Stochastic models for convective momentum transport

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The improved parameterization of unresolved features of tropical convection is a central challenge in current computer models for long-range ensemble forecasting of weather and short-term climate change. Observations, theory, and detailed smaller-scale numerical simulations suggest that convective momentum transport (CMT) from the unresolved scales to the resolved scales is one of the major deficiencies in contemporary computer models. Here a combination of mathematical and physical reasoning is utilized to build simple stochastic models which capture the significant intermittent upscale transports of CMT on the large scales due to organized unresolved convection from squall lines. Properties of the stochastic model for CMT are developed below in a test column model environment for the large scale variables. The effects of CMT from the stochastic model on a large scale convectively coupled wave in an idealized setting are presented below as a nontrivial test problem. Here the upscale transports from stochastic effects are significant and even generate a large scale mean flow which can interact with the convectively coupled wave.

stochastic model | convective momentum transport | atmospheric convection parameterization | tropical atmospheric convection

Abbreviations: CMT, convective momentum transport; GCM, general circulation model; CCW, convectively coupled wave

Moist convection in the tropics has a profound impact on the ability to predict extended range weather and short term climate change (1). The reason for this is the observed complex multi-scale features of organized, coherent, tropical convection across a wide range of scales varying from tens of kilometers and a few hours to the planetary scale of order 40,000 km on intraseasonal time scales with significant energy transfer across these scales (2–5). The current computer models for prediction of both weather and climate involve general circulation models (GCM) where the physical equations for these extremely complex flows are discretized in space and time and the effects of unresolved processes are parameterized according to various recipes. Typical mesh spacings of order 40 to 80 km are used for extended-range ensemble predictions and order 100 to 200 km for climate simulations; despite a large effort and some advances, the skill of contemporary GCMs in capturing these large scale patterns in the tropics is modest in the best circumstances (6). Contemporary observations (7–9), theory (10–13), and cloud resolving numerical simulations (14, 15) all point to the role of convective momentum transport (CMT) as one of the main mechanisms where organized moist convection on smaller scales affects the wave patterns on larger scales; the dynamic effects of CMT are poorly resolved by contemporary GCMs (16) although recent deterministic parameterization has improved the mean climatology (17, 18) and the El Niño Southern Oscillation (19). The main goal of this contribution is to develop a simple stochastic model to capture unresolved features of CMT.

The motivation for such a stochastic model comes from the observations of CMT (7, 8); these detailed observations show that, in the mean, CMT is downscale (damping on the large scales) and weak, but the fluctuations about the mean are very large and can have intense bursts of upscale transport (amplification on the large scales). These are exactly the types of circumstances where suitable coarse-grained stochastic models are able to capture the intermittent impact of smaller scale events on the larger scales. Theory and applications for such types of coarse-grained stochastic models have been developed recently (20–24). In fact, it is increasingly apparent that suitable stochastic parameterization is an important strategy to improve the fidelity of unresolved features in contemporary weather and climate models for a variety of physical processes (24). Here we develop and test a simple stochastic model for CMT which accounts for the novel physical features of energy transfer in this context. For our purposes here, we utilize the equations for the zonal momentum u in the large scale dynamics of a hydrostatic, two-dimensional, Boussinesq fluid to illustrate CMT; this equation is given by

$$\partial_t u + \partial_x(u^2) + \partial_z(wu) + \partial_x p = -\partial_z(\overline{w'u'}) \equiv F_{CMT}. \qquad \textbf{[1]}$$

In **[1]**, w is the large scale vertical velocity and p is the pressure. The right hand side of **[1]** is the force due to CMT, F_{CMT}, from the turbulent averaging of the fluctuations u' and w' on the smaller unresolved scales. Most contemporary GCMs parameterize this effect as cumulus friction on the large scales, i.e.,

$$-\partial_z(\overline{w'u'}) \approx -d_c(u - \hat{u}), \qquad \textbf{[2]}$$

where $d_c > 0$ is a damping constant and $\hat{u}$ is the vertical average of u. Such a parameterization strategy is broadly consistent with detailed observations that CMT is damping on the large scales in the mean and, in fact, scattered upright convection on small scales mixes momentum and induces damping; however, **[2]** ignores the key observational fact that CMT can transport energy upscale intermittently through convective organization on the unresolved smaller scales. Such effects are captured by the simple stochastic models developed next.

