eigenvector signal is displayed here using a base longitude of 125°W. This was also the location of the NOAA ship *Ronald H. Brown* during the CCKW event studied in detail by Straub and Kiladis (2002).

Before discussing the results, we note again that the eigenvector signal K(x, t) has been defined using the eigenvectors of shallow water theory. It is thus reasonable to expect that the winds and geopotential height associated with this signal should be in good agreement with the theory. However, the power spectrum of K(x, t) is substantially red, so it is not clear a priori that propagation at reasonable free wave speeds will arise from lag-lead regression. This is essentially the opposite of the situation in studies that define a wave signal using the eigenvalues of shallow water theory. In such studies, propagation at free wave speeds is reasonable to expect from the composite due to space-time filtering, but it is not clear a priori that the wave structures that arise from lag-lead regression will be in agreement with the theory.

Figure 4 shows a lag-lead regression of OLR anomalies averaged from 0° to 15°N associated with the Kelvin eigenvector signal at 125°W. Negative OLR anomalies propagate across the Pacific Ocean, beginning in the Pacific warm pool at approximately day -6and reaching the west coast of South America by day 3. The anomalies move with a speed of roughly $16 \,\mathrm{m \, s^{-1}}$ in good agreement with the linear theory and with previous studies of Kelvin waves in the presence of an ITCZ located north of the equator (Dias and Pauluis 2011; Yasunaga and Mapes 2014). We note that the spectral peak of the Kelvin eigenvector signal shown in Fig. 3 indicates that the Kelvin signal may propagate faster, a discrepancy that may be due to one of several reasons. For one, the power spectrum shown in Fig. 3 makes use of data from all longitudes and seasons, while the regression considers the signal at a single longitude during May-October. Also, as no filtering was applied to the OLR being regressed in Fig. 4, there is no guarantee a priori that OLR correlated with the Kelvin eigenvector signal must move at an identical speed. The signal is also associated with positive OLR anomalies in the Indian Ocean and Maritime Continent that propagate eastward at approximately $6 \,\mathrm{m \, s^{-1}}$, indicative perhaps of the inactive or dry phase of the boreal summer intraseasonal oscillation. Straub and Kiladis (2003a) found enhanced Kelvin-filtered OLR variance over the Pacific when the ISO is in its active convection phase, though the sign of the Kelvin-filtered OLR signal was not reported.

In Fig. 5, OLR and geopotential height and zonal and meridional winds at 200 hPa have been regressed against the base Kelvin eigenvector signal at 125°W. The convective activity associated with this signal is located almost entirely north of equator in a narrow range of latitudes, centered at approximately 10°N corresponding with the location of the boreal summer ITCZ (Dias and Pauluis 2011). These convective, geopotential, and wind signals are also similar to those of the composite obtained by Straub and Kiladis (2002) who regressed dry variables against a space-time-filtered convective proxy. Thus, while the Kelvin signal is calculated using the symmetric basis function $\phi_0(y)$, off-equatorial convection is strongly associated with the eigenvector signal. Upper-level zonal divergence anomalies propagate eastward as well, appearing to lead the convective activity by approximately 10° in longitude. The geopotential height looks very much like the linear theory at day 0, with some resemblance at day -3 as well. Note that some meridional flow also appears to exist between the convection and the equator on day 0 and day +3. The direction of this upper-level meridional flow is consistent with previous modeling studies showing that Kelvin waves passing through an ITCZ to the north of the equator contain some meridional circulation (Dias and Pauluis 2009). In the Southern Hemisphere a series of circulation centers are strongly correlated with the eigenvector signal, particularly on day -3 and day 0; these centers are reminiscent of wave trains identified by Straub and Kiladis (2003b).

FIG. 5. A map of OLR, wind vectors, and geopotential height anomalies regressed onto the May–October Kelvin eigenvector signal at 125°W (denoted by the red line) for (a) day -6, (b) day -3, (c) day 0, and (d) day +3. Winds and geopotential height are at 200 hPa. Negative (positive) OLR anomalies are depicted by shading (bold gray contour). The three levels of shading represent OLR anomalies < -3, -5, and $-7 W m^{-2}$; the bold gray contour represents OLR anomalies of $3 W m^{-2}$. Positive (negative) geopotential height anomalies are depicted by solid (dashed) black contours; contours are shown for intervals of 2 m with the zero contour omitted. Only wind vectors that are statistically significant are shown. The maximum wind speed is (a) 1.8, (b) 2.6, (c) 3.5, and (d) 4.4 m s^{-1}.

FIG. 6. A map of wind vectors and geopotential height anomalies at the equator regressed onto the May–October Kelvin eigenvector signal at $125^{\circ}W$ (denoted by the red line) for (a) day -6, (b) day -3, (c) day 0, (d) day +3. Positive (negative) geopotential height anomalies are depicted by thick (thin) contours; contours are shown for intervals of 1 m with the zero contour omitted. Wind vectors are only plotted for the lowest 12 levels; heights above these are in gray, depicting the lower stratosphere. Only wind vectors that are statistically significant are shown. The maximum wind speed is (a) 1.3, (b) 1.5, (c) 3.5, and (d) 4.3 m s⁻¹. The vertical wind component is multiplied by a factor of 1000.

