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Publisher's Editorial

The Faffufnik–Chaim Yankel Effect

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In the United States, as in many countries and more global entities, in order to receive funding for a major project, one has to submit a proposal. As anyone who has ever written one knows, writing a proposal is an unnatural act. Normally literate persons are reduced to using words such as "input" as a verb, as well as "facilitating" and "orientating," and talking about "stakeholders" and "meta-cognition." But large projects often require large budgets, and as painful as the process may be, we write the proposals and fill out the myriad forms required.

Most of the money that comes from federal sources in the United States for math education projects is given in grants from the National Science Foundation (NSF). NSF uses a peer-review process to determine which projects are funded. Panels of approximately six people are formed to review a set of proposals. The proposals in each panel are graded and compared with the grades from several other panels that are convened at the same time. The programs are ordered by grade, and funding proceeds on that basis. What actually happens is that on a first pass, a number of projects are graded highly enough to be assured of funding; a number are graded so low that they are immediately declined; and there is a group in the middle (said to be "on the bubble") whose fate is decided some time later when the final yearly budget for these programs is negotiated. The criteria for reviewing proposals, as specifically cited in NSF guidelines, are "intellectual merit" and "broader impacts."

The Consortium for Mathematics and Its Applications (COMAP) has been submitting proposals and administering projects for over 29 years. In the "good old days," if one had a good idea and a good staff of people to carry out that idea, then funding usually depended upon impressing one of

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the program officers who worked at the Foundation. Outside reviews were mostly handled by mail and were considered advisory. The bottom line was that if the NSF program officer thought that a project should be funded, it was. Admittedly, this created something of an old-boy network. People and institutions with a good track record of success tended to continue to receive funding, while those who were not yet members of the club had a hard time joining. This has given way to the more overtly democratic process described above where the reviewers' opinions rule.

It should also be said that if one goes back 20 years or so, most of the principal investigators (PIs) on mathematics education projects were Ph.D. mathematicians who had, so to speak, "given their youth to the devil and were giving their old age to the Lord." In other words, they had taken an interest in mathematics education later in their careers. And, to be honest, many other mathematics educators were persons who originally pursued careers as research mathematicians but were unable to complete their degrees. In any event, the PIs on these projects had extremely strong mathematics backgrounds.

In the United States at least, this has changed significantly. Mathematics education is now a well-established field unto itself and, in many cases, people highly successful in the field have relatively weak mathematical training. Increasingly, they are the principal investigators on new projects in mathematics education and they are the reviewers. They help decide what projects get funded and what projects don't. And, increasingly, they are responsible for the *Faffufnik–ChaimYankel Effect (FCE)*. What exactly is the FCE?

Years ago, a typical review of a COMAP proposal would read, "This is an excellent idea with an excellent staff with an excellent track record; we recommend this project for funding." The FCE refers to more typical current reviews that read, "This is an excellent idea with an excellent staff with an excellent track record. However, we have to recommend against funding because they don't make any reference to the seminal research papers of Faffufnik, nor do they plan to use the statistical protocols of Chaim Yankel." The reviewers may very well be students of Faffufnik and / or Chaim Yankel.

Of course, there are some sour grapes here. I am not a member of the Faffufnik and Chaim Yankel club. And now, as opposed to the good old days, it is the members of this club who get funded. But there is much more to be discussed. There appears to be an underlying assumption that mathematics education projects must proceed in the following way.

- First, they must be based upon research. Therefore, we heavily quote the results of prior research (see the papers of Faffufnik).
- Then, based upon that research, we make a new research hypothesis and test it with a small number of students. If at all possible, we make this experiment as close to a "gold standard" double-blind medical approach as possible.

- Then, using certain statistical protocols (see the work of Chaim Yankel), we conclude that there is some measurable effect and write a new proposal to test this effect on a larger population.
- This process is then iterated.

This is now a necessary condition for funding—independent of content and the strength of the ideas being considered.

The problem is that while this may very well help to make mathematics education research be seen as more of an established discipline, it is a criterion divorced from classroom practice. And we forget that we separate our efforts in education from the classroom at our peril. There has to be a way for good ideas that hold the promise of increasing student learning to be funded and for good people to work on them. Mathematics education is an art as well as a science, and it cannot be reduced to a set of research protocols and statistical tests and procedures. It is simply not possible to prove that an approach to teaching and learning will be effective before the fact.

Education, as a scientific discipline, is a young field with an active community focused on R&D—research on learning coupled with the development of new and better curriculum materials. In truth, however, much of the work is better described as D&R—informed and thoughtful development followed by careful analysis of results. It is in the nature of the enterprise that we cannot discover what works before we create the what. Curriculum development, in particular, is best related to an engineering paradigm. To test the efficacy of an approach, we must analyze needs, examine existing programs, build an improved model program, and then test it—in the same way that we build scale models to design a better bridge or building. This kind of iterative D&R leads to new and more effective materials and new pedagogical approaches that better incorporate the growing body of knowledge of cognitive science.