A Simple Stochastic Model for CMT

The stochastic model for CMT developed here involves a three-state continuous-time Markov jump process (25) for the small scale dynamics at each large scale spatio-temporal location (x, t) with transition rates depending in a suitable fashion on the local values of the large scale variables at (x, t) (20–24). Depending on the current small scale state, the strength and nature of the CMT in **[1]** is specified as developed below. Thus these stochastic models have a small additional computational cost. Recall that we are interested in parameterizing effects of CMT on scales of order 50–200 km. It is well-known that the large-scale low-level shears often organize moist deep convection into squall lines on these scales and furthermore that squall lines transport CMT upscale (11, 14, 15, 26). The stochastic models capture the statistics of this process. There are three phases in this process which can affect CMT, labelled by 1, 2, and 3:

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1. *Dry regime*. Weak or no cumulus friction. Favored for dry environments, regardless of shear.
2. *Upright convection regime*. Stronger cumulus friction. Favored for moist, weakly sheared environments.
3. *Squall line regime*. Intense CMT, either upscale or downscale depending on the shear. Favored for moist, sheared environments.

The transition rates between the regimes will depend on the large scale resolved variables such as the velocity u, the potential temperature θ, and the lower and middle troposphere cloud heating functions, Q_c and Q_d, resp. Another important parameter, Λ, measures whether the lower troposphere is moist or dry with $\Lambda = 1$ denoting a dry state and $\Lambda = \Lambda_* < 1$ a moist lower troposphere which is needed to precondition for deep convection (27–29). Denote the discrete, stochastic regime variable by

$$r_t = 1 \text{ (dry)}, \qquad r_t = 2 \text{ (conv.)}, \qquad r_t = 3 \text{ (squall)} \qquad \textbf{[3]}$$

Let the transition rate from regime i to regime j be T_{ij}. Plausible physical choices for the transition rates are

$$T_{12} = \frac{1}{\tau_r}\mathcal{H}(Q_d)e^{\beta_\Lambda(1-\Lambda)}e^{\beta_Q Q_d} \qquad \text{dry} \rightarrow \text{conv.} \qquad \textbf{[4]}$$

$$T_{13} = 0 \qquad \text{dry} \rightarrow \text{squall} \qquad \textbf{[5]}$$

$$T_{21} = \frac{1}{\tau_r}e^{\beta_\Lambda \Lambda}e^{\beta_Q(Q_{d,ref}-Q_d)} \qquad \text{conv.} \rightarrow \text{dry} \qquad \textbf{[6]}$$

$$T_{23} = \frac{1}{\tau_r}\mathcal{H}(|\Delta U_{low}| - |\Delta U|_{min})e^{\beta_U|\Delta U_{low}|}e^{\beta_Q Q_c} \qquad \text{conv.} \rightarrow \text{squall} \qquad \textbf{[7]}$$

$$T_{31} = T_{21} \qquad \text{squall} \rightarrow \text{dry} \qquad \textbf{[8]}$$

$$T_{32} = \frac{1}{\tau_r}e^{\beta_U(|\Delta U|_{ref}-|\Delta U_{low}|)}e^{\beta_Q(Q_{c,ref}-Q_c)} \qquad \text{squall} \rightarrow \text{conv.} \qquad \textbf{[9]}$$

where $\mathcal{H}(x)$ is a Heaviside function defined as 0 for $x \leq 0$ and 1 for $x > 0$. We use exponentials in the transition rates to get sensitive dependence on the large scale variables. The quantity ΔU_{low} measures the large scale low level shear which needs to be sufficiently large to allow for a squall line transition. Note that $T_{13} = 0$, which is reasonable since some initial upright convection should form before a squall line fully develops. There are two Heaviside functions in the transition rates T_{12} and T_{23} in **[4]** and **[7]**. They ensure, respectively, that (i) a transition from a dry state to a convective state cannot occur unless some mid-level cloud heating Q_d is present, and (ii) a transition from upright convection to squall lines cannot occur unless the low-level shear is sufficiently high. These conditions should be sufficient for preventing inappropriate regime transitions. Explicit formulas for Q_d, Q_c, ΔU_{low}, Λ, τ_r, and all other parameters needed in **[4]**-**[9]** are presented in Table 1.

Each of the three convective regimes has a different effect on the large scale CMT in **[1]**. The CMT for the resolved large scale momentum equation takes the form

$$F_{CMT} = -\partial_z(\overline{w'u'}) = \begin{cases} -d_1(U-\hat{U}) & \text{for} \quad r_t = 1 \\ -d_2(U-\hat{U}) & \text{for} \quad r_t = 2 \\ F_3 & \text{for} \quad r_t = 3 \end{cases} \qquad \textbf{[10]}$$