Figure 6 shows the vertical structure of the composite Kelvin wave at the equator. The composite has a strongly first baroclinic mode appearance over a wide range of longitudes; on day 0 and day +3, the first baroclinic structure appears to extend around the globe, with the sign changes occurring at the location of the Kelvin wave signal and approximately 50° -90°E. Zonal and vertical winds show a strong circulation cell to the west of the Kelvin eigenvector signal maximum, with strong upper-level divergence and weak lower-level convergence moving with the signal. From day -3 to day +3, a vertical tilt from the upper troposphere to the lower stratosphere can be seen in geopotential height, consistent with other studies that have regressed variables

against a convective proxy (Wheeler et al. 2000; Straub and Kiladis 2002; Kiladis et al. 2009; Roundy 2012). We note that there are some zonally symmetric anomalies in the geopotential height; based on plots from a wider range of lags (not shown), these appear to be due to correlation between the eigenvector signal and intraseasonal variability.

Vertical tilts can be seen more clearly in time-height composites from radiosonde data. Figure 7 shows the vertical structure of the composite Kelvin wave when constructed using radiosonde data. More specifically, in this time-height composite, radiosonde zonal wind at the island of Majuro (7.1°N, 171.4°E) was regressed against the reanalysis Kelvin eigenvector signal at a base

FIG. 7. A time-height (pressure) cross section of zonal winds at Majuro regressed onto the May–October Kelvin eigenvector signal at 172.5°E. Positive (negative) anomalies are depicted by thick (thin) contours; contours are shown for intervals of $0.125 \,\mathrm{m\,s^{-1}}$ with the zero contour omitted.

longitude of 172.5°E. The island of Majuro was chosen for comparison with Fig. 8 of Kiladis et al. (2009). In Fig. 7, the vertical tilt in the troposphere is more pronounced than in Fig. 6, and it is also more pronounced than the tilt seen in a time-height composite produced by replacing the radiosonde zonal winds with reanalysis zonal winds (not shown). We note that the contour interval used in Fig. 7 is somewhat small, consistent with some other studies of vertical tilts, in order to more readily identify any vertical tilts present in the composites [e.g., see Fig. 8 of Kiladis et al. (2009), which uses radiosonde data, and Fig. 2 of Roundy (2012), which uses reanalysis data]. These results suggest that vertical tilts can indeed be associated with the Kelvin eigenvector signal, even though the signal is constructed using a first-baroclinic-mode method.

These composites indicate that the Kelvin eigenvector signal K(x, t) is correlated to convection propagating with the theoretical wave speed. To further assess the robustness of this correlation, the signal is next compared with space-time-filtered OLR calculated using the method of Wheeler et al. (2000). Details of the method can be found in Wheeler et al. (2000), but we highlight here that symmetric OLR is used to create the Kelvin and Rossby signals while antisymmetric OLR is used for the MRG signal; also, see section 6 for a discussion of the wavenumber-frequency regions used for creating the space-time-filtered OLR signal. A Hovmöller diagram of the two signals from 1 May 1997 through 1 February 1998, along with raw OLR averaged from 0° to 15°N, is shown in Fig. 8a. This time period includes the CCKW event studied in Straub and Kiladis (2002) and includes the case study of space-time-filtered OLR considered in Wheeler and Weickmann (2001). Both methods identify many of the same wave events; see, for example, July and August at approximately 125°W, and 90°E–90°W in December and January. Differences do exist, however; the eigenvector signal identifies some instances of OLR anomalies that do not propagate at the expected speed, and some instances of Kelvin-like propagation of dry variables only, for example, September. The filtered OLR also identifies some events that the eigenvector signal does not, for example, November and December from 135° to 90°W.

A time series of the two signals at 125°W is shown in Fig. 8d; each time series was normalized by dividing by its standard deviation. While there are distinct differences in the signals, many of the strong CCKW events identified by filtered OLR correspond to dips in the eigenvector signal.

The pattern correlation between the two signals is shown in Fig. 8b for all longitudes and for the entire 33-yr period studied here. Over the Pacific ITCZ, the agreement is strongest in the boreal summer, while in the Indian Ocean region, agreement is best in boreal winter. When the pattern correlation is computed using only times when the normalized filtered OLR signal is less than -1, the correlation improves to approximately 0.6 in the Pacific ITCZ region during boreal summer. The pattern correlation was also computed for various lags between the signals. While the pattern correlation is highest for 0-day lag at many longitudes, at some longitudes the pattern correlation is highest for a 1-day lag, that is, when filtered OLR lags behind the eigenvector signal by 1 day.