I wish to be clear. I recognize that Faffufnik has done important research. I recognize that Chaim Yankel's protocols can help quantify our results. We must learn from the past, and theoretical frameworks are important for future work. But we also must recognize that quoting Faffufnik and ChaimYankel is not a substitute for imagination, creativity, and the application of common sense. The problems of mathematics education are difficult and will require the work of many people over a long period of time. We cannot afford to lose sight of this, even as mathematics education becomes a more-established research discipline.

Acknowledgment

This editorial is adapted from the author's talk at the International Commission on Mathematical Instruction (ICMI) meeting in Rome, Italy, 2008.

About the Author

Solomon Garfunkel, previously of Cornell University and the University of Connecticut at Storrs, has dedicated the last 30 years to research and development efforts in mathematics education. He has served as project director for the Undergraduate Mathematics and Its Applications (UMAP) and the High School Mathematics and Its Applications (HiMAP) Projects funded by NSF, and directed three telecourse projects including *Against All Odds: Inside Statistics*, and *In Simplest Terms: College Algebra*, for the Annenberg/CPB Project. He has been the Executive Director of COMAP, Inc. since its inception in 1980. Dr. Garfunkel was the project director and host for the series, *For All Practical Purposes: Introduction to Contemporary Mathematics*. He was the Co-Principal Investigator on the ARISE Project, and is currently the Co-Principal Investigator of the CourseMap, ResourceMap, and WorkMap projects. In 2003, Dr. Garfunkel was Chair of the National Academy of Sciences and Mathematical Sciences Education Board Committee on the Preparation of High School Teachers.

About This Issue

Paul J. Campbell
Editor

This issue runs longer than a regular 92-page issue, to more than 200 pages. However, not all of the articles appear in the paper version. Some appear only on the *Tools for Teaching* 2009 CD-ROM (and at `http://www.comap.com` for COMAP members), which will reach members and subscribers later and will also contain the entire 2009 year of *Journal* issues.

All articles listed in the table of contents are regarded as published in the *Journal*. The abstract of each appears in the paper version. Pagination of the issue runs continuously, including in sequence articles that do not appear in the paper version. *So if, say, p. 250 in the paper version is followed by p. 303, your copy is not necessarily defective!* The articles on the intervening pages are on the CD-ROM.

We hope that you find this arrangement agreeable. It means that we do not have to procrusteanize the content to fit a fixed number of paper pages. We might otherwise be forced to select only two or three Outstanding MCM papers to publish. Instead, we continue to bring you the full content.

Modeling Forum

Results of the 2009 Mathematical Contest in Modeling

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Introduction

A total of 1,675 teams of undergraduates from hundreds of institutions and departments in 14 countries, spent the first weekend in February working on applied mathematics problems in the 25th Mathematical Contest in Modeling.

The 2009 Mathematical Contest in Modeling (MCM) began at 8:00 P.M. EST on Thursday, February 5 and ended at 8:00 P.M. EST on Monday, February 9. During that time, teams of up to three undergraduates researched, modeled, and submitted a solution to one of two open-ended modeling problems. Students registered, obtained contest materials, downloaded the problem and data, and entered completion data through COMAP's MCM Website. After a weekend of hard work, solution papers were sent to COMAP on Monday. The top papers appear in this issue of *The UMAP Journal*, together with commentaries.

In addition to this special issue of *The UMAP Journal*, this year—for the first time—COMAP has made available a special supplementary "2009 MCM-ICM CD-ROM" containing the press releases for the two contests, the results, the problems, and original versions of the Outstanding papers that appear here in edited form. Information about ordering the CD-ROM is at `http://www.comap.com/product/?idx=1025` or from (800) 772–6627.

Results and winning papers from the first 24 contests were published in special issues of *Mathematical Modeling* (1985–1987) and *The UMAP Journal* (1985–2008). The 1994 volume of *Tools for Teaching*, commemorating the tenth anniversary of the contest, contains the 20 problems used in the first 10 years of the contest and a winning paper for each year. That volume and the special

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MCM issues of the *Journal* for the last few years are available from COMAP. The 1994 volume is also available on COMAP's special *Modeling Resource* CD-ROM. Also available is *The MCM at 21* CD-ROM, which contains the 20 problems from the second 10 years of the contest, a winning paper from each year, and advice from advisors of Outstanding teams. These CD-ROMs can be ordered from COMAP at `http://www.comap.com/product/cdrom/index.html`.