where $\hat{U}$ is the barotropic wind, d_j is small (and positive) or zero, and F_3 will be an upscale eddy momentum flux specified below. Here d_1 and d_2 will have the common value $d_1 = d_2 = 1/\tau_u$ and represent the cumulus friction known to occur in either dry or moist upright convective environments. To calculate F_3, the upscale CMT for the squall line regime, we utilize an exactly solvable multi-scale model which captures these features (12, 13, 30, 31). In this model, there is a balance ($w' = S'_\theta$) between the vertical velocity w' and the potential temperature source S'_θ, which represents convective heating. As a simple model of S'_θ for a tilted wave, consider a two-dimensional (x-z) setup and a heat source with two phase-lagged baroclinic modes: $S'_\theta = k\cos[kx-\omega t]\,\sqrt{2}\sin(z)+\alpha k\cos[k(x+x_0)-\omega t]\,\sqrt{2}\sin(2z)$. Two key parameters here are α, the strength of the second baroclinic heating, and x_0, the lag between the heating in the two vertical modes. The vertical velocity is then given by weak temperature gradient (WTG) balance, $w' = S'_\theta$, and the zonal velocity is given by the continuity equation, $u'_x + w'_z = 0$:

$$\begin{aligned} u'(x,z,t) &= -\sin[kx-\omega t]\,\sqrt{2}\cos(z) \\ &\quad - 2\alpha\sin[k(x+x_0)-\omega t]\,\sqrt{2}\cos(2z) \qquad \textbf{[11]} \\ w'(x,z,t) &= k\cos[kx-\omega t]\,\sqrt{2}\sin(z) \\ &\quad + \alpha k\cos[k(x+x_0)-\omega t]\,\sqrt{2}\sin(2z) \qquad \textbf{[12]} \end{aligned}$$

With this form of u' and w', the eddy flux divergence is explicitly calculated as

$$\partial_z(\overline{w'u'}) = \frac{3\alpha k}{2}\sin(kx_0)[\cos(z)-\cos(3z)] \qquad \textbf{[13]}$$

Notice that a wave with first and second baroclinic components generates CMT that affects the first and *third* baroclinic modes (12, 13). Thus, we model the CMT in the squall line regime by

$$F_3 = -\partial_z(\overline{w'u'}) = \kappa[\cos(z)-\cos(3z)]. \qquad \textbf{[14]}$$

Notice from **[13]**–**[14]** that F_3 depends on α (the strength of the stratiform heating relative to the deep convective heating) and on x_0 (the spatial lag between stratiform and deep convective heating). We will ignore the specific dependence of κ on α and x_0 and choose κ as a function of three quantities: ΔU_{mid}, $\Delta U_{mid}\Delta U_{low}$, and Q_d. Their detailed definitions are given in Table 1. The coefficient κ is defined here by

$$\kappa = \begin{cases} -\left(\frac{Q_d}{Q_{d,ref}}\right)^2 \frac{\Delta U_{mid}}{\tau_F} & \text{if} \quad \Delta U_{mid}\Delta U_{low} < 0 \\ 0 & \text{if} \quad \Delta U_{mid}\Delta U_{low} > 0 \end{cases} \qquad \textbf{[15]}$$

Note that this definition allows CMT for jet shears but not for uniform shears as in observations. A quadratic dependence on Q_d is appropriate, since $w'u'$ depends quadratically on S^θ. Notice also that this definition of κ depends on $|\Delta U_{low}|$ only in the nonlinear switch – when $\kappa \neq 0$ it does not vary with the magnitude of $|\Delta U_{low}|$. While the presentation above includes heating with only two vertical baroclinic modes, the model could be generalized to include arbitrary vertical structures. Next we calibrate the model in a simple column model testing environment.

Performance of the Stochastic Model for CMT in a Test Column Model

Here we calibrate the stochastic model for CMT in the simplest idealized setting with a single large scale grid point, i.e., a stochastic column model (24). The equations solved are simply $\partial u/\partial t = F_{CMT}$, where F_{CMT} is the stochastic CMT described above in **[10]** and **[14]**–**[15]**, and where u has a vertical structure with three baroclinic modes: $U(z,t) = \sum_{j=1}^{3} u_j(t)\sqrt{2}\cos(jz)$.

The thermodynamic variables are specified in the following way. The value of Λ is frozen as $\Lambda = 0.4$, while the values of the low-level and deep convective heating parameters, Q_c and Q_d, involve random non-overlapping bursts. The bursts are chosen to represent a random series of heating events from convectively coupled waves (2, 4). The time between successive bursts is picked from a Poisson distribution with mean of 1 day, and the amplitude of each burst is chosen from a Gaussian distribution with mean of 10 K/d and standard deviation of 2 K/d. This is shown for the first 50 days of the simulation in Fig. 1a. The low-level heating $Q_c(t)$ is set equal to $Q_d(t)$ for this column model case. Note that the heat sources are imposed functions and

do not allow any thermodynamic responses to changes in velocity; to include a thermodynamic response in a simple way, the upscale CMT is not allowed to exceed 10 m s^{-1} d^{-1}.

