The MJO skeleton model with observation-based background state and forcing

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The MJO skeleton model, a nonlinear oscillator model, has previously been shown to have solutions that exhibit several features in common with observed MJOs. In prior studies, model solutions have been found using mathematically simple (constant or sinusoidal) identical moistening and radiative forcing functions. Here we investigate whether this model can also produce realistic regional variability. To do this, observation-based forcing functions are prescribed for latent heat flux and radiative cooling, and model solutions-both linear and nonlinear-are studied. In the stochastic nonlinear model, solutions reproduce the climatological mean convective activity well and the climatological variance reasonably well for such a simple model. In the linearized model, the solutions are found to contain additional structure including a realistic wave envelope of convective activity centered over the warm pool. These linear solutions are then used to identify and compare significant MJO activity in both reanalysis data and stochastic model solutions. Additional potential uses for these analytical and numerical solutions are discussed.

Key Words: Madden-Julian oscillation, tropical intraseasonal oscillation, MJO skeleton model

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1. Introduction

The Madden-Julian oscillation (MJO) has been studied extensively since its discovery by Madden and Julian (1971, 1972). The MJO, sometimes referred to as the 30-60 day oscillation, has been detected in many atmospheric variables including, e.g., zonal winds, pressure, potential temperature, precipitation, and outgoing longwave radiation (OLR), among others, at locations around the globe (Madden and Julian 1994). The effects of the MJO are farreaching as not only is it connected with other weather systems in the tropics, e.g. the Indian monsoon, but it also affects medium and long-range global weather forecasts (Zhang 2005; Lau and Waliser 2012). Given these wide-ranging effects, creating models capable of producing a realistic MJO is an important and ongoing challenge.

As the MJO is the dominant component of intraseasonal variability in the tropical atmosphere, one important test of how well a model produces a realistic MJO is whether it exhibits realistic tropical convective variability on intraseasonal timescales. Some general circulation models (GCMs), especially those which include the effects of small-scale convective processes within the larger wave envelope of convective activity, have recently shown some success in producing realistic variability (see, e.g., Benedict and Randall (2009), Khouider *et al.* (2011)), but this has generally been a difficult test for most GCMs (Lin *et al.* 2006; Kim *et al.* 2009; Waliser *et al.* 2009;

Gottschalck *et al.* 2010; Sperber and Kim 2012). This test must of course also be applied to low-dimensional models of the MJO which attempt to provide insight into the structure of the MJO while containing as little complexity as possible.

One such low-dimensional model, developed by Majda and Stechmann (2009, 2011), has successfully reproduced several primary features of the Madden-Julian oscillation: slow eastward phase speed $\approx 5 \text{ m s}^{-1}$, dispersion relation $d\omega/dk \approx 0$, and horizontal quadrupole vortex structure. The simplest form of the model describes the interaction between the first baroclinic mode of winds and potential temperature, lower tropospheric moisture, and the planetary scale wave envelope of convection in the tropics. Motivated by the stochastic nature of tropical convection, the model has been updated recently to include stochastic convective activity (Thual *et al.* 2014, accepted). The model contains a minimal number of parameters including two prescribed forcing functions: one for latent heat flux and one for radiative cooling.

Solutions to this model have previously been found with mathematically simple (either constant or sinusoidal) forcing functions. Also, the radiative cooling and moisture forcing have previously been chosen to be equal for the sake of simplicity; equal forcing terms do not allow for a Walker circulation, however, even when these prescribed functions have zonal variations. Here we prescribe forcing functions which are motivated by reanalysis datasets. Based on recent results suggesting that outgoing long wave radiation (OLR) is proportional to diabatic heating (Stechmann and Ogrosky 2014), radiative cooling is estimated by a weighted difference of OLR and precipitation data. For latent heat flux, daily reanalysis data is used. Using these observationally-based forcing functions, we examine how well the stochastic skeleton model solutions exhibit realistic (i) climatological mean convective activity and (ii) climatological variance.

Another important test for low-dimensional models such as the skeleton model is whether the theoretical description they offer of the MJO's structure is discernible in reanalysis and/or observational data during periods of significant MJO activity. Models that pass this test offer a means for identifying MJOs which share a common structure with the theory, providing an additional contribution to the ongoing conversation about how best to identify the MJO (Wheeler and Hendon 2004; Kiladis et al. 2005; Straub 2013; Kiladis et al. 2014).

To this end, linear solutions to the deterministic form of the skeleton model have previously been found in the presence of a uniform radiative-convective equilibrium. Their signal has recently been identified in periods of significant observed MJO activity through a method of data projection (Stechmann and Majda 2015). Here, the zonally-varying background state created by the forcing functions described above is used to find linear solutions that contain additional structure, including a wave envelope centered over the warm pool. We then examine the degree to which the signal of these extended linear solutions is seen in anomalies from a seasonal cycle using reanalysis data. Using anomalies isolates the behavior of the model structures in reanalysis data on intraseasonal or shorter timescales, in contrast to the long-time average used to estimate the model parameters.

The rest of the paper is thus organized as follows. Section 2 contains a review of the different forms of the skeleton model and their solutions previously reported in the literature. Section 3 describes the data and methods used, and contains a comparison of the steady-state solution of the model with reanalysis data. Linear solutions to the model are found in Section 4 using an observationally-motivated background state; these solutions are then used to identify the MJO in observational and reanalysis data. Solutions to the stochastic model are found in Section 5 and compared with observations. Discussion of the results is given in Section 6 and a summary of the main findings of the paper is given in Section 7. Some details of the procedure used to calculate the linear solutions and additional comments regarding these solutions are given in the appendix.

2. MJO skeleton model

We first review the different forms of the skeleton model and their solutions previously studied in the literature.

2.1. Deterministic skeleton model

The MJO skeleton model, originally proposed by Majda and Stechmann (2009), is a simple nonlinear oscillator model which couples a Matsuno-Gill type model, without damping, to two additional evolution equations: one for lowertropospheric moisture, and one for the wave envelope of convective activity. The model uses a vertical truncation so that only the first baroclinic mode is retained, i.e. u(x, y, z, t) = $\sqrt{2u^*(x,y,t)}\cos(z)$, etc., so that (dropping stars for ease of notation)

$$u_t - yv - \theta_x = 0, \tag{1a}$$

$$yu - \theta_y = 0, \tag{1b}$$

$$\theta_t - u_x - v_y = Ha - s^{\sigma}, \tag{1c}$$

$$q_t - \bar{Q}(u_x + v_y) = -\bar{H}a + s^q, \qquad (1d)$$

$$a_t = \Gamma(q - q_a)a, \tag{1e}$$

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where (u, v, θ, q, a) represent zonal and meridional winds, potential temperature, lower-tropospheric moisture, and the planetary scale envelope of convective activity, respectively. The equations (1) have been nondimensionalized by standard reference scales, see, e.g., Stechmann and Majda (2015). Equations (1a)-(1c) are the equatorial long-wave equations and represent the dry dynamics, while the variables q and a are included to represent moist convective processes. Equations (1d)-(1e) govern the model interaction between moisture anomalies and convective activity. Equation (1e) is the simplest equation that encapsulates the observation that anomalies in lower tropospheric moisture tend to lead anomalies in convective activity (Myers and Waliser 2003); this equation was originally proposed by Majda and Stechmann (2009). Note that the only nonlinearity in the model is the qa term in (1e).

There are a minimal number of model parameters: $\bar{H} \approx 0.22$ is a constant heating rate prefactor, $s^{\theta}(x, y)$ is the radiative cooling rate, $\tilde{Q} = 0.9$ is a background vertical moisture gradient, $s^q(x, y)$ is a moistening term, q_a is a background moisture state (often taken to be 0 for simplicity), and Γ is a growth/decay rate of the wave envelope of convective activity.

The model (1) can be simplified further by expressing each of the variables and forcing functions as a linear combination of parabolic cylinder functions, e.g.,

$$u(x,y,t) = \sum_{m=0}^{\infty} u_m(x,t)\phi_m(y),$$
(2)

where ϕ_m is the *m*-th parabolic cylinder function. Substituting (2) and analogous expressions for the other variables and forcing terms into (1) results in an infinite set of systems of equations for the spectral coefficients u_m , v_m , etc. This system is best solved by defining characteristic variables $r_m = \frac{1}{\sqrt{2}}(u_m - \theta_m)$ and $l_m = \frac{1}{\sqrt{2}}(u_m + \theta_m)$, see, e.g. (Matsuno 1966; Gill 1980; Majda 2003); these variables can further be transformed into wave variables K and R_m by

$$K = r_0, \qquad R_m = \sqrt{m+1}r_{m+1} - \sqrt{m}l_{m-1}, \qquad (3)$$

where K represents the amplitude of the Kelvin wave, and R_m represents the amplitude of the *m*-th Rossby wave.