One additional feature apparent in Fig. 8d is that there are more "very large" negative anomalies present in the filtered OLR signal than are present in the dry wave signal. A comparison of the number of "extreme events," defined as a signal less than some cutoff, for example, -3, confirms that the filtered OLR signal identifies more of these "very strong" Kelvin wave events. The frequency of these large negative anomalies in the filtered OLR signal is likely due to the negative skewness of OLR distributions in this part of the world.

b. Rossby wave

We next present results for the Rossby eigenvector signal $R_1(x, t)$. Previous studies have indicated that Rossby waves over the Pacific ITCZ are predominantly barotropic, while Rossby waves over the Pacific warm pool during boreal summer have a largely first baroclinic vertical structure (Kiladis and Wheeler 1995; Yang et al. 2007a); potential factors contributing to these different vertical structures have been shown to include the strength of coupling with convection and vertical shear

FIG. 8. (a) Hovmöller plot of OLR anomalies (shading) averaged from 0° to 15°N from 1 May 1997 through 1 Feb 1998. Red contours denote a Kelvin-filtered OLR signal of -15 W m^{-2} ; black contours denote a Kelvin eigenvector signal of -0.45. (b) Pattern correlation of the time series between each signal as a function of longitude; solid blue denotes the annual correlation, dashed green denotes May–October, and dotted red denotes November–April. (c) As in (b), but the pattern correlation is calculated using only times where the normalized Kelvin-filtered OLR signal is <-1. (d) Time series of the two wave signals at 125°W for 1997, normalized by dividing each series by its standard deviation.

(see, e.g., Yang et al. 2007a; Wang and Xie 1996). Motivated in part by these observations, OLR and dry variables are regressed against the Rossby eigenvector signal at 140°E.

Figure 9 shows the lagged regression of OLR averaged from 2.5° to 12.5°N against the Rossby eigenvector signal. Negative OLR anomalies propagate westward across the warm pool, beginning just west of the date line and breaking up while passing over the Philippines. The anomalies move with a speed of roughly $5 \,\mathrm{m\,s^{-1}}$ in good agreement with the linear theory for free waves.

In Fig. 10, OLR and geopotential height and zonal and meridional winds at 200 hPa have been regressed against the Rossby eigenvector signal at 140°E. The convective activity associated with the signal lies on the equator at day -6 and moves westward and slightly northward.

FIG. 9. As in Fig. 4, but for OLR averaged from 2.5° to 12.5°N regressed against the May–October Rossby eigenvector signal at 140°E. The five levels of shading represent OLR anomalies < -3.5, -4.5, -5.5, -6.5, and -7.5 W m⁻²; the two contour levels represent OLR anomalies of 3.5 and 4.5 W m⁻².

Rossby gyres can be seen in both the circulation and height on each side of the equator on days -3 through +3; when dynamical fields at 850 hPa are regressed onto the signal, the resulting composite has essentially the same features with opposite signs near the equator (not shown). The convection is approximately aligned with the gyre north of the equator at day 0, consistent with the findings of Yang et al. (2007a), who show that Rossby waves in boreal summer in the Eastern Hemisphere have convection located in the center of an upper-level anticyclone. The upper-level height anomalies to the east of the signal maximum exhibit peaks along the equator, similar to structures identified in other studies; see, for example, Kiladis et al. (2009), who find a similar "forced Kelvin-like response" to equatorial heating when regressing against a spacetime-filtered cloudiness indicator. The location of several gyres in the Southern Hemisphere is also in good agreement with the corresponding composite in Fig. 17c of Kiladis et al. (2009).

Figure 11 shows the vertical structure of the composite Rossby wave at the equator. The structure is clearly first baroclinic in both winds and height, consistent with both the vertical projection used here and with the structures identified in other studies, such as the one by Yang et al. (2007a), who found that Rossby waves in boreal summer in the Eastern Hemisphere have a predominantly first baroclinic vertical structure. The upperlevel divergence on day 0 is centered at approximately 160°E, consistent with Fig. 10 and with the theoretical wave structure at the equator. Based on plots from a wider range of lags (not shown), the presence of zonally symmetric anomalies in the geopotential height appears to be due to coupling between the eigenvector signal and intraseasonal variability.

Figure 12 shows the agreement between the Rossby eigenvector signal and Rossby-filtered OLR again using the method of Wheeler et al. (2000). A Hovmöller diagram depicts both signals as well as OLR averaged from 10°S to 5°N from May 1992 through October 1992, coinciding with the time period studied in Yang et al. (2003, 2007a,b,c). While some events are identified by both methods, the signals often identify different events from one another. Also noticeable is that while the eigenvector signal propagates largely westward, the propagation speeds for the largest signal anomalies are not all in agreement with the linear theory for planetaryscale free waves. A time series of the two signals at 140°E is shown in Fig. 12d after normalization. While some of the strong OLR events are also captured by eigenvector signal, there is considerably less agreement than in the Kelvin case.

The pattern correlation between the two signals is shown in Fig. 12b for all longitudes. Unlike the Kelvin wave, the agreement is consistently poor over the Pacific ITCZ. This is likely consistent with previous studies that have identified most Rossby wave activity in this region as being barotropic, while Rossby waves over the warm pool tend to have a largely first baroclinic vertical structure (Kiladis and Wheeler 1995; Wang and Xie 1996; Wheeler et al. 2000; Yang et al. 2007a). The Rossby wave pattern correlation is more sensitive to longitude than either the Kelvin or MRG wave pattern correlations, consistent with findings of other studies (Kiladis et al. 2009). Over the warm pool region, the agreement is not significantly different during different seasons, with a pattern correlation of 0.2–0.3. When the pattern correlation is computed using only times when the normalized filtered OLR signal is less than -1, the correlation improves to approximately 0.4-0.5 over the warm pool during boreal summer. Strong pattern correlation also appears to exist over the very eastern Pacific Ocean. Last, unlike the Kelvin-filtered OLR, the distribution of signal amplitudes over the entire time period (not shown) indicates that the Rossby-filtered OLR signal has a comparable number of large events to the eigenvector signal.