The initial velocity profile is the jet shear shown in Fig. 1c as the thick solid line, which is defined in terms of vertical modes as $u_1 = 10$ m/s, $u_2 = -10$ m/s, $u_3 = 0$. For this column model case, the model relaxes back to this initial profile instead of a barotropic wind $\hat{U}$ as shown in **[10]** for $r_t = 1$ or 2. The time evolution of u_j, $j = 1, 2, 3$, is shown in Fig. 1b for a duration of 600 days. The fluctuations about the initial conditions occur with $u_1(t) - u_1(0) = -(u_3(t) - u_3(0))$ due to the form of the CMT in **[14]**. The velocity mostly fluctuates with 10 m/s $< u_1 <$ 15 m/s, but there are also intermittent bursts where u_1 becomes as large as 20 m/s. The jet of the velocity profile becomes more intense and moves to lower levels when u_1 intermittently reaches values of 15–20 m/s, as shown in Fig. 1c. The jet profiles in Fig. 1c are representative of those shown in the time series in Fig. 1b, with u_2 always at the value -10 m/s and u_3 fluctuations always following u_1 fluctuations due to the form of the CMT in **[14]**.

This column model test case demonstrates the intermittent bursts that can occur with this model. Other cases were tested where the heating function bursts in Fig. 1a had fixed amplitudes and/or a fixed period between bursts, and the results were similar to those shown in Fig. 1. These tests also serve as a calibration of the model parameters. The test shown in the next section allows variations of model variables in the horizontal; furthermore, the heating functions Q_c and Q_d will be interactive functions of the model variables, thereby allowing feedback between CMT and the model thermodynamics.

Stochastic CMT for a Large Scale Convectively Coupled Wave

An interesting, important, and nontrivial test of the stochastic model for CMT is its effect on an organized synoptic scale convectively coupled wave (CCW) (4, 15). Here we represent the CCW through an idealized multi-cloud model which captures key features of the observational record in its simplest nonlinear formulation (27–29). The multicloud model is the following set of eight equations

$$\frac{\partial u_1}{\partial t} - \frac{\partial \theta_1}{\partial x} = F^1_{CMT} \quad \textbf{[16]}$$

$$\frac{\partial u_2}{\partial t} - \frac{\partial \theta_2}{\partial x} = F^2_{CMT} \quad \textbf{[17]}$$

$$\frac{\partial u_3}{\partial t} = F^3_{CMT} \quad \textbf{[18]}$$

$$\frac{\partial \theta_1}{\partial t} - \frac{\partial u_1}{\partial x} = H_d + \xi_s H_s + \xi_c H_c - R_1 \quad \textbf{[19]}$$

$$\frac{\partial \theta_2}{\partial t} - \frac{1}{4}\frac{\partial u_2}{\partial x} = H_c - H_s - R_2 \quad \textbf{[20]}$$

$$\frac{\partial \theta_{eb}}{\partial t} = \frac{1}{h_b}(E - D) \quad \textbf{[21]}$$

$$\frac{\partial q}{\partial t} + \tilde{Q}\frac{\partial}{\partial x}(u_1 + \tilde{\lambda} u_2) = -P + \frac{1}{H_T}D - \frac{\partial}{\partial x}[q(u_1 + \tilde{\alpha} u_2)] \quad \textbf{[22]}$$

$$\frac{\partial H_s}{\partial t} = \frac{1}{\tau_s}(\alpha_s P - H_s) \quad \textbf{[23]}$$

The variables u_j are the jth baroclinic mode velocity, θ_j are the jth baroclinic mode potential temperature, θ_{eb} is the boundary layer equivalent potential temperature, and q is the vertically integrated water vapor. The source terms for these equations include cloud heating from three cloud types: deep convective heating, H_d, stratiform heating, H_s, and congestus heating, H_c. The radiative cooling is R_j, evaporation is E, and downdrafts are D. These are all *interactive* source terms that are defined in terms of the model variables. The definitions of H_d, H_c, Q_d, and Q_c are

$$H_d = \frac{1-\Lambda}{1-\Lambda^*}Q_d \quad \textbf{[24]}$$

$$H_c = \alpha_c \frac{\Lambda - \Lambda^*}{1-\Lambda^*}Q_c \quad \textbf{[25]}$$

$$Q_d = \left[\bar{Q} + \frac{1}{\tau_{conv}}(a_1\theta_{eb} + a_2 q - a_0(\theta_1 + \gamma_2\theta_2))\right]^+ \quad \textbf{[26]}$$

$$Q_c = \left[\bar{Q} + \frac{1}{\tau_{conv}}(\theta_{eb} - a_0'(\theta_1 + \gamma_2'\theta_2))\right]^+ \quad \textbf{[27]}$$

See (29) for definitions of the other source terms.