The model (1) can now be truncated at any desired meridional mode. The simplest form of the skeleton model can be found by using a meridional truncation retaining only K, R_1 , q_0 , and a_0 ,

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$$K_t + K_x = -\frac{1}{\sqrt{2}}(\bar{H}A - S^{\theta}),$$
 (4a)

$$R_t - \frac{1}{3}R_x = -\frac{2\sqrt{2}}{3}(\bar{H}A - S^{\theta}),$$
(4b)

$$Q_t + \frac{\bar{Q}}{\sqrt{2}}K_x - \frac{\bar{Q}}{6\sqrt{2}}R_x = \frac{\bar{Q}}{6}(\bar{H}A - S^{\theta}) - \bar{H}A + S^q, \quad (4c)$$
$$A_t = \Gamma(Q - Q_A)A, \quad (4d)$$

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where we have used R to denote R_1 for simplicity, $Q = q_0$, $A = a_0$, $S^{\theta} = s_0^{\theta}$ and $S^q = s_0^q$, and $\Gamma = 0.6$ will be the prescribed growth/decay rate. The presence of the radiative cooling S^{θ} in the moisture equation is due to the $\tilde{Q}v_y$ term in (1d).

The only model parameters whose values have not yet been specified here are the cooling and moistening rates S^{θ} and S^{q} . In most previous studies these terms were taken to be constants, i.e. $S^{\theta} = S^{q} = const.$ (Majda and Stechmann 2009, 2011; Chen and Stechmann submitted; Thual et al. 2014, accepted). The case where these terms are equal and vary sinusoidally with longitude was also considered by Majda and Stechmann (2011) and Thual et al. (2014, accepted). A brief overview of the model solutions previously found with equal S^{θ} and S^{q} is given next.

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2.2. Solutions to the deterministic model

Each of the four variables K, R, Q, and A in the model (4) can be decomposed into a steady background component and anomalies, i.e.

$$K = K_s(x) + K_*(x,t), \qquad R = R_s(x) + R_*(x,t),$$

$$Q = Q_s(x) + Q_*(x,t), \qquad A = A_s(x) + A_*(x,t).$$
(5)

After substituting (5) into (4), the background state is determined by

$$\partial_x K_s = -\frac{1}{\sqrt{2}} (\bar{H}A_s - S^\theta), \tag{6a}$$

$$\partial_x R_s = 2\sqrt{2}(\bar{H}A_s - S^\theta),\tag{6b}$$

$$\frac{Q}{\sqrt{2}}\partial_x K_s - \frac{Q}{6\sqrt{2}}\partial_x R_s = \frac{Q}{6}(\bar{H}A_s - S^\theta) - (\bar{H}A_s - S^q), \quad (6c)$$

$$Q_s = Q_A. \tag{6d}$$

As pointed out by Majda and Klein (2003), it follows that for a steady-state solution to exist in such a model without damping, it must be the case that

$$\overline{\overline{H}A_s} = \overline{S^\theta} = \overline{S^q},\tag{7}$$

where $\overline{f} = \int_0^L f \, dx$; here L is the circumference of the earth. Note that this condition is not required for solutions to exist in the traditional Matsuno-Gill model because of the damping used there. Also, the Matsuno-Gill model is frequently solved on the real line in the literature (see, e.g., Gill (1980); Gill and Rasmusson (1983)), while here the model at hand is solved on the periodic domain [0, L]. Combining (6a)-(6c) leads to

$$\bar{H}A_s(x) = \frac{S^q(x) - \tilde{Q}S^\theta(x)}{1 - \tilde{Q}}.$$
(8)

Thus the model background amplitude of convective activity is a weighted sum of the prescribed cooling and moistening rates. If these rates are prescribed as constants with longitude, the background convective state is also constant. If they are prescribed by zonally varying functions but still equal to one another, the background convective state is also zonally varying, but no background Walker circulation can exist; this can be seen by setting $S^q(x) = S^{\theta}(x)$ in equations (8) and (6).

If these rates are taken as equal but varying with longitude, i.e. $S^{\theta}(x) = S^{q}(x)$, (8) reduces to the background state $\overline{H}A_{s} =$ $S^{\theta} = S^{q}$ considered by Majda and Stechmann (2011) and Thual et al. (2014, accepted). Substitution of (8) into (6a) and (6b) yields zonally-varying background K and R. This background state was studied by Stechmann and Ogrosky (2014). It was shown that the background state of K and R could be accurately predicted when outgoing longwave radiation (OLR) was used as a proxy for $\bar{H}A_s - S^{\theta}$, i.e., the sum of latent and radiative heating. While other heat sources certainly contribute to the total heating, e.g. eddy sensible heat fluxes, these are neglected here (as they are in some other simple models of the tropical atmosphere, see, e.g., Khoudier and Majda (2006)) due to the dominant role played by latent and radiative heat processes in the tropics. For this reason, the phrase 'total diabatic heating' will sometimes be used here to describe the sum of latent and radiative heating.

After removing the background state from (4), the anomalies are governed by

$$\partial_t K_* + \partial_x K_* = -\frac{H}{\sqrt{2}} A_*, \tag{9a}$$

$$\partial_t R_* - \frac{1}{3} \partial_x R_* = -\frac{2\sqrt{2H}}{3} A_*, \qquad (9b)$$

$$\partial_t Q_* + \frac{Q}{\sqrt{2}} \partial_x K_* - \frac{Q}{6\sqrt{2}} \partial_x R_* = \left(\frac{Q}{6} - 1\right) \bar{H} A_*, \qquad (9c)$$

$$\partial_t A_* = \Gamma Q_* (A_* + A_s), \qquad (9d)$$

where the effects of the background state are felt entirely through the A_s term in (9d). If the anomalies remain small compared to the base state at all times and locations, i.e. $A_* \ll A_s$, then the nonlinearity Q_*A_* in (9d) can be neglected, making (9) a linear model. The linear solutions to (9) with $A_s = const$. were studied by Majda and Stechmann (2009), one of which bears a striking resemblance in phase speed, dispersion relation, and structure to the observed MJO.

When the nonlinear term Q^*A^* is retained in (9), the solutions contain additional features also seen in observations of the MJO. Transient solutions to the nonlinear model (4) were found numerically by Majda and Stechmann (2011) for both the trivial and zonally-varying background state. These solutions contain all the features of the linear solutions, and also exhibit localized standing oscillations and interactions between the MJO and dry Kelvin and Rossby waves reminiscent of observations. Analytical traveling wave solutions to the nonlinear model (9) were found by Chen and Stechmann (submitted). In addition to the features seen in the linear solutions, these nonlinear solutions exhibit intense active convective phases followed by longer suppressed convective phases.

2.3. Stochastic skeleton model and its solutions

While the deterministic skeleton model (4) produces an MJO with the planetary envelope A of synoptic convective activity, details of the convective activity are left unresolved. These synopticscale processes can impact the MJO; in order to account for their effects, the skeleton model has recently been updated to include a stochastic parameterization of these synoptic-scale processes (Thual *et al.* 2014). Specifically, a stochastic birth-death process that governs the evolution of the wave envelope A was added by defining a random variable

$$a = \Delta a\eta, \tag{10}$$

where η is a nonnegative integer and Δa is a fixed step size. The probability of a given state η evolves according to the master equation

$$\partial_t P(\eta) = [\lambda(\eta - 1)P(\eta - 1) - \lambda(\eta)P(\eta)] + [\mu(\eta + 1)P(\eta + 1) - \mu(\eta)P(\eta)], \quad (11)$$

where λ and μ are the upward and downward rates of transition and have been chosen so that the dynamics of the original model (4) are essentially recovered on average.

Solutions to the stochastic model were found numerically by Thual *et al.* (2014) with both a trivial and a zonally-varying background state. These solutions were found to exhibit realistic convective variability, and individual MJO events occurred in the context of intermittent wavetrains similar to those seen in nature. Solutions to the model truncated at five meridional modes in the presence of a seasonal cycle were recently found as well, and exhibit meridionally asymmetric events reminiscent of the seasonal behavior of observed MJOs (Thual *et al.* accepted).