c. MRG wave

Convectively coupled MRG waves have been shown to have highest variability over the western and central Pacific ITCZ during boreal summer and fall (Roundy and Frank 2004a). Composites of these waves propagate

FIG. 10. As in Fig. 5, but for the May–October Rossby eigenvector signal at 140°E and including (e) a day +6, and with four levels of shading representing OLR anomalies < -4, -6, -8, and -10 W m^{-2} ; the bold gray contour represents OLR anomalies of 4 W m⁻². The maximum wind speed is (a) 2.2, (b) 2.9, (c) 4.9, (d) 3.4, and (e) 2.9 m s^{-1} .

westward in accordance with the theory, though as they move over the western Pacific toward the Maritime Continent, previous studies have shown these composites turn to the northwest (Kiladis et al. 2009). Here, OLR and dry variables are regressed against the MRG eigenvector signal at 140°W located in the central Pacific.

Figure 13 shows the lagged regression of OLR averaged from 2.5° to 12.5°N against the MRG eigenvector signal at 140°W. Negative OLR anomalies propagate westward at approximately 25 m s^{-1} across the Pacific ITCZ with an apparent eastward group velocity. The phase velocity is in good agreement with the linear theory and other studies of MRG waves. There is also some low-frequency convective activity in the western Pacific that is correlated with the MRG eigenvector signal (not shown); the impact of the MJO on the MRG signal is discussed briefly in section 6.

In Fig. 14, OLR and geopotential height and zonal and meridional winds at 200 hPa have been regressed against the base MRG eigenvector signal at 140°W. The convective activity north of the equator is located at about 7.5°N and propagates westward as expected. This region of convection has a meridional tilt consistent with previous studies [see, e.g., Kiladis et al. (2009) for a composite based on cloudiness as well as an EOF]. Upper-level equatorward flow through the convection can be seen on day -2 and day 0. The circulation shows a strong MRG signal on all days shown here, with circulation gyres centered either over the equator or slightly south of the equator. The height anomalies are also in good agreement with the theory on day -2 and day 0; a full wavelength is clearly visible in the height signal on day 0, suggesting that the dry structure has zonal wavenumber k = 6 or 7, slightly larger than the estimate in Kiladis et al. (2009). As with the Kelvin eigenvector, there are strong geopotential anomalies in the Southern Hemisphere associated with the MRG eigenvector signal. On days -1 and 0, these anomalies, combined with the eigenvector signal near the equator, form a wave train similar to the one identified in Kiladis et al. (2016) during the boreal summer.

FIG. 11. As in Fig. 6, but for the May–October Rossby eigenvector signal at 140° E, and including (e) a day +6, and geopotential height contours are shown for an interval of 2 m. The maximum wind speed is (a) 2.2, (b) 2.9, (c) 3.5, (d) 3.6, and (e) 2.5 m s^{-1} .

FIG. 12. As in Fig. 8, but averaged from 10°S to 5°N from May through October 1992 and with red contours denoting a Rossby-filtered OLR signal of -10 W m^{-2} and black contours denoting a Rossby eigenvector signal of -0.5, and with (d) the time series of the two signals at 140°E for 1992, normalized by dividing each series by its standard deviation.

Figure 15 shows the vertical structure of the composite MRG wave at 7.5°N. Note the narrow longitudinal extent of the statistically significant winds in the upper troposphere; the lower-level winds are not as highly statistically significant as the upper-level winds. Upper-level zonal convergence aligning with the region of convective activity can be seen for all days shown, consistent with the linear theory. While the vertical structure in the troposphere is predominantly first baroclinic, it also exhibits a pronounced "boomerang structure" in the height field despite the first baroclinic projection method used here. This is consistent with vertical tilts found in other studies of MRG waves closer to the date line (see, e.g., Kiladis et al. 2009).

Figure 16 shows the agreement between the MRG eigenvector signal and MRG-filtered OLR. A Hovmöller plot of May–October 1997 shows that while some events are identified by both methods, the signals largely differ in the events they identify. The narrow range of frequencies retained by the OLR filtering method for a given wavenumber results in some otherwise MRG-like OLR signals being missed, for example,

FIG. 13. As in Fig. 4, but for OLR averaged from 2.5° to 12.5°N regressed against the May–October MRG eigenvector signal at 140°W. The three levels of shading represent OLR anomalies < -1, -1.5, and -2 W m^{-2} ; the three contours represent OLR anomalies of 1, 1.5, and 2 W m^{-2} .

during September near 100°W; this event is identified by the eigenvector signal. The eigenvector signal also sometimes propagates eastward, for example, during May near 120°W.

A time series of the two signals at 140°W is shown in Fig. 16d after normalization. The narrow frequency range used in creating the filtered OLR signal is apparent in the striking regularity of its frequency of oscillations. Like the Rossby wave, there are periods of significant disagreement between the signals.