The equations in **[16]**–**[23]** are those of the multicloud model of (29) with the following changes. A few source terms have been changed from (29). The congestus heating, H_c, which is shown in **[25]**, is treated diagnostically here by taking the limit $\tau_c \to 0$ in (29). Also, the parameters γ_2' in Q_c and γ_2 in Q_d take different values here: $\gamma_2' = 2$ and $\gamma_2 = 0.1$. Using a large value of γ_2' emphasizes the second baroclinic mode and gives Q_c the characteristics of a low-level convective available potential energy (CAPE) closure. The parameter τ_{conv} takes a value of 1 hour here instead of $\tau_{conv} = 2$ hours as it was in (29). This change in τ_{conv} reduces the wavelength of the most unstable waves from 4000 to 1500 km, thereby reducing artificial effects of the wave wrapping around the 6000 km periodic domain and interacting with itself. An equation for u_3 has also been included here in **[18]**. Besides these changes mentioned above, the parameter values used here are all the same as those in the standard case of (29).

The multicloud model is used here with the stochastic CMT model described above, which enters into **[16]**–**[18]** through the terms F^j_{CMT}, which are the components of F_{CMT} from **[10]** and **[14]**–**[15]** in the jth baroclinic mode. The simulation shown here uses a periodic domain of width 6000 km to represent a single CCW, and a grid spacing of $\Delta x = 50$ km is used to represent a typical grid spacing used in contemporary GCMs. The initial conditions are a spatially uniform radiative-convective equilibrium with a small perturbation.

Fig. 2 shows the CCW that develops from the initial perturbation. For the first 10 days of the simulation, the CCW is weak, and the squall line regime $r_t = 3$ is never reached. After time $t = 10$ days, the deep convective heating, H_d, reaches values larger than 10 K/d, the squall line regime ($r_t = 3$) of the stochastic CMT model is often accessed, and upscale CMT generates u_3 intermittently. On the other hand, upscale CMT has a significant but less obvious effect on u_1, which is coupled more strongly than u_3 to the dynamics of the CCW.

Fig. 3 shows the structure of the CCW averaged in a reference frame moving with the wave at -17.5 m/s from time $t = 20$ to 30 days. This CCW has key features in agreement with observations, including the propagation speed and the left-to-right tilt with height (4, 27–29). Fig. 4 shows the structure of this wave average for u_1, u_2, and u_3. Also shown in Fig. 4 is the wave average for a simulation *without* the stochastic CMT model, i.e., with simple cumulus friction as in **[2]** replacing the stochastic CMT model. The stochastic CMT model produces a nontrivial mean wind that can be seen in the plot of the difference in Fig. 4c. In turn, this mean wind can interact significantly with the CCW as time progresses (32).

Concluding Discussion

A simple stochastic model for CMT was developed and tested. The model represents the convective regime at each large scale spatio-temporal location (x, t) by a three-state continuous-time Markov jump process, r_t. Three convective regimes are represented: dry regime, upright convection regime, and squall line regime. Transition rates between regimes depend on the large scale resolved variables such as the velocity u, the potential temperature θ, and the low- and mid-level cloud heating functions, Q_c and Q_d, resp. The CMT from unresolved

scales acting at the large scale spatio-temporal location (x, t) depends on the convective regime at (x, t). During the dry and upright convection regimes, the resolved scales are damped due to cumulus friction, and during the squall line regime, the resolved scales are forced by upscale momentum transport.

The stochastic model for CMT was tested for a single large scale grid point, i.e., as a stochastic column model. The model produced intermittent bursts in the velocity with physically reasonable values. This test is also useful as a calibration of the model parameters. The stochastic model was also tested on a 6000 km domain with 50 km grid spacing for a large scale convectively coupled wave. The stochastic model for CMT produced a nontrivial mean wind in comparison to a simulation without the stochastic CMT model.

While the tests shown in this paper involved a simplified vertical structure with three baroclinic modes, the stochastic model for CMT can be generalized to include a more general vertical structure using the WTG formulas in **[11]–[13]**. Furthermore, the authors plan to make other extensions of this model in the future. Another important direction is the inclusion of two horizontal spatial variables, x and y; several additional important effects are present with two horizontal spatial directions (7, 8, 33). Another interesting direction is to include the nonlinear multi-scale interaction of the CCW and mean flow (32).