All of these previous studies of the skeleton model found solutions using equal forcing functions, i.e. $S^{\theta} = S^{q}$. The rest of this paper will be concerned with solutions to the model when $S^{\theta}(x) \neq S^{q}(x)$.

3. Data and Methods

We next seek to construct estimates for each of the model variables K, R, Q, and A, and the model parameters S^{θ} and S^{q} through use of reanalysis and observational data.

H. R. Ogrosky and S. N. Stechmann

3.1. Data

For the dry variables and lower tropospheric moisture, NCEP/NCAR reanalysis daily zonal winds, geopotential height and specific humidity are used, respectively (Kalnay *et al.* 1996). GPCP daily precipitation data is used to estimate the strength of convective heating (Huffman *et al.* 2012). For the moistening rate S^q , NCEP/NCAR reanalysis daily latent heat net flux is used, and for total diabatic heating $\overline{H}A - S^{\theta}$, NOAA daily interpolated outgoing longwave radiation (OLR) is used (Liebmann and Smith 1996). All of these datasets have a horizontal spatial resolution of $2.5^{\circ} \times 2.5^{\circ}$ except for the latent heat net flux which has a spatial resolution of $1.875^{\circ} \times 1.875^{\circ}$ and the GPCP data which has a spatial resolution of $1^{\circ} \times 1^{\circ}$. The time period used in this study is from 1 January 1997 to 31 December 2013, which coincides with the availability of the GPCP precipitation data.

In order to construct observed values for the model variables and parameters, we rely heavily on the approach outlined by Stechmann and Majda (2015); this method will be summarized next.

3.2. Model variables K and R

The model variables K and R are constructed from zonal winds and geopotential height in three steps: (1) a vertical mode truncation to move from 3D (x, y, z) to 2D (x, y), (2) a meridional mode truncation to move to 1D (x), and (3) conversion of primitive variables u and θ to equatorial wave variables K and R.

The first step, a vertical mode truncation, is achieved by associating the pressure levels of 850 hPa and 200 hPa with the bottom, z = 0, and top, $z = \pi$, of the troposphere, respectively. When the velocity is expressed as the sum of the barotropic mode u_{BT} and a first baroclinic mode u_{BC} , the first baroclinic component can be approximated by

$$u_{\rm BC}(x,y,t) = \frac{u(850 \text{ hPa}) - u(200 \text{ hPa})}{2\sqrt{2}}$$
(12)

Geopotential height Z can be used to estimate θ by use of hydrostatic balance, $\partial Z/\partial p = -\theta$, so that

$$\theta_{\rm BC}(x, y, t) = -\frac{Z(850 \text{ hPa}) - Z(200 \text{ hPa})}{2\sqrt{2}}.$$
 (13)

All variables have been made dimensionless through the scales in Stechmann and Majda (2015).

Next, to achieve the meridional projection, the parabolic cylinder functions $\phi_m(y)$ are used. These functions form an orthonormal basis; the first three are

$$\phi_0(y) = \frac{1}{\pi^{1/4}} e^{-y^2/2},\tag{14a}$$

$$\phi_1(y) = \frac{1}{\pi^{1/4}} \frac{1}{\sqrt{2}} 2y e^{-y^2/2},$$
(14b)

$$\phi_2(y) = \frac{1}{\pi^{1/4}} \frac{1}{2\sqrt{2}} (4y^2 - 2)e^{-y^2/2}.$$
 (14c)

The first baroclinic variables $u_{BC}(x, y, t)$ and $\theta_{BC}(x, y, t)$ can be expanded in terms of the basis functions $\phi_m(y)$, as in, e.g., (2), where the spectral coefficients $u_m(x, t)$ are found by projecting u_{BC} onto each basis function $\phi_m(y)$:

$$u_m(x,t) = \int_{-\infty}^{\infty} u_{\rm BC}(x,y,t)\phi_m(y)\,dy.$$
 (15)

Similar formulas apply for θ .

The final step in identifying the model variables K and R in the data is achieved through the definitions of K and R in terms of

 u_m and θ_m :

$$K(x,t) = \frac{1}{\sqrt{2}}(u_0 - \theta_0),$$
(16a)

$$R(x,t) = -\frac{1}{\sqrt{2}}(u_0 + \theta_0) + (u_2 - \theta_2).$$
(16b)

The derivation of the definitions (16) is summarized in Stechmann and Majda (2015) and Majda (2003). The Kelvin wave amplitude K is thus proportional to $\exp(-y^2/2)$ and decays away from the equator, while the first Rossby wave amplitude R includes offequatorial gyres that arise due to the $\phi_2(y)$ terms in (16b).

3.3. Model variables Q and A

The variable Q represents the lower tropospheric anomalies qin water vapor. To estimate Q, anomalies in reanalysis specific humidity data are first nondimensionalized by the reference scale $L_v/c_p\tilde{\alpha}$, where $L_v = 2.5 \times 10^6$ J kg⁻¹ is the latent heat of vaporization, $c_p = 1006$ J kg⁻¹ K⁻¹ is the specific heat of dry air at constant pressure, and $\tilde{\alpha} = 15$ K is the reference potential temperature scale. Next, a weighted average of the dimensionless specific humidity at three levels near the bottom of the troposphere is calculated to move from 3D (x, y, z) to 2D (x, y), and a meridional mode truncation is calculated to move to 1D (x). The weighted average is given by

$$q_{\rm LT}(x, y, t) = \frac{1}{4}q(925 \text{ hPa}) + \frac{1}{2}q(850 \text{ hPa}) + \frac{1}{4}q(725 \text{ hPa}).$$
(17)

This lower tropospheric moisture is then projected onto the meridional modes by the same approach as (15), and the model variable Q is taken as

$$Q(x,t) = \int_{-\infty}^{\infty} q_{\rm LT}(x,y,t)\phi_0(y) \, dy.$$
 (18)

We note that the weights and levels used here are meant to identify lower free tropospheric moisture, consistent with the observations which motivated development of the model, see, e.g., Myers and Waliser (2003). Adding boundary layer moisture to the model is an area of ongoing research.

Before constructing an estimate of A, a comment on the meaning of A is in order. The variable A represents the planetary-scale envelope of convective activity; the A equation is a phenomenological equation based on the observation that deep convection tends to lag positive anomalies in lower tropospheric moisture, see, e.g., Myers and Waliser (2003). With this viewpoint, a measure of convective heating is needed to construct A; while there are multiple ways this could be done, one method would be to use an estimate of tropical rainfall, such as GPCP precipitation data.

However, a second viewpoint arises if the A equation is taken to represent the idea that positive anomalies in total diabatic heating lag positive anomalies in lower tropospheric moisture. With this viewpoint in mind, the model equations (4) would take the form

$$K_t + K_x = -\frac{1}{\sqrt{2}}\bar{H}\tilde{A},\tag{19a}$$

$$R_t - \frac{1}{3}R_x = -\frac{2\sqrt{2}}{3}\bar{H}\tilde{A},$$
 (19b)

$$Q_t + \frac{Q}{\sqrt{2}}K_x - \frac{Q}{6\sqrt{2}}R_x = \left(\frac{Q}{6} - 1\right)\bar{H}\tilde{A} + S^q - S^\theta, \quad (19c)$$

$$HA_t = \Gamma(Q - Q_A)(HA + S^o), \qquad (19d)$$

where $\bar{H}\tilde{A} = \bar{H}A - S^{\theta}$ represents total diabatic heating in contrast to the convective heating only which is represented by $\bar{H}A$. If this viewpoint of the model is adopted, OLR or any other



Figure 1. Time-averaged (a) background convective state $\bar{H}A_s(x)$ calculated from GPCP data, (b) diabetic heating $\bar{H}A_s(x) - S^{\theta}(x)$ calculated from OLR, (c) S^{θ} estimated from $\bar{H}A_s - (\bar{H}A_s - S^{\theta})$, and (d) S^q calculated from latent heat flux. Data averaged from 1 January 1997 through 31 December 2013; annual, DJF, and JJA data shown in black, red, and blue, respectively. All quantities are dimensionless.

dataset which is often used as a proxy for total diabatic heating would be a logical choice for constructing A.