The pattern correlation between the two signals is shown in Fig. 16b for all longitudes. The agreement is best in the Eastern Hemisphere and the Pacific ITCZ, with a correlation of approximately 0.2–0.3. The agreement is slightly better in boreal summer than boreal winter in most of these regions; virtually no agreement exists near South America during any season. When the pattern correlation is computed using only times when the normalized filtered OLR signal is less than -1, the correlation improves slightly over the warm pool during boreal summer. Last, similar to the Kelvin-filtered OLR, the MRG-filtered OLR identifies more "extreme events" than the eigenvector signal.

5. Wave structures associated with eigenvector and/or convective signals

The comparison of the eigenvector signals with filtered OLR indicates that the two signals sometimes identify the same convectively coupled wave events and sometimes do not. Times when the signals both identify strong events would seem to indicate strong coupling between convection and the dry fields in a manner consistent with the linear theory. It is of interest to know what is being identified by each signal when the other signal is not strong.

To explore this question, each of the variables OLR, winds, and geopotential height were again regressed against a wave signal, but for three subsets of the entire 33-yr record. In the first regression, the base signal used is the eigenvector signal, and only days when the normalized eigenvector signal was less than -1 and the normalized filtered OLR signal was greater than -1were used. In the second regression, the base signal was the filtered OLR signal; only days when the filtered OLR signal was less than -1 and the eigenvector signal was greater than -1 were used. Last, in the third regression the base signal was again the filtered OLR signal, but this time only days when both signals were less than -1 were used to create the composite. The number of days used to create each of the composites in this section ranges from approximately 200 days up to 800 days.

a. Kelvin wave

The Kelvin wave is examined first. The case where the Kelvin eigenvector signal is strong but filtered OLR is weak is shown in Fig. 17a. The winds and height are in good agreement with the theory, though some meridional tilt in the winds and height appear to the east of the eigenvector signal. There is essentially no convection present in the composite; it thus appears that the filtered OLR signal was weak on most of these days because of little convective activity rather than "incorrect" propagation.

The case where the filtered OLR signal is strong but the eigenvector signal is weak is shown in Fig. 17b. A region of convective activity is centered at approximately the base longitude (125°W) and 7.5°N. Away from the base longitude there is some upper-level zonal outflow, consistent with a Kelvin structure. There is also, however, strong meridional flow through the center of the convective activity; these winds (as well as weaker winds away from the equator, not shown) have more in common with a Rossby-like structure than a Kelvin structure. Two local maxima can be seen in the geopotential height contours at approximately 15°N and 15°S, also similar to a Rossby-like structure. These features are somewhat consistent with the forced Kelvin response studied by Gill (1980). Yang et al. (2007b) found a similar composite structure over the Pacific ITCZ that they termed "eastward-moving Rossby waves." They found that these waves travel eastward at

FIG. 14. As in Fig. 5, but for fields regressed against the May–October MRG eigenvector signal at 140°W for (a) day -1, (b) day 0, (c) day +1, and (d) day +2. The four levels of shading represent OLR anomalies < -1, -1.5, -2, and -2.5 W m⁻²; the bold gray contour represents OLR anomalies of 1 W m⁻². Geopotential height contours are shown for intervals of 0.5 m, with the zero contour omitted. The maximum wind speed is (a) 2.8, (b) 3.6, (c) 3.1, and (d) 2.0 m s⁻¹.

FIG. 15. As in Fig. 6, but for fields regressed against the May–October MRG eigenvector signal at 7.5°N for (a) day -1, (b) day 0, (c) day +1, and (d) day +2. Geopotential height contours are shown for an interval of 0.4 m with the zero contour omitted. The maximum wind speed is (a) 1.6, (b) 1.5, (c) 1.2, and (d) 0.8 m s⁻¹.

approximately $16 \,\mathrm{m\,s}^{-1}$, consistent with the linear theory for Kelvin waves.

signals are strong than when only the filtered OLR signal is strong.

Last, the case where both signals are strong is shown in Fig. 17c; as expected, both the dry variables and OLR are in good agreement with the theory and previous studies based on filtered OLR. Strong convective activity at the base point longitude centered at approximately 10°N exists, and both winds and height from 10°S to 10°N are in good agreement with the theory, though some meridional winds are also present. Note that stronger convection can be seen for times when both

b. Rossby wave

For the Rossby wave, the composite for times when only the eigenvector signal is strong show circulation and height anomalies in good agreement with the theory near the base point longitude, with upper-level outflow and height troughs along the equator extending across the Pacific Ocean and into South America; see Fig. 18a. Two off-equatorial gyres can be seen in the circulation at

FIG. 16. As in Fig. 8, but averaged from 2.5° to 12.5°N from May through October 1997, and with red contours denoting an MRG-filtered OLR signal of -7 W m^{-2} and black contours denoting an MRG eigenvector signal of -0.25, and with (d) the time series of the two wave signals at 140°W for 1997, normalized by dividing each series by its standard deviation.

the base longitude, consistent with the theory. A weak convective signal is located at approximately 15N centered in one of the gyres.

Figure 18b shows the composite structure for times when only the filtered OLR signal is strong. A region of convective activity associated with the signal is centered at approximately 7.5°N and 140°E. A strong gyre to the south of the equator can be seen in both circulation and height anomalies, shifted to the east relative to Fig. 18a, but otherwise the dynamical fields associated with the filtered OLR signal are significantly weaker than those associated with the eigenvector signal.