This year, the two MCM problems represented significant challenges. The author of Problem A, Daniel Solow of Case Western Reserve University, Cleveland, OH, was also one of the final judges. His problem, "Designing a Traffic Circle," asked teams to use a model to determine how best to control traffic flow in, around, and out of a circle, clearly stating the objective(s) and summarizing the conditions for use of various types of traffic-control methods. Problem B, "Energy and the Cellphone," was written by Joe Malkevitch of York College in Jamaica, NY. What is the long-term consequence of large-scale usage of cellphones in terms of electricity use by the battery and the charger? Teams were asked to take into account the fact that cellphones last much less time (they get lost and break) than phones for landlines and to suggest an optimal way (in terms of an energy perspective) to provide phone service to a "Pseudo U.S.," a country of 300 million people with about the same economic status as the current U.S. but with no landlines or cellphones.

In addition to the MCM, COMAP also sponsors the Interdisciplinary Contest in Modeling (ICM) and the High School Mathematical Contest in Modeling (HiMCM). The ICM runs concurrently with MCM and for the next several years will offer a modeling problem involving an environmental topic. Results of this year's ICM are on the COMAP Website at `http://www.comap.com/undergraduate/contests`; results and Outstanding papers appeared in Vol. 30 (2009), No. 2. The HiMCM offers high school students a modeling opportunity similar to the MCM. Further details about the HiMCM are at `http://www.comap.com/highschool/contests`.

2009 MCM Statistics

- 1,675 teams participated
- 7 high school teams (<1%)
- 350 U.S. teams (21%)
- 1,325 foreign teams (79%), from Australia, Canada, China, Finland, Germany, Hong Kong, Hungary, Indonesia, Ireland, Mexico, Singapore, South Africa, United Kingdom
- 9 Outstanding Winners (<1%)
- 294 Meritorious Winners (18%)
- 298 Honorable Mentions (18%)

- 1,074 Successful Participants (63%)

Problem A: Designing a Traffic Circle

Many cities and communities have traffic circles—from large ones with many lanes in the circle (such as at the Arc de Triomphe in Paris and the Victory Monument in Bangkok) to small ones with one or two lanes in the circle. Some of these traffic circles position a stop sign or a yield sign on every incoming road, which gives priority to traffic already in the circle; some position a yield sign in the circle at each incoming road to give priority to incoming traffic; and some position a traffic light on each incoming road (with no right turn allowed on a red light). Other designs may also be possible.

The goal of this problem is to use a model to determine how best to control traffic flow in, around, and out of a circle. State clearly the objective(s) you use in your model for making the optimal choice as well as the factors that affect this choice. Include a Technical Summary of not more than two double-spaced pages that explains to a traffic engineer how to use your model to help choose the appropriate flow-control method for any specific traffic circle. That is, summarize the conditions under which each type of traffic-control method should be used. When traffic lights are recommended, explain a method for determining how many seconds each light should remain green (which may vary according to the time of day and other factors). Illustrate how your model works with specific examples.

Problem B: Energy and the Cellphone

This question involves the "energy" consequences of the cellphone revolution. Cellphone usage is mushrooming, and many people are using cellphones and giving up their landline telephones. What is the consequence of this in terms of electricity use? Every cellphone comes with a battery and a recharger.

Requirement 1

Consider the current U.S., a country of about 300 million people. Estimate from available data the number H of households, with m members each, that in the past were serviced by landlines. Now, suppose that all the landlines are replaced by cellphones; that is, each of the m members of the household has a cellphone. Model the consequences of this change for electricity utilization in the current U.S., both during the transition and during the steady state. The analysis should take into account the need for charging the batteries of the cellphones, as well as the fact that cellphones do not last as long as landline phones (for example, the cellphones get lost and break).

Requirement 2

Consider a second "Pseudo U.S."—a country of about 300 million people with about the same economic status as the current U.S. However, this emerging country has neither landlines nor cellphones. What is the optimal way of providing phone service to this country from an energy perspective? Of course, cellphones have many social consequences and uses that landline phones do not allow. A discussion of the broad and hidden consequences of having only landlines, only cellphones, or a mixture of the two is welcomed.

Requirement 3

Cellphones periodically need to be recharged. However, many people always keep their recharger plugged in. Additionally, many people charge their phones every night, whether they need to be recharged or not. Model the energy costs of this wasteful practice for a Pseudo U.S. based on your answer to Requirement 2. Assume that the Pseudo U.S. supplies electricity from oil. Interpret your results in terms of barrels of oil.

Requirement 4

Estimates vary on the amount of energy that is used by various recharger types (TV, DVR, computer peripherals, and so forth) when left plugged in but not charging the device. Use accurate data to model the energy wasted by the current U.S. in terms of barrels of oil per day.

Requirement 5

Now consider population and economic growth over the next 50 years. How might a typical Pseudo U.S. grow? For each 10 years for the next 50 years, predict the energy needs for providing phone service based upon your analysis in the first three requirements. Again, assume electricity is provided from oil. Interpret your predictions in term of barrels of oil.