For the rest of this study, we adopt the first viewpoint, and use GPCP data to estimate $\overline{H}A$ in the following way. Precipitation data is recorded in units of mm d⁻¹. The energy released by $m \text{ mm } d^{-1}$ of precipitation at a given location increases the temperature of the surrounding column of air at a rate

$$\bar{H}A = \left(\frac{g\rho_w L_v}{p_0 c_p}\right)m,\tag{20}$$

where $p_0 = 1.013 \times 10^5$ kg m⁻¹ s⁻² is the mean atmospheric pressure at mean sea level, $\rho_w = 10^3$ kg m⁻³ is the density of water, g = 9.8 m s⁻² is acceleration due to gravity, and (20) has units of K d⁻¹. An average precipitation of 1 mm d⁻¹ thus corresponds to a heating rate of ≈ 0.24 K d⁻¹, or a dimensionless heating rate of $\bar{H}A \approx 0.0054$ when scaled by the characteristic heating rate of 45 K d⁻¹. This 2D (x, y) heating rate is converted to 1D (x) by projecting onto the meridional modes in the same manner as the primitive variables u, θ , etc.; A is estimated by the leading meridional mode, i.e.

$$\bar{H}A(x,t) = \bar{H}A_0(x,t). \tag{21}$$

3.4. Estimating A_s , S^{θ} , and S^q

In order to solve the linearized system in (9), an estimate of the background state $\bar{H}A_s$ is needed. We estimate $\bar{H}A_s$ by taking a long-time average of precipitation data. Figure 1(a) shows the annual, December-January-February (DJF), and June-July-August (JJA) dimensionless estimated $\bar{H}A_s(x)$ from averaging GPCP data from 1997 through 2013. The region of highest precipitation occurs over the maritime continent and western Pacific ocean (\approx 80E-180), while local maxima also exist near the Amazon (\approx 80W-50W, most pronounced during the DJF season) and the Congo (\approx 20E-30E). The standard deviation (with respect to longitude) of the annual $\bar{H}A_s$ is 0.0141.

In the full model equations (4) and their stochastic version, we instead must specify the radiative cooling and latent heating functions, S^{θ} and S^{q} , respectively. In a recent paper (Stechmann and Ogrosky 2014), the authors found that when OLR was used to estimate $\bar{H}A_s - S^{\theta}$, the Kelvin wind response given by (6a) was predicted over various timescales with remarkable accuracy. We here adopt the same approach, and use OLR to estimate dimensional $\bar{H}A_s - S^{\theta}$ by

$$\bar{H}A_s - S^\theta = -H_{\text{OLR}} \cdot \text{OLR}_{0a}, \qquad (22)$$

where OLR_{0a} is understood to represent leading meridional mode anomalies from the zonal mean, and where

$$H_{\rm OLR} = 0.056 \text{ K d}^{-1} (\text{W m}^{-2})^{-1}.$$
 (23)

One reason for not considering the zonal mean component of heating here is that we have an accurate estimate of total diabatic heating variations (via OLR variations), but not the zonally uniform component of diabatic heating. The annual, DJF, and JJA averages of dimensionless $\bar{H}A_s - S^{\theta}$ are shown in Figure 1(b). While the overall pattern is similar to precipitation in Figure 1(a), there is higher local variation in OLR near the Congo region (and Amazon to a lesser extent) relative to the maritime continent than is seen in precipitation. The standard deviation of annual $\bar{H}A_s - S^{\theta}$ is 0.0247.

To estimate S^{θ} , one could use an algorithm such as the Hydrologic Cycle and Earth's Radiation Budget (HERB) algorithm (L'Ecuyer and McGarragh 2010). Here, as an alternative, we estimate radiative cooling in the following way: S^{θ} is estimated by the difference between total diabatic heating and convective heating, i.e.

$$S^{\theta} = \bar{H}A_s - (\bar{H}A_s - S^{\theta}); \qquad (24)$$

the result is shown in Figure 1(c). Note that this is not a residual of two poorly estimated quantities as total diabatic heating can be accurately estimated from OLR (Stechmann and Ogrosky 2014).

The zonal variations in estimated radiative cooling S^{θ} are similar in magnitude to those in the estimated convective heating $\overline{H}A_s$; annual S^{θ} has a standard deviation of 0.0129. While it may seem surprising that this estimate of radiative heating variations is the same order of magnitude as condensational heating variations, independent estimates of radiative heating have also identified large zonal variations (L'Ecuyer and McGarragh 2010). For the stochastic model, the annual curve will be used to inform S^{θ} after smoothing by retaining only the first 8 Fourier modes.

For S^q , we use reanalysis latent heat flux data, averaged over the same time period and projected onto the leading meridional mode,

$$S^{q} = H_{\rm LHF} \cdot \rm LHF_{0}.$$
⁽²⁵⁾

The proportionality constant H_{LHF} is estimated by noting that in the model the convective heating represented by $\bar{H}A_s$ should, in the zonal mean, approximately balance the mean cooling due to evaporation, so that

$$H_{\rm LHF} \approx \frac{\overline{\bar{H}A_s}}{\overline{\rm LHF_0}} \approx 0.0088 \text{ K d}^{-1} ({\rm W m}^{-2})^{-1},$$
 (26)

where $\overline{LHF_0}$ is the zonal mean of the leading meridional mode of LHF. The annual, DJF, and JJA averages of S^q are shown in Figure 1(d). The zonal variations of the latent heating source term S^q are smaller in magnitude than the estimates for radiative and convective heating; annual S^q has a standard deviation of 0.0061. For the stochastic model, the annual curve will be used to inform S^q after smoothing by retaining only the first 8 Fourier modes. A summary of the datasets used to construct the steady-state model variables and forcing functions is given in Table 1.

Before finding model solutions, we pause to see how well the steady-state described by (8) holds with the datasets used here.



Figure 2. (a) Left-hand side (LHS) and right-hand side (RHS) of equation (27) averaged from 1 January 1997 through 31 December 2013. (b) Same as (a) but smoothed to retain 3 Fourier modes only. LHS is estimated from GPCP precipitation data using (20). RHS is computed using LHF and OLR data.

Model variable/	Observational surrogate	Relationshin
	suitogate	Relationship
$\bar{H}A_s$	GPCP precipitation	(20)
$\bar{H}A_s - S^{\theta}$	OLR	(22)
S^{θ}	OLR, GPCP precipitation	(24)
S^q	LHF	(25)

Table 1. Datasets used as surrogates for model variables and forcing functions. Equation numbers describe how the model variables and forcing functions were estimated by the datasets.

Equation (8) can be rearranged to

$$\bar{H}A_s(x) = S^q(x) + \tilde{Q} \left[\bar{H}A_s(x) - S^\theta(x) \right].$$
 (27)

Figure 2(a) compares the left-hand and right-hand sides of (27) which represent observed precipitation and a weighted combination of observed latent heating and OLR, respectively; they have a root-mean-square error (RMSE) of 0.0123 and pattern correlation of 0.96. These have been smoothed in Figure 2(b) by retaining only the mean and first three Fourier modes; this smoothed version will be used in Section 4. We note that while OLR and GPCP precipitation data are highly correlated on their own (with a pattern correlation of 0.92) the introduction of LHF to the right-hand side does improve the agreement slightly.

4. Linear theory

Next, linear solutions are found to (9) using a varying background state of convective activity, $A_s(x)$. In this case, each linear eigenmode is not a perfect sinusoid but an oscillation that is localized near the western Pacific warm pool region. As an example application, these eigenmodes will be used to identify the MJO in reanalysis and OLR data.

4.1. The linearized equations

Each of the quantities K_* , R_* , Q_* , and A_* in the linearized form of (9) can be expressed as a superposition of plane-waves, e.g.,

$$K_{*}(x,t) = \sum_{k=-k_{m}}^{k_{m}} \hat{K}_{k} e^{i\left(\frac{2\pi kx}{L} - \omega t\right)},$$
(28)

with similar expressions for R_* , Q_* , and A_* , while the steady background state estimated in Section 3.4 can be decomposed into its Fourier modes

$$A_{s}(x) = \hat{S}_{0} + \sum_{k=1}^{k_{m}} \alpha \left[\hat{S}_{k} e^{i\left(\frac{2\pi kx}{L}\right)} + \hat{S}_{-k} e^{-i\left(\frac{2\pi kx}{L}\right)} \right], \quad (29)$$

where $\alpha \leq 1$ is a parameter introduced to decrease the magnitude of the zonal variations in A_s relative to the mean of A_s ; the significance of this parameter will be discussed in Section 4.2.



Figure 3. Background convective state $\overline{H}A_s(x)$ used in Section 4.