The composite for times when both signals are strong shows a very strong convective signal to the north of the equator (see Fig. 18c). The dry variables are in reasonable agreement with the theory, showing strong gyres both north and south of the equator in both circulation and height.

c. MRG wave

Figure 19a shows the composite structure for times when only the MRG eigenvector signal is strong. The

FIG. 17. (a) A map of OLR, wind vectors, and geopotential height anomalies regressed onto the May–October Kelvin eigenvector signal at 125°W (denoted by the red line) using only times where the normalized eigenvector signal was <-1 and the normalized Kelvin-filtered OLR signal was >-1. Negative OLR anomalies depicted by shading. The three levels of shading represent OLR anomalies <-10, -15, and -20 W m⁻². Positive (negative) height anomalies depicted by solid (dashed) black contours; contours are shown for intervals of 2 m with the zero contour omitted. Only statistically significant wind vectors are shown. (b) As in (a), but each field is regressed onto the May–October Kelvin-filtered OLR signal using only times where the normalized filtered OLR signal was <-1 and the normalized eigenvector signal was >-1. (c) As in (b), but only using times when both normalized signals were <-1 or both normalized signals were >1. The maximum wind speed is (a) 5.2, (b) 2.2, and (c) 4.5 m s⁻¹.

circulation and height anomalies are in good agreement with the theory near the base point. The statistically significant winds extend over a much narrower range of longitudes than in the corresponding Kelvin and Rossby composites, and some disagreement with the theory can be seen in the circulation and height anomalies to the east of the base point and north of the equator. Also, the center of the circulation to the west of the signal peak is actually located to the south of the equator. Not surprisingly, at the base longitude very little convection is associated with the eigenvector signal, though convective activity can be seen near the South Pacific convergence zone as well as the Maritime Continent; the latter could perhaps be an indication of an association between the eigenvector signal and the boreal summer intraseasonal oscillation.

The composite for times when only the filtered OLR signal is strong show a strong convective signal in the expected location (see Fig. 19b). Virtually no winds or height anomalies are associated with this wave signal.

Figure 19c shows the composite structure for times when both signals are strong. A region of convective

activity is centered at 10°N and the expected longitude. Winds and height are again in good agreement with the theory, with similar exceptions to those noted above for the eigenvector signal.

6. Combining temporal filtering with the eigenvector projection method

In the previous section, it was shown that differences between the eigenvector signal and filtered OLR exist for all three wave types; these differences are most significant for the Rossby and MRG waves. Thus far, no temporal filtering has been used in creating the eigenvector signals examined in the previous section (though it was used in making the filtered OLR signals that the eigenvectors signals were checked against in the previous section). We next create space-time-filtered versions of the eigenvector signals in order to assess how much the lack of temporal filtering contributes to these differences.

To make a direct comparison with the filtered OLR signal used in the previous section, the same regions of

FIG. 18. As in Fig. 17, but for the Rossby signals. Geopotential height contours are shown for intervals of 4 m with the zero contour omitted. The maximum wind speed is (a) 7.9, (b) 2.4, and (c) 5.1 m s^{-1} .

wavenumber-frequency space are used for filtering the eigenvector signals (see Fig. 20; Wheeler et al. 2000). The space-time-filtered signals were calculated by first removing the mean value for each characteristic variable r, l, and v, and at every longitude the first and last 10% of days are tapered to zero using a cosine tapering function. A spatial Fourier transform is then taken, and the eigenvector signals are created at each point in time by projecting the characteristic variables onto the eigenvector structures. A temporal Fourier transform is then taken of this data. Next, temporal filtering is applied by retaining only those Fourier coefficients that lie within the appropriate regions of wavenumberfrequency space (see Fig. 20). Taking the inverse Fourier transform (in both space and time) results in the filtered eigenvector signals.

These filtered signals are shown in Fig. 21 from 1 July 2009 through 30 June 2010. The filtered signals each evolve with the expected phase and group velocities, though with smaller amplitude than the unfiltered signals since much information has been filtered out. Comparing Fig. 21 with the unfiltered signals during the same time period in Fig. 2 shows that the filtering smooths signal anomalies that do not propagate with the expected speed from the free wave theory. We note that multiple methods have been proposed for space–time filtering to identify CCEWs; see, e.g., Roundy and Schreck (2009) for a definition of Kelvin, Rossby, and

MRG waves that makes use of broader regions of wavenumber–frequency space. Use of such filters retains more of the original signal while allowing for larger deviations from the theoretical free wave speeds.

The degree to which these modified signals correspond with filtered OLR can be assessed through calculating the pattern correlation of the time series of each signal at each longitude. Figure 22 shows the pattern correlation of the unfiltered eigenvector signals with filtered OLR (left column, identical to Figs. 8b, 12b, and 16b), and the filtered eigenvector signals with filtered OLR (middle column). For all three wave types, the pattern correlation increases with the use of space–time filtering.