The stochastic model for CMT developed and tested here is a promising approach given recent results of other CMT models. Mixing–entrainment models of CMT have shown some skill in improving the climate mean state, i.e., the Hadley circulation (17, 18), as well as the El Niño Southern Oscillation (19). However, these models require a computationally expensive pressure calculation. The stochastic model for CMT presented here is a less expensive alternative that does not require an expensive pressure calculation besides having the additional attractive feature of representing intermittent upscale CMT. This stochastic CMT model's ability to capture *intermittent* effects from unresolved scales also distinguishes it from other stochastic parameterizations for different aspects of cumulus convection (34, 35).

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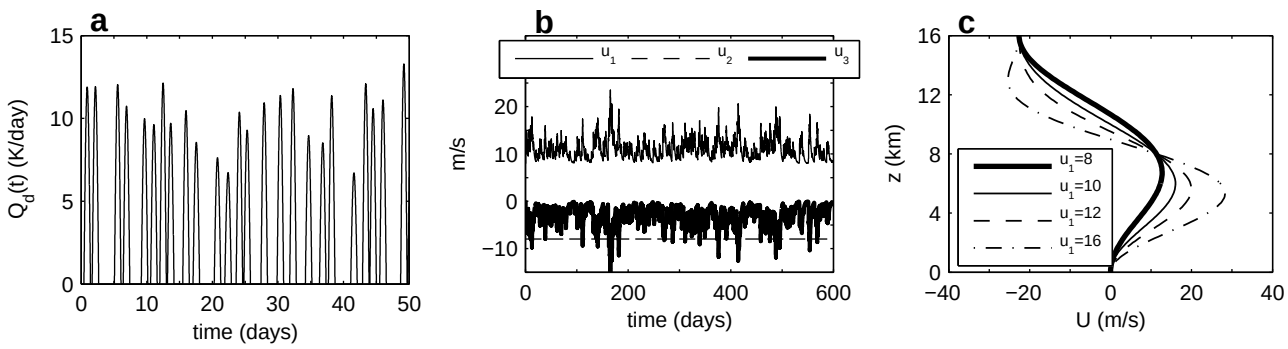

Fig. 1. Performance of the stochastic model for CMT in a test column model. (a) Mid-level cloud heating, $Q_d(t)$, for the first 50 days of the simulation. (b) Velocity u_j of the jth vertical baroclinic mode for the first 600 days of the simulation. (c) Velocity profiles representative of the velocities shown in (b).