Substitution of (28) and (29) into (9) yields a system of $8k_m + 4$ equations due to the four variables that are each expanded in terms of $2k_m + 1$ wavenumbers k with $-k_m \le k \le k_m$. This is a single system of equations where each wavenumber is coupled to every other wavenumber through the background state $A_s(x)$. In practice, the background state is smoothed to retain only the first N Fourier modes, so that $\hat{S}_j = 0$ for |j| > N; in all cases presented below, N = 1 or 3.

The case of a constant background state studied by Majda and Stechmann (2009) is recovered if we take N = 0, i.e. if $\hat{S}_j = 0$ for $j \neq 0$. In this case a system of 4 equations for wavenumber k_{i_1} decouples from the system for k_{i_2} , with $i_1 \neq i_2$, so that the problem becomes $2k_m + 1$ eigenvalue problems, each for a different wavenumber; each of these can then be solved separately, and the resulting eigenvectors each contain contributions from one wavenumber only. Here, in contrast, N > 0 and each system of four equations is coupled to the other systems through the background state A_s . Additional details and further discussion of the problem and solution procedure are given in the Appendix.

Figure 3 shows the background state for the four combinations of α and N values used to generate the results discussed here: the constant background state $\alpha = 0$ and three zonally-varying background states. Other combinations of parameter values were studied as well, but these four combinations appear sufficient to demonstrate both the effects of a zonally-varying background state and the sensitivity of solutions on the parameters α and N.

4.2. MJO eigenmodes

The solutions to (9) are linear modes of 4 types: dry Kelvin modes, dry Rossby modes, moist Rossby modes, and MJO modes (Majda and Stechmann 2009). Each MJO mode has an averaged-wavenumber, \bar{k} , which can be calculated by

$$\bar{k} = \sum_{k=-k_m}^{k_m} |k| \sqrt{\hat{K}_k^2 + \hat{R}_k^2 + \hat{Q}_k^2 + \hat{A}_k^2}.$$
 (30)

In the case with uniform $A_s = \overline{A_s}$, \overline{k} is an integer for each mode; in the case with zonally varying $A_s(x)$, \overline{k} is a real number that assesses on average what wavenumber contains the most power. The MJO modes can then be ordered by average wavenumber, i.e. they will be denoted by

$$\hat{\mathbf{e}}_1, \, \hat{\mathbf{e}}_2, \dots$$
 (31)

so that $\hat{\mathbf{e}}_1$ refers to the MJO mode with smallest \bar{k} , $\hat{\mathbf{e}}_2$ refers to the MJO mode with \bar{k} larger than that of $\hat{\mathbf{e}}_1$ and smaller than \bar{k} for all other MJO modes, and so on. In some places in the



Figure 4. (a) Phase speed ω/\bar{k} and (b) oscillation frequency $\omega(k)$ for uniform background state $\alpha = 0$ (black crosses), and varying background state, $\alpha = 0.1$, N = 1 (red circles).

text, the alternate notation MJO-1, MJO-2, etc., may be used. As the skeleton model was proposed to describe the planetary-scale dynamics of the MJO, it is the lowest-wavenumber modes, i.e. $\hat{\mathbf{e}}_j$ for say j = 1 - 4, that are of the most relevance here; only these first few modes with small \bar{k} will be studied here.

We first consider the frequencies of the MJO modes with uniform or varying background state. The case of a uniform background state was studied by Majda and Stechmann (2009), where it was shown that the MJO modes have a frequency that is roughly constant with k, and a phase speed of $\approx 4 - 7$ m s⁻¹ for k = 2 - 4, features in common with the observed MJO. This constant background case is depicted by the black crosses in Figure 4. When the modes are found in the presence of a varying background state with $\alpha = 0.1$, N = 1, the phase speed for the MJO-j modes for j = 2, 3, ..., are very similar to their counterparts in the constant background case, see Figure 4(a).

The difference between the uniform and varying cases can be seen more clearly in the structure of the MJO modes. The uniform MJO-2 mode is shown in Figure 5. Positive (negative) anomalies in convective activity are depicted by red (blue) shaded regions. The individual variables K, R, Q, and A are shown as functions of x in panels (c), (e), (g), and (i), while the contribution each wavenumber k makes to each variable is shown in panels (d), (f), (h) and (j). This mode retains its shape as it propagates eastward around the globe with approximate phase speed 5.6 m s⁻¹.

The varying MJO-2 mode is shown in Figure 6 with $\alpha = 0.1$ and N = 1. Note that many of the features seen in the uniform modes are reproduced in the varying modes, e.g. the convective activity A is in quadrature with the other three variables K, R, and Q, and A lags behind Q, R and a pair of anticyclones, and leads K and a pair of cyclones. However there is additional structure created by the coupling between wavenumbers; both the amplitudes and phases of the components of different wavenumbers are fixed relative to each other. This additional structure creates a wave envelope with maximum amplitude centered over the maximum of the background convective heating



Figure 5. Uniform background state MJO-2 mode, i.e. $\alpha = 0$; $\omega \approx 0.0235$ cpd, $\bar{k} = 2.0$. Convective anomalies are depicted by shading. (a) Positive (negative) pressure anomalies are depicted by solid (dashed) lines. (b) Positive (negative) moisture anomalies are depicted by solid (dashed) lines. (c) K, (e) R, (g) Q, and (i) A are shown; the contribution made to each by each wavenumber is shown in (d), (f), (h), and (j), respectively.

anomaly located at approximately 140E as in Figure 3. This envelope has a base of support stretching from roughly 60E to 160W, spanning the Indian ocean, maritime continent, and western Pacific ocean. Consistent with Figure 4, the varying background state also has the effect of shifting power to higher wavenumbers and lowering the phase speed. The average wavenumber for the varying MJO-2 mode is $\bar{k} \approx 3.0$ resulting in a narrower base of support for individual convective anomalies, while for the uniform mode MJO-2 (depicted in Figure 3 in Majda and Stechmann (2009)), there is only a contribution from k = 2.

To further emphasize the difference between the uniform and varying cases, the propagation of the varying MJO-2 mode's convective anomalies is depicted in Figure 7. Linear disturbances propagate eastward through this envelope with phase speed ≈ 4 m s⁻¹, increasing in strength over the Indian Ocean, reaching maximum amplitude near the western Pacific warm pool, and decaying rapidly after crossing the date line. In contrast, the sinusoidal uniform modes have anomalies over the global tropics.

We also note that a background state with zonal variations small compared with the zonal mean, e.g. $\alpha = 0.1$, is sufficient to see significant change in the structure and phase speed of the relevant MJO modes. Increasing the amplitude further results in a wave envelope with narrower base of support through which the linear disturbances propagate, and a higher average wavenumber \bar{k} . Figure 8(a) shows the MJO-2 mode with $\alpha = 0.5$; note that each



Figure 6. Same as Figure 5 but for varying background state MJO-2 mode with $\alpha = 0.1, N = 1; \omega \approx 0.0245$ cpd, $\bar{k} \approx 3.0$.



Figure 7. Propagation of varying MJO-2 mode convective anomalies (shading) with $\alpha = 0.1$, N = 1. (a) t = 0 d, (b) t = 10 d, (c) t = 20 d, (d) t = 30 d.



Figure 8. Varying background state MJO-2 mode with $\alpha = 0.5$, N = 1. $\omega \approx 0.267$ cpd and $\bar{k} \approx 4.2$. Convective anomalies are depicted by shading. (a) Positive (negative) pressure anomalies are depicted by solid (dashed) lines. (b) Positive (negative) moisture anomalies are depicted by solid (dashed) lines.

positive convective anomaly covers a smaller range in longitude than when $\alpha = 0.1$ in Figure 6. We also note that this sensitivity is also seen, to a slightly lesser degree, in the moist Rossby modes, but is not present in the dry modes. In fact, the dry Kelvin and Rossby modes (not shown) are almost entirely unaffected by the background convective state.

Finally, these variations are also sensitive to the number of Fourier modes retained in the smoothed background state. Figure 9 shows the MJO-2, MJO-3, and MJO-4 modes with $\alpha =$ 0.1 and N = 3. The MJO-2 mode is similar to the MJO-2 modes with N = 1 but with a slightly narrower wave envelope. The MJO-3 mode is similar to the MJO-3 mode with N = 1 (not shown), but with narrower base of support for individual anomalies and a slightly broader base of support in the wave envelope. The MJO-4 mode is the lowest-wavenumber mode to show a second region of linear anomaly activity; linear disturbances propagate through the envelope centered over the warm pool as with the other modes, decrease in strength over the Pacific, but regain some strength as they approach South America and cross over the Amazon region. This reflects the local maximum in this region in the background convective state with N = 3, see Figure 3.