We note that it is also possible to use filtering on a single variable rather than the eigenvector structures considered here. To assess whether use of the eigenvectors rather than a single dry variable, for example, zonal winds, improves the agreement with OLR, the pattern correlation of a filtered-wind signal is plotted for each wave. For the Kelvin and Rossby waves, filtered symmetric zonal winds are used; the winds are averaged from 10°S to 10°N for the Kelvin wave and 5°S to 10°N for the Rossby wave. For the MRG wave, filtered symmetric meridional winds averaged from 2.5° to 12.5°N are used. The pattern correlations are shown in the right-hand column of Fig. 22. While the pattern correlations between the filtered univariate signals and

FIG. 19. As in Fig. 17, but for the MRG signals. The three levels of shading represent OLR anomalies < -4, -6, and -8 W m^{-2} . Geopotential height contours are shown for intervals of 2 m, with the zero contour omitted. The maximum wind speed is (a) 9.3 and (c) 3.3 m s^{-1} .

filtered OLR are higher than that of the unfiltered eigenvector signals and filtered OLR, for most regions and seasons the correlation is not as strong as when the filtered eigenvector signal is used.

This improvement in agreement with filtered OLR obtained by using the eigenvector structures rather than winds alone is shown in Fig. 23 by taking the difference in the pattern correlations in Fig. 22. Specifically, the difference between the right-hand column and middle column of Fig. 22 is shown in Fig. 23 for each of the three waves. The filtered eigenvector signals exhibit higher pattern correlation with filtered OLR than do their corresponding filtered univariate signals at most longitudes and during most seasons. This higher level of agreement is strongest for the Rossby wave, though we note that the Rossby filtered wind signal might be improved by forming a different metric out of the winds (such as, for example, the difference between symmetric zonal winds at 0° and 15°N). We also note that extensive

phase shift testing was not done, as a primary goal of the paper was to assess the degree to which observed structures couple with convection in accordance with quasi-equilibrium theory; further fine tuning of the signals is left to future work.

7. Discussion

Thus far, the results have primarily focused on the May–October period when the MJO is relatively inactive. During the boreal winter, however, the MJO can be expected to have a significant impact on some or all of the wave signals studied here (e.g., Straub and Kiladis 2003a; Roundy and Frank 2004b; Zhang 2005; Roundy 2008; Dias et al. 2013). Evidence of these impacts was seen in Fig. 2, where the MJO appears to have a significant effect on all three eigenvector signals. Two MJO events appear to project strongly onto the Kelvin eigenvector and weakly onto the Rossby eigenvector

FIG. 20. Regions of the zonal wavenumber–frequency space used for space–time filtering here (see also Wheeler et al. 2000), for Kelvin (black), Rossby (blue), and MRG (red) waves. Dispersion curves shown are for equivalent depths of 8, 12, 25, 50, and 90 m.

during November–February, and evidence of this can be seen in other years with documented MJO activity (not shown). The MRG signal attains its strongest values in early February during the conclusion of an MJO event; this correlation between the MRG signal and MJO termination was also seen in other years as well (not shown).

The fact that the MJO must impact these signals can also be anticipated from straightforward mathematical considerations. Specifically, while the MJO is sometimes classified as a CCEW, its presence is not predicted in the classical shallow water theory. Since the projection method here results in variables that are simply a new set of basis functions, and since all of the reanalysis data are projected onto these basis functions, the MJO must therefore be projected onto some subset of these basis functions (Roundy and Schreck 2009).

While the MJO's convective envelope can project onto each of the wave modes, it has also been shown that the MJO can be composed of CCEWs. The types of CCEWs present within the MJO and their strength depends on the event (see, e.g., Dias et al. 2013). A brief examination of the eigenvector signals during boreal winter indicates that some MJO events clearly contain convectively coupled Kelvin and Rossby waves within their wave envelope. Some events, however, simply

FIG. 21. The (a) K, (b) R_1 , and (c) MRG filtered eigenvector signals from 1 Jul 2009 through 30 Jun 2010 so that only information corresponding to wavenumber–frequency combinations lying in the corresponding region of Fig. 20 is retained.

FIG. 22. (a),(d),(g) Pattern correlation between the eigenvector signals of section 4 with the corresponding filtered OLR signals as a function of longitude for the Kelvin, Rossby, and MRG waves, respectively. (b),(e),(h) Pattern correlation between the space-time-filtered eigenvector signals with the corresponding filtered OLR signals. (c),(f),(i) Pattern correlation between space-time-filtered univariate dry signals with the corresponding filtered OLR signals.

project onto these eigenvectors with no clear association with any convective anomalies traveling like free Kelvin or Rossby waves. Similarly, the conclusion of MJO events often corresponds with a high-amplitude MRG eigenvector signal near the date line. Some of these signals are clearly associated with convection propagating like free MRG waves, while some of these signals propagate much slower than expected; the latter signals contribute to the low-frequency, moderatewavenumber power peaks seen in the power spectrum plots of Fig. 3c.