The Results

The solution papers were coded at COMAP headquarters so that names and affiliations of the authors would be unknown to the judges. Each paper was then read preliminarily by two "triage" judges at either Appalachian State University (Traffic Circle Problem) or at the National Security Agency (Cellphone Energy Problem). At the triage stage, the summary and overall organization are the basis for judging a paper. If the judges' scores diverged for a paper, the judges conferred; if they still did not agree, a third judge evaluated the paper.

Additional Regional Judging sites were created at the U.S. Military Academy and at the Naval Postgraduate School to support the growing number of contest submissions.

Final judging took place at the Naval Postgraduate School, Monterey, CA. The judges classified the papers as follows:

	Outstanding	Meritorious	Honorable Mention	Successful Participation	Total
Traffic Circle Problem	4	192	165	763	1124
Cellphone Energy Problem	5	102	133	311	551
	9	294	298	1074	1675

The 9 papers that the judges designated as Outstanding appear in this special issue of *The UMAP Journal*, together with commentaries. We list those teams here and the Meritorious teams (and advisors) at the end of this report; the list of all participating schools, advisors, and results is in the **Appendix**.

Outstanding Teams

Institution and Advisor	**Team Members**

Traffic Circle Papers

"A Simulation-Based Assessment of Traffic Circle Control"	
Harvard University	Christopher Chang
Cambridge, MA	Zhou Fan
Clifford H. Taubes	Yi Sun
"One Ring to Rule Them All: The Optimization of Traffic Circles"	
Harvey Mudd College	Aaron Abromowitz
Claremont, CA	Andrea Levyi
Susan E. Martonosi	Russell Melick
"Three Steps to Make the Traffic Circle Go Round"	
Tsinghua University	Zeyuan Zhu
Beijing, China	Tianyi Mao
Jun Ye	Yichen Huang

"Pseudo-Finite Jackson Networks and Simulation: A Roundabout Approach to Traffic Control"
University of Colorado
Boulder, CO
Anne Dougherty
Anna Lieb
Anil Damle
Geoffrey Peterson

Cellphone Energy Papers

"Mobile to Mobil: The Primary Energy Costs for Cellular and Landline Telephones"
Clarkson University
Potsdam, NY
Joseph Skufca
Nevin Brackett-Rozinsky
Katelynn Wilton
Jason Altieri

"Energy Implications of Cellular Proliferation in the U.S."
College of Idaho
Caldwell, IDA
Michael P. Hitchman
Benjamin Coate
Zachary Kopplin
Nate Landis

"Modeling Telephony Energy Consumption"
Cornell University
Ithaca, NY
Alexander Vladimirsky
Amrish Deshmukh
Rudolf Nikolaus Stahl
Matthew Guay

"America's New Calling"
Southwestern University
Georgetown, TX
Richard Denman
Stephen R. Foster
J. Thomas Rogers
Robert S. Potter

"Wireless Networks: An Easy *Cell*"
University of Delaware
Newark, DE
Louis Rossi
Jeff Bosco
Zachary Ulissi
Bob Liu

Awards and Contributions

Each participating MCM advisor and team member received a certificate signed by the Contest Director and the appropriate Head Judge.

INFORMS, the Institute for Operations Research and the Management Sciences, recognized the teams from the University of Colorado–Boulder (Traffic Circle Problem) and Cornell University (Cellphone Energy Problem) as INFORMS Outstanding teams and provided the following recognition:

- a letter of congratulations from the current president of INFORMS to each team member and to the faculty advisor;
- a check in the amount of $300 to each team member;
- a bronze plaque for display at the team's institution, commemorating their achievement;
- individual certificates for team members and faculty advisor as a personal commemoration of this achievement;
- a one-year student membership in INFORMS for each team member, which includes their choice of a professional journal plus the *OR/MS Today* periodical and the INFORMS society newsletter.

The Society for Industrial and Applied Mathematics (SIAM) designated one Outstanding team from each problem as a SIAM Winner. The teams were from Harvard University (Traffic Circle Problem) and Southwestern University (Cellphone Energy Problem). Each of the team members was awarded a $300 cash prize, and the teams received partial expenses to present their results in a special Minisymposium at the SIAM Annual Meeting in Denver, CO in July. Their schools were given a framed hand-lettered certificate in gold leaf.

The Mathematical Association of America (MAA) designated one Outstanding North American team from each problem as an MAA Winner. The teams were from Harvey Mudd College (Traffic Circle Problem) and Clarkson University (Cellphone Energy Problem). With partial travel support from the MAA, the teams presented their solution at a special session of the MAA Mathfest in Portland, OR in August. Each team member was presented a certificate by an official of the MAA Committee on Undergraduate Student Activities and Chapters.