4.3. Identifying the MJO in reanalysis data

In Stechmann and Majda (2015), the uniform MJO modes were used to identify the MJO in reanalysis data. This was achieved in two steps: first, reanalysis data $U(x,t) = (K, R, Q, A)^{T}(x,t)$ was projected onto each of the low-wavenumber MJO modes, i.e.

$$\mathbf{MJOS}_{k}^{\wedge}(t) = \hat{\mathbf{e}}_{k}^{\dagger} \hat{\mathbf{U}}_{k}(t), \qquad (32)$$

for k = 1 - 3, where [†] refers to the conjugate transpose and $\hat{\mathbf{U}}_k$ are the Fourier coefficients of U. Thus for each wavenumber k we have a measure of the strength and phase of the MJO-k signal. Second, taking the inverse Fourier transform of (32) gives the realvalued scalar quantity MJOS(x, t), referred to as the MJO skeleton signal. The reanalysis data was found to contain a strong signal of this theoretical structure at times and locations where welldocumented observed MJOs have occurred over the last thirty years.

The first step described above, i.e. projecting reanalysis data onto individual MJO modes, may be extended to the varying background case with little modification. We now consider

8



Figure 10. (a) Evolution of the complex-valued $\text{MJOS}_j(t)$ for j = 2 from 1 July 2009 through 30 June 2010. Here MJOS_j has been rescaled by its standard deviation; points outside the circle are greater than one standard deviation away from the mean. (b) Evolution of MJOSA_j and (c) MJOSP_j for j = 2 over the same time period. Thick red lines denote the period from 1 November 2009 through 26 February 2010.



Figure 9. Varying background state MJO-2, 3, and 4 modes with $\alpha = 0.1$, N = 3. (a) MJO-2, $\omega \approx 0.245$ cpd and $\bar{k} \approx 3.6$; (b) MJO-3, $\omega \approx 0.239$ cpd and $\bar{k} \approx 6.0$; (c) MJO-4, $\omega \approx 0.236$ cpd and $\bar{k} \approx 8.8$. Convective anomalies are depicted by shading. Positive (negative) pressure anomalies are depicted by solid (dashed) lines.

reanalysis data

$$\hat{\mathbf{U}} = [\hat{\mathbf{U}}_{-k_m}, \ \hat{\mathbf{U}}_{-k_m+1}, \ \dots \ \hat{\mathbf{U}}_{k_m-1}, \ \hat{\mathbf{U}}_{k_m}]^{\mathrm{T}},$$
 (33)

with

$$\hat{\mathbf{U}}_{k} = [\hat{K}_{k}, \ \hat{R}_{k}, \ \hat{Q}_{k}, \ \hat{A}_{k}]^{\mathrm{T}}.$$
 (34)

This data is projected onto a single MJO eigenvector $\tilde{\mathbf{e}}_j$ in a manner similar to (32),

$$MJOS_{j}(t) = \tilde{\mathbf{e}}_{j}^{\dagger} \hat{\mathbf{U}}(t)$$

= MJOSA_{j}(t) exp[iMJOSP_{j}(t)], (35)

resulting in a complex-valued scalar function of time, where the magnitude of this signal is denoted $MJOSA_j(t)$ and the phase of the signal is denoted by $MJOSP_i(t)$. (Note that here we use



Figure 11. (a,b) Same as Figure 10(a,b) but from 1 July 2007 through 30 June 2008. (c) MJOSA signal as in Stechmann and Majda (2015) during the same time period. Thick red lines denote December 2007 and January 2008. (d,e) Wavenumber-2 and 3 components of MJOSA, respectively.

MJOS, rather than MJOS^{\wedge} as in (Stechmann and Majda 2015), to denote the signal in Fourier space). The results in this section were found using the MJO modes with $\alpha = 0.1$ and N = 1, and using reanalysis data smoothed to retain only wavenumbers $k = \pm 1, \pm 2, \pm 3$. In all figures, the signal MJOS_j has been normalized by its standard deviation over the sixteen year period 1 July 1997 through 30 June 2013; an MJOSA value of one thus indicates a signal one standard deviation away from the complex mean of approximately zero.

The evolution of $MJOS_i$, $MJOSA_i$ and accumulated $MJOSP_i$ is shown in Figure 10 for j = 2 from 1 July 2009 through 30 June 2010. This period overlaps with the Year of Tropical Convection (YOTC) (Moncrieff et al. 2012; Waliser et al. 2012). The most significant MJO-like activity during YOTC occurred during the 2009-10 DJF season. The MJOSA₂ signal in panel (b) is strongest in early November and late January through February of this season. The variations in signal strength from late November through early January indicate that the observations only partially reflect the theoretical structure of the MJO skeleton model with varying background state during this time. In panel (c), an increase of 2π in the accumulated phase indicates the signal MJOS has completed one counterclockwise cycle around the complex plane. Periods of elevated MJOSA signal correspond with an upward trend in the accumulated phase MJOSP₂; the increase indicates eastward propagation. Note that the accumulated phase is only meaningful during time periods of a strong MJOSA signal. In panel (a), these two trends are shown together by plotting the daily values of the complex-valued signal MJOS₂.

The pattern correlation can be used to compare the MJOSA₂ signal with other MJO indices currently in use and is here defined as

$$PC(f,g) = \frac{\frac{1}{T} \int_0^T f(t)g(t)dt}{\left[\frac{1}{T} \int_0^T |f(t)|^2 dt\right]^{1/2} \left[\frac{1}{T} \int_0^T |g(t)|^2 dt\right]^{1/2}},$$
 (36)

where T is the length of time considered and f and g are two MJO indices defined on the time period [0, T]. During the time period shown in Figure 10, i.e. 1 July 2009 through 30 June 2010, the MJOSA₂ signal has a pattern correlation of 0.86 with the Real-time Multivariate MJO (RMM) index (Wheeler and Hendon 2004) and a pattern correlation of 0.87 with the OLR MJO Index (OMI) (Kiladis *et al.* 2014); the RMM and OMI have a pattern correlation of 0.90 with each other during the same time period. Note that in contrast to the RMM and OMI indices, the MJOSA₂ does not rely on empirical orthogonal functions (EOFs) and does not use temporal filtering.

The zonally-varying $MJOS_2$ and $MJOSA_2$ signals are shown for a second time period from 1 July 2007 through 30 June 2008 in Figure 11(a,b). Spikes can be seen in $MJOSA_2$ in the months of December 2007 and January 2008 (shown in red) suggesting significant MJO activity. The elevated signal during the months of December and January corresponds with an upward trend in the accumulated phase $MJOSP_2$ indicating eastward propagation (not shown). We note that these features are also pronounced in modes $MJOS_3$ (not shown).

Figure 11(c) shows the MJOSA signal from Stechmann and Majda (2014) during the same time period, and panels (d) and (e) show the wavenumber 2 and 3 components of this signal, respectively. As with the MJOSA₂ signal, a prolonged elevated MJOSA signal is seen throughout December and January. This elevated signal is seen most clearly when using all three uniform MJO-*j* modes for j = 1, 2, 3 (panel c); the individual components MJO-2 (panel d) and MJO-3 (panel e) show an elevated signal as well, though less pronounced than the total signal.

While many similarities exist between the uniform and varyingbackground signals, including a pattern correlation of 0.92 during the year shown in Figure 11, they are not identical for at least two reasons. First, the MJOSA is elevated whenever the theoretical structure of the skeleton model is present at any longitudes. In contrast, the MJOSA₂ is elevated when this theoretical structure is present at longitudes in the base of support of the wave envelope in Figure 6, i.e. roughly 60E to 160W. Restricting the search for the theoretical structure of the skeleton model to those longitudes where the MJO is most active can be seen as a conceptual improvement over the original data projection technique. Second,



Figure 12. S^q and S^{θ} used for the stochastic model simulations.

the MJOSA signal in Stechmann and Majda (2014) uses OLR, rather than precipitation data, for convective heating A. During this time period, the MJOSA₂ signal has a pattern correlation of 0.88 with the RMM index (Wheeler and Hendon 2004), and a pattern correlation of 0.89 with the OMI index (Kiladis *et. al.* 2014).