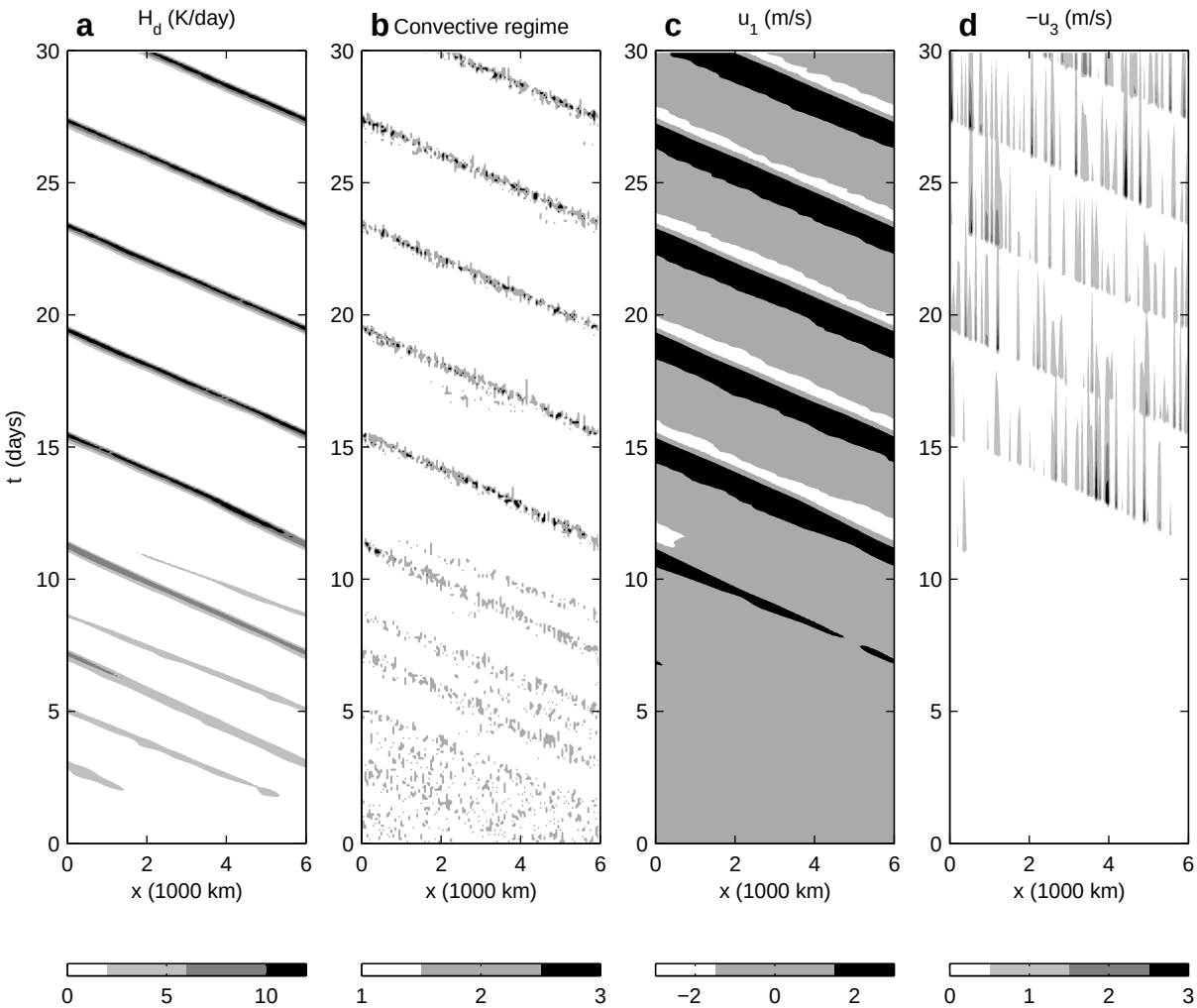

Fig. 2. Simulation of a convectively coupled wave using the multicloud model with the stochastic model for CMT. (a) Deep convective heating, H_d. (b) Convective regime, r_t, which takes discrete values 1, 2, and 3 corresponding to a dry regime, upright convective regime, and squall line regime, resp. (c) First baroclinic mode velocity, u_1. (d) (Negative) third baroclinic mode velocity, $-u_3$.

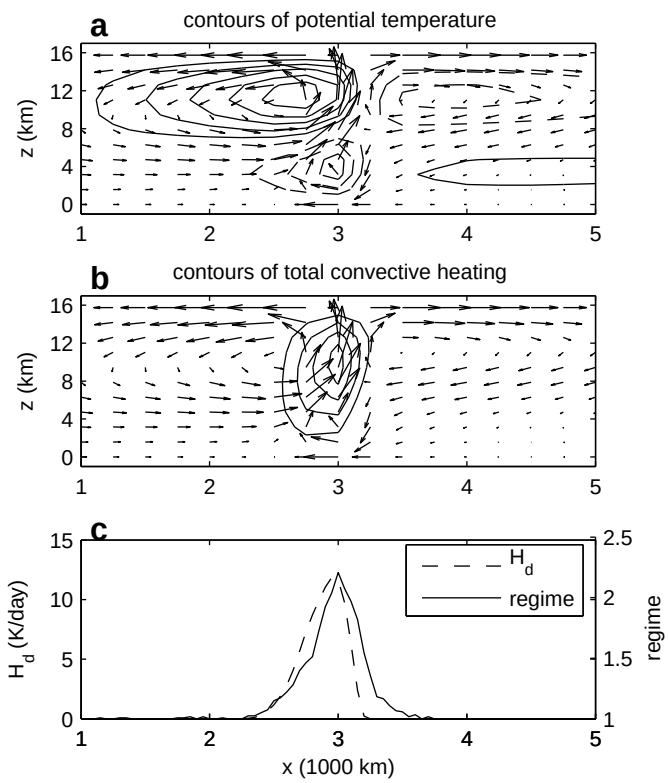

Fig. 3. Structure of the convectively coupled wave averaged in a reference frame moving with the wave at -17.5 m/s from time $t = 20$ to 30 days. Solid (dashed) contours denote positive (negative) anomalies, with the zero contour left out. The contour interval is 0.2 K for potential temperature and 4 K/day for convective heating. The maximum horizontal velocity shown is 6.8 m/s, and the maximum vertical velocity shown is 9.1 cm/s.