Ben Fusaro Award

One Meritorious or Outstanding paper was selected for each problem for the Ben Fusaro Award, named for the Founding Director of the MCM and awarded for the sixth time this year. It recognizes an especially creative approach; details concerning the award, its judging, and Ben Fusaro are in Vol. 25 (3) (2004): 195–196. The Ben Fusaro Award winners were the

University of Iowa (Traffic Circle Problem) and Lawrence Technological University (Cellphone Energy Problem).

Judging

Director
Frank R. Giordano, Naval Postgraduate School, Monterey, CA

Associate Director
William P. Fox, Dept. of Defense Analysis, Naval Postgraduate School, Monterey, CA

Traffic Circle Problem

Head Judge
Marvin S. Keener, Executive Vice-President, Oklahoma State University, Stillwater, OK

Associate Judges
William C. Bauldry, Chair, Dept. of Mathematical Sciences, Appalachian State University, Boone, NC (Head Triage Judge)
Kelly Black, Mathematics Dept., Union College, Schenectady, NY
Karen D. Bolinger, Mathematics Dept., Clarion University of Pennsylvania, Clarion, PA (SIAM Judge)
Patrick J. Driscoll, Dept. of Systems Engineering, U.S. Military Academy, West Point, NY
J. Douglas Faires, Youngstown State University, Youngstown, OH
Ben Fusaro, Dept. of Mathematics, Florida State University, Tallahassee, FL
Jerry Griggs, Mathematics Dept., University of South Carolina, Columbia, SC (Problem Author)
Steve Horton, Dept. of Mathematical Sciences, U.S. Military Academy, West Point, NY (MAA Judge)
Mario Juncosa, RAND Corporation, Santa Monica, CA (retired)
Michael Moody, Olin College of Engineering, Needham, MA (SIAM Judge)
John L. Scharf, Mathematics Dept., Carroll College, Helena, MT (Ben Fusaro Award Judge)
Dan Solow, Mathematics Dept., Case Western Reserve University, Cleveland, OH (INFORMS Judge)
Michael Tortorella, Dept. of Industrial and Systems Engineering, Rutgers University, Piscataway, NJ
Richard Douglas West, Francis Marion University, Florence, SC
Dan Zwillinger, Raytheon Company, Sudbury, MA

Cellphone Energy Problem

Head Judge
Maynard Thompson, Mathematics Dept., University of Indiana,
Bloomington, IN

Associate Judges
Peter Anspach, National Security Agency, Ft. Meade, MD
(Head Triage Judge)
Jim Case (SIAM Judge)
Veena Mendiratta, Lucent Technologies, Naperville, IL
Peter Olsen, Johns Hopkins Applied Physics Laboratory, Baltimore, MD
David H. Olwell, Naval Postgraduate School, Monterey, CA
(INFORMS Judge)
Kathleen M. Shannon, Dept. of Mathematics and Computer Science,
Salisbury University, Salisbury, MD (SIAM Judge)
Marie Vanisko, Dept. of Mathematics, Carroll College, Helena, MT
(Ben Fusaro Award Judge)

Regional Judging Session at U.S. Military Academy

Head Judges
Patrick J. Driscoll, Dept. of Systems Engineering, and
Steve Horton, Dept. of Mathematical Sciences,
United States Military Academy (USMA), West Point, NY

Associate Judges
Tim Elkins, Dept. of Systems Engineering, USMA
Michael Jaye, Dept. of Mathematical Sciences, USMA
Darrall Henderson, Sphere Consulting, LLC
Steve Horton, Dept. of Mathematical Sciences, USMA
Tom Meyer, Dept. of Mathematical Sciences, USMA
Scott Nestler, Dept. of Mathematical Sciences, USMA

Regional Judging Session at Naval Postgraduate School

Head Judge
William P. Fox, Dept. of Defense Analysis, and Frank Giordano,
Naval Postgraduate School (NPS), Monterey, CA

Associate Judges
Greg Mislik, Matt Boensel, and Pete Gustitis
—all from the Naval Postgraduate School, Monterey, CA

Triage Session for Traffic Circle Problem

Head Triage Judge
William C. Bauldry, Chair, Dept. of Mathematical Sciences,
Appalachian State University, Boone, NC

Associate Judges
Jeffry Hirst, Rick Klima, Mark Ginn, and Tracie McLemore Salinas
—all from Dept. of Mathematical Sciences, Appalachian State University, Boone, NC

Triage Session for Cellphone Energy Problem

Head Triage Judges
Peter Anspach, National Security Agency (NSA), Ft. Meade, MD
Jim Case

Associate Judges
Other judges from inside and outside NSA, who wish not to be named.

Sources of the Problems

The Traffic Circle Problem was contributed by Daniel Solow (Case Western Reserve University), who was also one of the final judges, and the Cellphone Energy Problem by Joe Malkevitch (York College of CUNY).