Additional steps and modifications to the projection method used here were considered by the authors as well. For one, OLR and lower tropospheric moisture data were incorporated into the projection method (not shown) in an attempt to identify eigenvector signals associated with convective activity. Specifically, the constraints

$$OLR = c_1 (\nabla \cdot \mathbf{u}), \tag{14a}$$

$$OLR = c_2 q, \qquad (14b)$$

were imposed, where q is lower tropospheric moisture, and the proportionality constants c_1 and c_2 may be estimated by, for example, simple least squares fitting. Imposing constraint (14a) filters the data so that lower-level convergence and OLR are linearly related; imposing constraint (14b) results in OLR being in phase with lower tropospheric moisture. Several variants of the method outlined here were also tried (not shown). Imposing the constraints had some effect on the Rossby and MRG signals, but the impact was not significant for any of the waves.

FIG. 23. (a) Difference between Figs. 22b and 22c. (b) Difference between Figs. 22e and 22f. (c) Difference between Figs. 22h and 22i.

Other modifications to the projection method were considered as well. The results presented here show the eigenvector signals calculated by retaining only zonal wavenumbers k = 1-10, though other spatial filters were considered. The signals were calculated here by removing a 120-day running mean to reduce the

impact of low-frequency activity like ENSO on the signals; other length running means were considered as well. Also, in order to align peaks in the signal amplitudes with the expected convective anomalies, the zonal Fourier coefficients of each signal were rotated in the complex plane as described in section 3. A variety of angles for this rotation were considered; while some choices other than the ones used in the results presented here do improve the pattern correlation with filtered OLR signals, these improvements are slight and it is unclear that they offer any real improvement in CCEW identification.

Many other methods for identifying CCEWs rely on space-time filtering, restricting their use to near-realtime filtering (e.g., Wheeler and Weickmann 2001). Methods that do not rely on temporal filtering could thus be an important tool for CCEW prediction (Roundy and Schreck 2009). As no temporal filtering was used in creation of the eigenvector signals, the method presented here could be used in combination with any data assimilation method. Alternately, the space-timefiltered version of these signals could potentially be a useful component of near-real-time prediction methods as they provide information about the structure of the dry fields and their coupling with convection. Since space-time filtering of the signals does improve the strength of the association with convection, the eigenvector projection method used here will likely be most powerful when used in concert with other techniques, for example, convective proxies, space-time filtering, and EOFs.

In addition, we note that the method used here has relied on choosing an equivalent depth a priori; many space-time-filtering methods ascribe large regions of wavenumber-frequency space to each wave type in an effort to create signals that incorporate waves of as many equivalent depths as possible; see, e.g., Roundy and Schreck (2009). Also, while the method has been applied here using one particular first baroclinic structure, other choices of vertical basis functions may be used as well.

Finally, as has already been noted above, the methods used here are not able to successfully identify all convectively coupled Kelvin, Rossby, and MRG waves. Observed waves exhibit behavior that is not strictly symmetric or antisymmetric; these waves may not project cleanly onto the theoretical structures used here. Even for regions, seasons, and wave types that have "correct" composites, individual events may vary widely. The current study has focused solely on the Kelvin, Rossby, and MRG waves; it would be interesting to apply the methods used here to study the higherfrequency gravity waves as well. We also note that the method used here should not be expected to identify waves that are not strongly coupled to deep convection.

8. Conclusions

To conclude, a new method has been used for studying the structure and propagation of CCEWs. The method uses only dry variables of shallow water theory, that is, no convective proxy. Furthermore, in contrast with many previous studies that make use of the eigenvalues of the theory, the method here employs a projection onto the wave eigenvectors of the linear theory; no temporal filtering is used.

This method is able to successfully identify CCEWs that propagate with roughly the expected velocities reasonably well for such a simple approach, especially during boreal summer when the MJO is relatively inactive. The method is most successful at identifying convectively coupled Kelvin waves, while the agreement between this method and space-time-filtered OLR is not as good for the Rossby and MRG waves. The composite Kelvin wave propagates across the central and eastern Pacific with the expected eastward speed of $\approx 16 \,\mathrm{m \, s^{-1}}$ and is associated with convection north of the equator. Across the ITCZ, the signal is reasonably well correlated with space-time-filtered OLR. This composite is fairly robust to changes in the base longitude used for the regression within the eastern and central Pacific; for base longitudes closer to the Pacific warm pool, Maritime Continent, and Indian Ocean, the resulting composite exhibits more MJO-like propagation.

The composite Rossby wave propagates with the expected westward speed of $\approx 5 \,\mathrm{m \, s}^{-1}$ and is associated with convection north of the equator. The signal is somewhat correlated with filtered OLR across the Pacific warm pool region; no correlation exists over the central and eastern Pacific. The composite MRG wave exhibits the expected westward phase velocity and eastward group velocity and is also associated with convection north of the equator. This signal is somewhat correlated with filtered OLR across the entire Pacific Ocean. The Rossby composite is somewhat more sensitive to changes in the base longitude, while the MRG composite is fairly consistent using a base longitude anywhere in the central and eastern Pacific.

The absence of temporal filtering suggests the method could potentially be a useful component of real-time prediction methods during boreal winter in the regions where it successfully identifies CCEWs. Furthermore, it was shown that temporal filtering of the eigenvector signals significantly improves the agreement with space– time-filtered OLR signals. It is thus possible that the method could also be useful for near-real-time methods of CCEW prediction that rely on space-time filtering. Finally, it was also shown that, compared to univariate dry signals, use of the full eigenvector structure provides some improved agreement with filtered OLR in most regions and seasons.

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