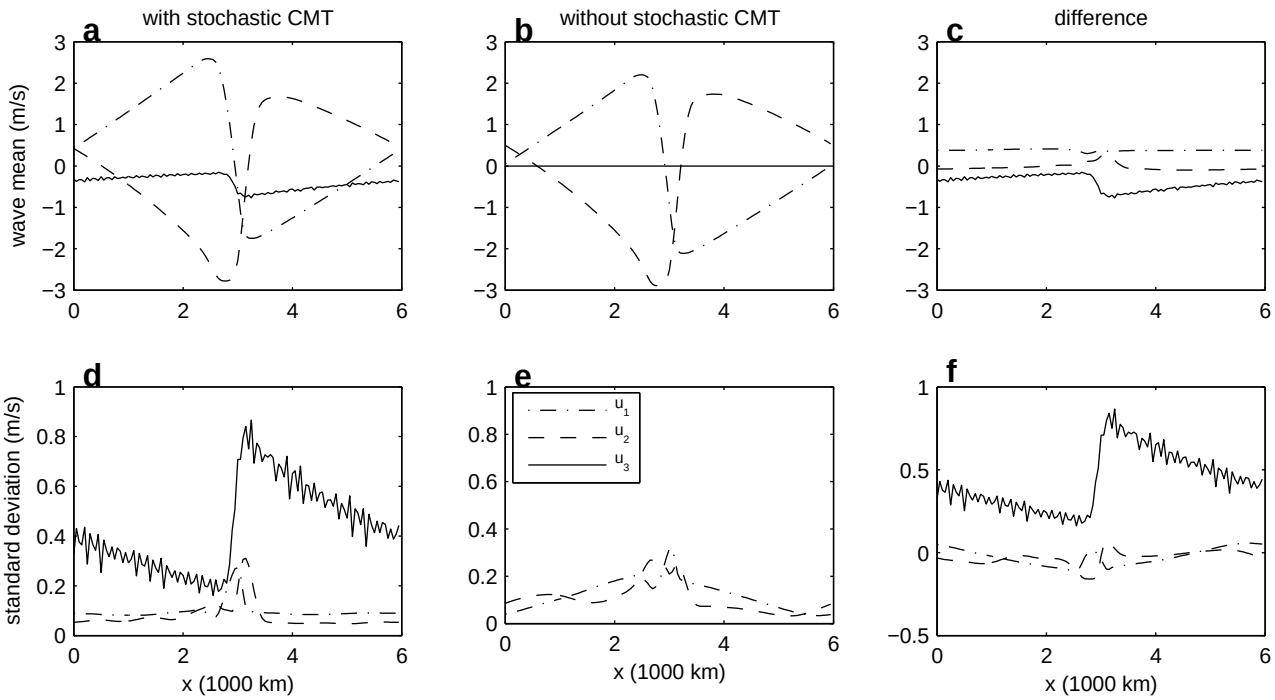

Fig. 4. Stucture of the CCW averaged in a reference frame moving with the wave for the results in Figs. 2–3 as well as for a simulation without the stochastic model for CMT. (a)–(c) Wave mean. (d)–(f) Standard deviation about the wave mean. Dash-dot lines are used for u_1, dashed lines for u_2, and solid lines for u_3.

Table 1. Parameters of the stochastic CMT model

Parameter	Description	Value
τ_r	Time scale of regime transition rates	8 h
β_Λ	Sensitivity of transition rates to changes in Λ	1
β_Q	Sensitivity of transition rates to changes in Q_d and Q_c	$(10\ \mathrm{K/d})^{-1}$
β_U	Sensitivity of transition rates to changes in low-level wind shear	$(10\ \mathrm{m/s})^{-1}$
$Q_{c,ref}$	Reference value of Q_c for transition rates	$10\ \mathrm{K/d}$
$Q_{d,ref}$	Reference value of Q_d for transition rates	$10\ \mathrm{K/d}$
$\lvert\Delta U\rvert_{ref}$	Reference value of low-level shear for transition rates	$20\ \mathrm{m/s}$
$\lvert\Delta U\rvert_{min}$	Minimum low-level shear needed for transition to squall line regime	$5\ \mathrm{m/s}$
d_1	Momentum damping rate for dry regime	$(3\ \mathrm{d})^{-1}$
d_2	Momentum damping rate for upright convection regime	$(3\ \mathrm{d})^{-1}$
τ_F	Time scale of upscale CMT for squall line regime	1.25 d
ΔU_{low}	$\mathrm{sgn}\ (U(z_*) - U(0))\ \max_{1\ \mathrm{km}<z<7\ \mathrm{km}} \lvert U(z) - U(0)\rvert$	
ΔU_{mid}	$\mathrm{sgn}\ (U(z_{**}) - U(z_*))\ \max_{7\ \mathrm{km}<z<13\ \mathrm{km}} \lvert U(z) - U(z_*)\rvert$	

z_* denotes the value of z where the maximum in ΔU_{low} is achieved.
z_{**} denotes the value of z where the maximum in ΔU_{mid} is achieved.