Acknowledgments

Major funding for the MCM is provided by the National Security Agency (NSA) and by COMAP. Additional support is provided by the Institute for Operations Research and the Management Sciences (INFORMS), the Society for Industrial and Applied Mathematics (SIAM), and the Mathematical Association of America (MAA). We are indebted to these organizations for providing judges and prizes.

We also thank for their involvement and support the MCM judges and MCM Board members for their valuable and unflagging efforts, as well as

- Two Sigma Investments. (This group of experienced, analytical, and technical financial professionals based in New York builds and operates sophisticated quantitative trading strategies for domestic and international markets. The firm is successfully managing several billion dollars using highly automated trading technologies. For more information about Two Sigma, please visit `http://www.twosigma.com` .)

Cautions

To the reader of research journals:

Usually a published paper has been presented to an audience, shown to colleagues, rewritten, checked by referees, revised, and edited by a journal editor. Each paper here is the result of undergraduates working on a problem over a weekend. Editing (and usually substantial cutting) has taken place; minor errors have been corrected, wording altered for clarity or economy, and style adjusted to that of *The UMAP Journal*. The student authors have proofed the results. Please peruse their efforts in that context.

To the potential MCM Advisor:

It might be overpowering to encounter such output from a weekend of work by a small team of undergraduates, but these solution papers are highly atypical. A team that prepares and participates will have an enriching learning experience, independent of what any other team does.

COMAP's Mathematical Contest in Modeling and Interdisciplinary Contest in Modeling are the only international modeling contests in which students work in teams. Centering its educational philosophy on mathematical modeling, COMAP uses mathematical tools to explore real-world problems. It serves the educational community as well as the world of work by preparing students to become better-informed and better-prepared citizens.

Meritorious Teams

Designations of departments named Mathematics, Mathematical Sciences, Mathematics and Computer Science, or the like are omitted.

Traffic Circle Problem (192 teams)

Beihang University, Beijing, China (Peng Lin Ping)
Beihang University, Advanced Engineering, Beijing, China (Wei Feng)
Beijing Institute of Technology, Beijing, China (Xue-Wen Li)
Beijing Institute of Technology, Beijing, China (Gui-Feng Yan)
Beijing Institute of Technology, Beijing, China (Chun-guang Xiong)
Beijing Institute of Technology, Beijing, China (Bing-Zhao Li)
Beijing Jiaotong University, Beijing, China (Bingtuan Wang)
Beijing Jiaotong University, Beijing, China (Peng Cao)
Beijing Language and Culture University, Computer Science, Beijing, China (Xiaoxia Zhao)
Beijing Language and Culture University, Computer Science, Beijing, China (Ping Yang)
Beijing Normal University, Beijing, China (Qing He)
Beijing Normal University, Beijing, China (Xianghui Shi)
Beijing University of Posts and Telecommunications, Computer Science and Technology, Beijing, China (Wenbo Zhang)

Beijing University of Posts and Telecommunications, Electronic Engineering, Beijing, China (Qing Zhou)
Beijing University of Posts and Telecommunications, Electronic Engineering, Beijing, China (Jianhua Yuan)
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Cellphone Energy Problem (102 teams)

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Editor's Note

The complete roster of participating teams and results has become too long to reproduce in the printed copy of the *Journal*. It can now be found at the COMAP Website, in separate files for each problem:

`http://www.comap.com/undergraduate/contests/mcm/contests/2009/results/MCM-A-Results-2009.pdf` and
`http://www.comap.com/undergraduate/contests/mcm/contests/2009/results/MCM-B-Results-2009.pdf`

The listings will also appear on the annual end-of-year CD-ROM.

A Simulation-Based Assessment of Traffic Circle Control

Christopher Chang
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Yi Sun

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Advisor: Clifford H. Taubes

Summary

The difficulty of evaluating the performance of a control system for a traffic circle lies largely in crucial dependence on the local interactions among individual drivers. Traffic circles are relatively small compared to highways and are therefore susceptible to blockages caused by lane changes, entrances, and exits. A complete model must account for effects of such individual car behavior. Existing models, however, do not track performance at the level of individual cars.

We propose a novel simulator-based approach to evaluating and selecting such control systems. We create a multi-agent discrete-time simulation of behavior under different control systems. The behavior of individual cars in our simulator is determined *autonomously* and *locally*, allowing us to capture the effects of local interactions. In addition, by modeling each car separately, we track the time spent in the traffic circle for each individual car, giving us a more specific measure of performance than the more-commonly-used aggregate rate of car passage.

Measuring the performance of several control strategies using both metrics, we find that the *rate of incoming traffic* and the *number of lanes* in the traffic circle are the major factors for optimal choice of a control system. Based on the simulated performance of traffic circles with varying values of these parameters, we have two different recommendations for traffic control systems based upon the rate of incoming traffic:

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- When the rate of incoming traffic is low, *entering cars should yield to cars already in the circle*.
- When the rate of incoming traffic increases beyond a certain threshold (which should be determined empirically), *traffic lights should control entering traffic and the outermost lane of the traffic circle*. These lights should be synchronized so that the time between successive lights turning green is the average time needed for a car to travel between them.