The second step in the technique used in (Stechmann and Majda 2015) combines the information from all the relevant MJO modes into a single quantity MJOS(x, t). Such a step could potentially be used here as well, although it is complicated by the zonally-varying base state $A_s(x)$. Furthermore, the need for a function MJOS(x, t) is lessened here since each individual eigenmode already has an interesting zonal structure that is not simply a single sinusoid. Lastly, by using a single eigenmode only, one can obtain a compact representation of the MJO in terms of just two numbers, its amplitude and phase.

5. Nonlinear stochastic model

We next turn to the stochastic form of the skeleton model. In Thual *et al.* (2014), the model was solved with equal forcing functions, i.e. $S^q(x) = S^{\theta}(x)$, where each function was either a constant or a single sinusoid. Here, we use forcing terms $S^q(x)$ and $S^{\theta}(x)$ that are informed by the estimates found in Section 3. Specifically, using the annual averages shown in Figure 1(c,d) as our starting point, two additional steps are taken: (i) only the first 8 Fourier modes are retained in order to include variations on planetary scales only, and (ii) the Fourier coefficients for $k = \pm 1 - 8$ are multiplied by a factor β , i.e. if \hat{S}_k^q is the wavenumber-*k* Fourier coefficient of S^q shown in Figure 1(d), then

$$\tilde{S}^{q} = \hat{S}_{0}^{q} + \beta \sum_{k=1}^{8} \left[\hat{S}_{k}^{q} e^{i\left(\frac{2\pi kx}{L}\right)} + \hat{S}_{-k}^{q} e^{-i\left(\frac{2\pi kx}{L}\right)} \right], \quad (37)$$

and \tilde{S}^q will be used as our forcing function (dropping tildes from here). An analogous formula for S^{θ} is used, with identical value for β as that used in (37). Figure 12 shows the forcing functions used to generate the solutions shown below; here $\beta = 0.1$. The addition of the parameter β will be discussed further in Section 6.

One might anticipate that these stochastic model solutions would contain features in common with both the uniformbackground stochastic solutions, e.g. the formation of intermittent MJO wavetrains, and the varying-background linear solutions discussed in Section 4, e.g. locally enhanced convective activity at longitudes with strong heating. It will be shown that this is indeed the case, and a quantitative comparison with reanalysis data will be made.

5.1. Convective activity

The model was solved using the numerical procedure described by Thual *et al.* (2014). The solver was run for 20,000 days which was enough time for the model solution to settle into a statistical



Figure 13. Howmoller plot of (a) observed $\overline{H}A$ and (b) model solution $\overline{H}A$. Observed data is shown for DJF seasons 2005/06 - 2011/12; model data is shown for a continuous 840-day period.

steady-state; such a simulation can be run relatively quickly, since 400 days of simulation time takes roughly only 1 minute of wallclock time on a typical laptop computer. The solution's convective activity $\overline{H}A$ is shown as a Hovmoller plot for an 840-day period near the end of the simulation in Figure 13(b).

The observed convective activity is shown in Figure 13(a) for seven boreal winters from 16 November 2005 through 15 March 2012, i.e. the lowest portion of the Hovmoller plot depicts the time period 16 November 2005 through 15 March 2006. Black horizontal lines separate the seven extended DJF seasons; note that the black lines are omitted in the model Hovmoller plot as a continuous 840-day period of time is displayed. In the observed data, several events of slow, eastward propagation can be seen, e.g. December 2007 through January 2008 and December 2009 through January 2010. Note that the observed and modeled convective activity should not be compared directly day by day, as each plot is a representative sample of the climatological convective variability; they should be compared statistically instead.

Both the models and observations in Figure 13 show much high frequency/wavenumber activity, particularly the observations; the higher noise level in observations is at least in part due to the higher spatial resolution of the reanalysis data (144 zonal gridpoints, $\Delta x \approx 280$ km) than that of the model simulation (64 gridpoints, $\Delta x \approx 625$ km). In order to focus on the planetary scale dynamics on intraseasonal timescales of relevance for the MJO, both the sixteen year period of 120-day extended DJF seasons described above for the observed data and sixteen 120-day segments of the numerical simulation were filtered. This was achieved by applying a cosine-tapering function to the first and last twelve days of each segment, and then taking the Fourier transform in both space and time. Only wavenumbers $k = \pm 1, \pm 2, \pm 3$ and frequencies $1/90 \le \omega \le 1/30$ cpd were retained; the remaining data was then transformed back to physical space.

The filtered data is shown in Figure 14 for observed and modeled $\bar{H}A$. While propagation can be seen in both the eastward and westward directions, the eastward propagation appears to be the dominant feature in both observed and modeled $\bar{H}A$. In the model data, the strongest anomalies appear between day 500 and 720 as two somewhat distinct periods of MJO activity. Individual



Figure 14. (a) Observed and (b) modeled $\overline{H}A$ filtered by removing the statistical steady-state and retaining only wavenumbers $k = \pm 1, \pm 2, \pm 3$ and frequencies $1/90 \le \omega \le 1/30$ cpd.



Figure 15. Power spectrum of (left) observed and (right) modeled $\overline{H}A$.

anomalies have similar strength and propagation speed to the observed anomalies. The intermittency in MJO activity seen in the model is also reminiscent of that in the observations, with several successive MJO events followed by periods of smaller anomalies. The largest anomalies in the observed data generally occur from roughly 60E through 150W; the strongest anomalies in the model solution occur in a similar range, though strong anomalies can also be seen near South America and the Atlantic Ocean, e.g. day 760 through 840. Note again that the observed and modeled convective activity should be compared statistically, rather than directly day by day.

The logarithm of the wavenumber-frequency spectrum is shown in Figure 15 for the entire sixteen-year period of observed $\bar{H}A$ and the last sixteen years of modeled $\bar{H}A$. The observed data represents anomalies from a seasonal cycle, calculated as the sum of the mean and first three seasonal harmonics. The spectrum was then calculated by splitting the anomalies into successive 120-day segments, removing the mean, tapering the first and last 10% of each segment, and averaging the power over all 120-day segments. The spectrum of observed $\bar{H}A$ shows eastward power in the MJO frequency range at low wavenumbers, as does the spectrum of modeled $\bar{H}A$. Both plots show considerably more power in the eastward than the westward direction for wavenumbers $k = \pm 1, \pm 2, \pm 3$.

The statistical steady state of the raw data and the variance of the filtered data are shown in Figure 16. The agreement in mean convection $\overline{H}A$ is very good, with a pattern correlation



Figure 16. (a) Steady state of observed $\overline{H}A$ (solid black line) and model-predicted $\overline{H}A$ (red dashed line). (b) Variance of filtered observed and model-predicted $\overline{H}A$.

of 0.96. The agreement in variance is good from approximately 50E to 170W, while the model overestimates the variance near South America and Africa. This agreement is somewhat sensitive to the choice of model parameter values, and to the choice of wavenumber-frequency box size in the filtering step. Note that both eastward and westward power were retained in the filtering technique described above.

5.2. Data projection

In Section 4, the extended linear MJO modes were used to identify a period of significant MJO activity through data projection. This same method is next used to identify significant MJO events in the stochastic model solutions. The model data is again filtered by removing all wavenumbers except $k = \pm 1, \pm 2, \pm 3$ and all frequencies except $1/90 \le \omega \le 1/30$ cpd. This filtered data is then projected onto the eigenmodes in the same manner as the observed data in Section 4. Results are shown here for the MJO-2 mode.

Figure 17 shows the evolution of MJOSA₂ and accumulated MJOSP₂ during a ten year period. The observed signal MJOSA₂ is shown for the ten year period from 1 July 2003 through 30 June 2013; the model signal is also shown for a ten-year period. Large fluctuations tend to occur on shorter timescales in the observed data than the model data. Both model and observations show a tendency towards eastward propagation, or positive phase accumulation, though periods of westward movement occur as well. Periods of extended eastward propagation in the model, e.g. day 1800 to 2000, have a frequency of roughly $\omega = 1/40$ cpd, consistent with the linear results. These periods are occasionally interrupted by (generally shorter) periods of westward propagation which slow the phase accumulation. This phase accumulation can be further quantified by calculating an average annual phase accumulation. When the stochastic model is run with a zonally-varying background state for an extended 20,000 day simulation, the average annual phase accumulation produced by the model is 35% of that seen in observations. When the model is run with a uniform background state (not shown), the resulting phase accumulation exhibits the same qualitative behavior but produces an average phase accumulation of only 19% of the observations.