For a low rate of incoming traffic, the circle is relatively clear of cars, so entering cars can merge in without blocking the road or slowing the flow. By making entering cars yield to cars in the circle, we maximize the total throughput of cars while maintaining average speed.

When incoming traffic saturates the circle, allowing cars to merge freely into the circle impedes the flow of others. While throughput is still quite high, our simulation predicts that each car will spend an extremely long time in the circle.

Instead, we recommend that traffic lights attenuate the incoming flow of cars. While cars must wait slightly longer to enter, the number of cars in the circle is limited, allowing those cars a reasonable speed. Our simulator predicts that this policy will allow fewer cars to travel through the circle at a much higher speed.

By viewing the performance of the control system at the level of the individual cars, our simulator distinguishes between the performance of these two systems in this case and select the correct system to use.

We therefore recommend as follows: For times with high occupancies and rates of incoming traffic, implement synchronized traffic lights; for other times, require entering cars to yield to cars in the circle. Under this system, the total throughput is maximized while still maintaining an acceptable level of individual performance.

Zhou Fan, Christopher Chang, and Yi Sun.

The text of this paper appears on pp. 227–245.

One Ring to Rule Them All: The Optimization of Traffic Circles

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Claremont, CA

Advisor: Susan E. Martonosi

Summary

Our goal is a model that can account for the dynamics of vehicles in a traffic circle. We mainly focus on the rate of entry into the circle to determine the best way to regulate traffic. We assume that vehicles circulate in a single lane and that only incoming traffic can be regulated (that is, incoming traffic never has the right-of-way).

For our model, the adjustable parameters are the rate of entry into the queue, the rate of entry into the circle (service rate), the maximum capacity of the circle, and the rate of departure from the circle (departure rate). We use a compartment model with the queue and the traffic circle as compartments. Vehicles first enter the queue from the outside world, then enter the traffic circle from the queue, and lastly exit the traffic circle to the outside world. We model both the service rate and the departure rate as dependent on the number of vehicles inside the traffic circle.

In addition, we run computer simulations to have a visual representation of what happens in a traffic circle during different situations. These allow us to examine different cases, such as unequal traffic flow coming from the different queues or some intersections having a higher probability of being a vehicle destination than others. The simulation also implements several life-like effects, such as how vehicles accelerate on an empty road but decelerate when another vehicle is in front of them.

In many cases, we find that a high service rate is the optimal way to maintain traffic flow, signifying that a yield sign for incoming traffic is most effective. However, when the circle becomes more heavily trafficked,

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a lower service rate better accommodates traffic, indicating that a traffic light should be used. Thus, a light should be installed in most circle implementations, with variable timing depending on the expected amount of traffic.

The main advantage of our approach is that the model is simple and allows us to see clearly the dynamics of the system. Also, the computer simulations provide more in-depth information about traffic flow under conditions that the model could not easily show, as well as enabling visual observation of the traffic. Some disadvantages to our approach are that we do not analyze the effects of multiple lanes nor stop lights to control the flow of traffic within the circle. In addition, we have no way of analyzing singular situations, such as vehicles that drive faster or slower than the rest of the traffic circle, or pedestrians.

Aaron Abromowitz, Andrea Levy, Russell Melick.

The text of this paper appears on pp. 247–260.

Three Steps to Make the Traffic Circle Go Round

Zeyuan Allen Zhu
Tianyi Mao
Yichen Huang

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Advisor: Jun Ye

Summary

With growing traffic, control devices at traffic circles are needed: signals, stop/yield signs, and *orientation signs*—a special sign that we designed.

We create two models—one macroscopic, one microscopic—to simulate transport at traffic circles. The first models the problem as Markov chain, and the second simulates traffic by individual vehicles—a "cellular-automata-like" model.

We introduce a multiobjective function to evaluate the control. We combine saturated capacity, average delay, equity degree, accident rate and device cost. We analyze how best to choose control the traffic circle, in terms of:

- placement of basic devices, such as lights and signs;
- installation of orientation signs, to lead vehicles into the proper lanes; and
- self-adaptivity, to allow the traffic to auto-adjust according to different traffic demands.

We examine the 6-arm-3-lane Sheriffhall Roundabout in Scotland and give detailed suggestions for control of its traffic: We assign lights with a 68-s period, and we offer a sample orientation sign.

We also test smaller and larger dummy circles to verify strength and sensitivity of our model, as well as emergency cases to judge its flexibility.