The daily values of $MJOSA_2$ are displayed in a histogram in Figure 18 for both observed and modeled data. The distribution of the modeled and observed signals are very similar including long tails that correspond to large-amplitude MJO events. We note that the distribution of signal amplitudes when the stochastic model is run with a uniform background state (not shown) is also similar to observations.

Some additional comparisons between the model and observed $MJOS_2$ signals were made by the authors, including a comparison of extended periods of high values of $MJOSA_2$; initial results show good agreement in the length and behavior of these large events between the model and reanalysis data. Further

comparisons of the model and observed signals using other standard MJO indices are currently being undertaken by collaborators.

6. Further discussion of sensitivity studies

Several comments should now be made. First, the data projections presented here were conducted a second time with OLR as a proxy for $\overline{H}A$, consistent with the alternative viewpoint of the meaning of $\overline{H}A$ discussed in Section 3. Using OLR rather than precipitation data did not significantly alter the main features of the projections described here.

Second, in addition to the parameter sensitivity studies already described in Section 4, the sensitivity of the stochastic model results to the model parameter values of Γ , $\overline{S^q}$, $\overline{S^{\theta}}$, and β was examined. As Γ sets the rate of growth/decay in the planetary scale envelope of convective activity, higher values of Γ result in a power shift to higher frequencies in convective activity, though there is little change in the variance over long time periods. Different values for the zonal means of the steady forcing functions, i.e. $\overline{S^q}$ and $\overline{S^{\theta}}$, tended to also shift the frequencies and had some impact on the ratio of eastward-to-westward power. This frequency dependence on the parameters Γ and $\overline{S^{\theta}}$ is consistent with the approximate frequency of MJOs in the linearized skeleton model, which was shown by Majda and Stechmann (2009) to be

$$\omega_{\rm MJO} \approx \sqrt{\Gamma \overline{S^{\theta}} (1 - \tilde{Q})}.$$
 (38)

Increasing β , i.e. increasing the amplitude of zonal variations in the forcing functions, results in stronger zonal variations in the variability and background state.

Finally, additional model terms were considered for the latent heat flux in the moisture evolution equation. For example, an initial investigation was made into incorporating a bulk parameterization; this is part of an ongoing area of investigation, though initial attempts have not significantly improved the agreement between model solutions and observations. An attempt was also made to incorporate the effects of evaporation-wind feedback; this is also an area of ongoing research.

7. Conclusions

To conclude, the MJO skeleton model has been assessed by comparison with reanalysis and observational data. Solutions to the model with varying forcing functions compare favorably with reanalysis data in several ways. Model solutions were also used to design new ways of analyzing observational data.

The presence of zonal variations in the steady forcing creates a realistic background state of convective activity. This background state creates additional realistic structure in the linear solutions including a wave envelope centered over the warm pool through which convective events reminiscent of the MJO pass. Signals of these linear solutions are seen in reanalysis data during periods of significant MJO activity, suggesting that this additional structure is indeed present during some MJOs. This MJO structure with warm-pool envelope is achieved here with a compact index defined by just two scalars: its amplitude MJOSA(t) and phase MJOSP(t) (as opposed to the function MJOS(x, t) defined in Stechmann and Majda (2015)).

Stochastic nonlinear simulations were also found using observation-based forcing, and the extent to which these simulations reproduce realistic zonal variations in the climatological mean state and intraseasonal variability was explored. It has been shown that these model solutions also contain additional structure created by the zonal variations in forcing including realistic convective variability throughout much of the tropics,



Figure 17. (a) MJOSA₂ and (b) accumulated MJOSP₂ for observed data from 1 July 2003 through 30 June 2013. (c) MJOSA₂ and (d) accumulated MJOSP₂ for model data over an identical length time period (approximately 3650 days).



Figure 18. Histogram of $MJOSA_2(t)$ values for (a) observed and (b) model data. Time periods are identical to those of Figure 17. The bin width is 0.05, and 80 bins were used spanning the range of 0 to 4.

a dominance of eastward power over intraseasonal timescales and planetary spatial scales, and MJO events localized within the linear wave envelope. The intermittency of the MJO events in these stochastic solutions is also reminiscent of those seen in observations.

One could also construct more complicated and realistic background states with the observational data than those presented here. For instance, adding time-variations to the forcing terms S^q and S^θ could allow for a seasonally-varying simulation, and adding additional meridional modes could allow for important additional off-equatorial features, some of which have been explored recently by Thual *et al.* (accepted) with equal forcing functions; these questions are left for future work.

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Appendix A. Computing the linear eigenmodes with a zonally-varying background state

Substitution of (28) and (29) into (9) yields a system of $8k_m + 4$ equations which can be written in terms of each wavenumber k

such that $-k_m \leq k \leq k_m$,

$$\omega \hat{K}_k = \frac{2\pi k}{L} \hat{K}_k - \frac{i}{\sqrt{2}} \hat{A}_k, \qquad (39a)$$

$$\omega \hat{R}_k = -\frac{2\pi k}{3L} \hat{R}_k - \frac{2i\sqrt{2}}{3} \hat{A}_k, \qquad (39b)$$

$$\omega \hat{Q}_k = \frac{2\pi k \tilde{Q}}{\sqrt{2}L} \hat{K}_k - \frac{2\pi k \tilde{Q}}{6\sqrt{2}L} \hat{R}_k + \left(\frac{\tilde{Q}}{6} - 1\right) i \hat{A}_k, \qquad (39c)$$

$$\omega \hat{A}_{k} = i \hat{Q}_{k} \hat{S}_{0} + \alpha \sum_{\substack{m+n=k, \ m \neq k \\ |m|, |n| \leq k_{m}}} i \hat{Q}_{m} \hat{S}_{n}.$$
(39d)

There are thus $2k_m + 1$ such wavenumbers coupled to each other through the equations (39d). These systems together comprise an eigenvalue problem,

$$(\mathbf{A} - \omega \mathbf{I})\hat{\mathbf{U}} = 0, \tag{40}$$

where I is the $4(2k_m + 1) \ge 4(2k_m + 1)$ identity matrix, and \hat{U} is a column vector with $4(2k_m + 1)$ entries,

$$\hat{\mathbf{U}} = [\hat{\mathbf{U}}_{-k_m}, \ \hat{\mathbf{U}}_{-k_m+1}, \ \dots \ \hat{\mathbf{U}}_{k_m-1}, \ \hat{\mathbf{U}}_{k_m}]^{\mathrm{T}},$$
 (41)

with

$$\hat{\mathbf{U}}_k = [\hat{K}_k, \ \hat{R}_k, \ \hat{Q}_k, \ \hat{A}_k]^{\mathrm{T}},$$
 (42)

and

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_{-k_m} & \mathbf{S}_1 & \mathbf{S}_2 & \dots & \mathbf{S}_{2k_m} \\ \mathbf{S}_{-1} & \mathbf{A}_{-k_m+1} & \mathbf{S}_1 & \dots & \mathbf{S}_{2k_m-1} \\ \mathbf{S}_{-2} & \mathbf{S}_{-1} & \mathbf{A}_{-k_m+2} & \dots & \mathbf{S}_{2k_m-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{S}_{-2k_m} & \mathbf{S}_{-2k_m+1} & \mathbf{S}_{-2k_m+2} & \dots & \mathbf{A}_{k_m} \end{bmatrix},$$
(43)

where

for $-2k_m \leq j \leq 2k_m$. As mentioned in the main text, the background state is smoothed to retain only the first N Fourier modes, so that $\hat{S}_j = 0$ for |j| > N.

The eigenvalue problem in (40) is solved numerically using MATLAB's eig function. We note that in spectral calculations, the linear solutions are susceptible to distortion due to the truncated system of Fourier modes. To ensure the robustness of the solutions, k_m must be taken large enough so that the lowest- \bar{k} MJO modes are insensitive to increasing k_m further. The convergence of the eigenvalues was tested by increasing k_m and tracking the change in both eigenvalues ω_j and the components of the eigenvectors \hat{e}_j for j < 6. The change in each of these values when k_m was increased by 1 was less than 10^{-14} for $k_m > 60$ in most results reported here; for cases where this change was greater than 10^{-14} , k_m was increased as necessary, up to a maximum value of $k_m = 100$. Consequently, the eigenmodes presented here are robust.

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