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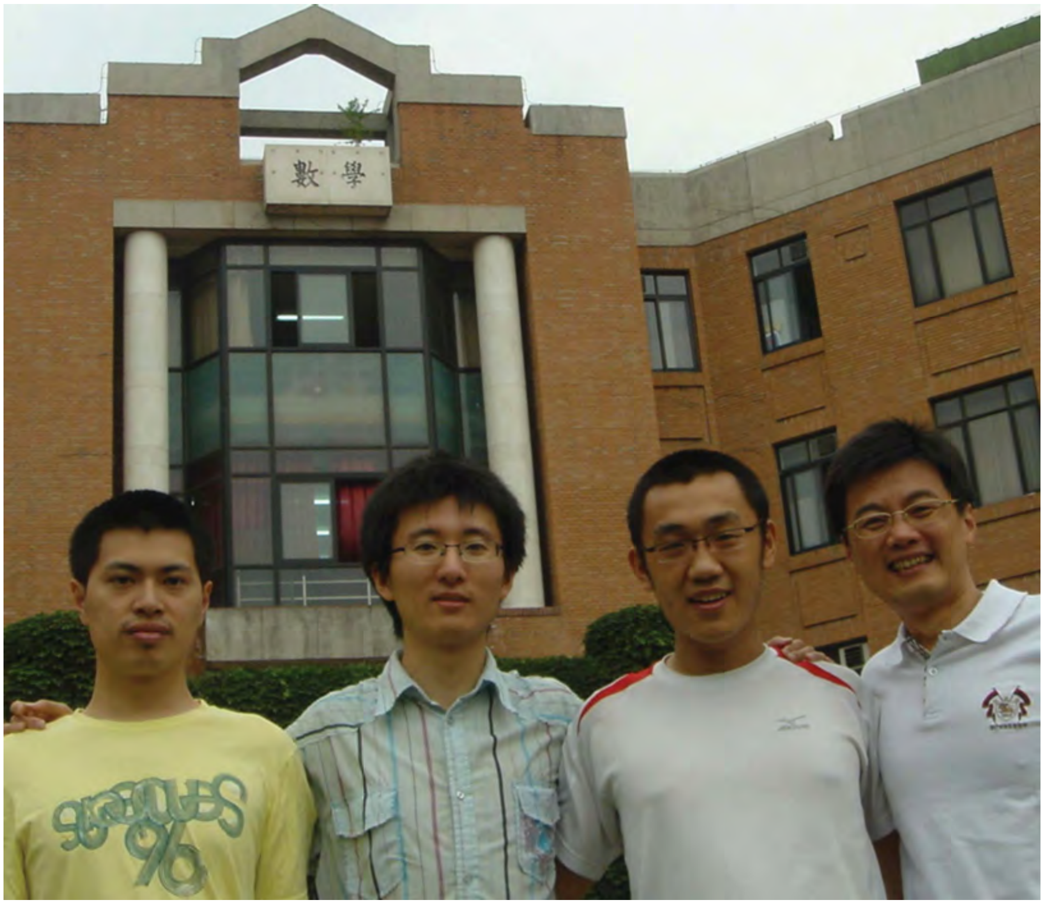

Yichen Huang, Zeyuan Allen Zhu, Tianyi Mao, and team advisor Jun Ye.

The text of this paper appears on pp. 261–280.

Pseudo-Finite Jackson Networks and Simulation: A Roundabout Approach to Traffic Control

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Advisor: Anne Dougherty

Summary

Roundabouts, a foreign concept a generation ago, are an increasingly common sight in the U.S. In principle, they reduce accidents and delays. A natural question is, "What is the best method to control traffic flow within a roundabout?" Using mathematics, we distill the essential features of a roundabout into a system that can be analyzed, manipulated, and optimized for a wide variety of situations. As the metric of effective flow, we choose time spent in the system.

We use Jackson networks to create an analytic model. A roundabout can be thought of as a network of queues, where the entry queues receive external arrivals that move into the roundabout queue before exiting the system. We assume that arrival rates are constant and that there is an equilibrium state. If certain conditions are met, a closed-form stationary distribution can be found. The parameters values can be obtained empirically: how often cars arrive at an entrance (external arrival rate), how quickly they enter the roundabout (internal arrival rate), and how quickly they exit (departure rate). We control traffic by thinning the internal arrival process with a "signal" parameter that represents the fraction of time that a signal light is green.

A pitfall of this formulation is that restricting the capacity of the round-

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about queue to a finite limit destroys the useful analytic properties. So we utilize a "pseudo-finite" capacity formulation, where we allow the roundabout queue to receive a theoretically infinite number of cars, but we optimize over the signal parameter to create a steady state in which a minimal number of waiting cars is overwhelmingly likely. Using lower bound calculations, we prove that *a yield sign produces the optimal behavior for all admissible parameter values*. The analytic solution, however, sacrifices important aspects of a real roundabout, such as time-dependent flow.

To test the theoretical conclusions, we develop a computer simulation that incorporates more parameters: roundabout radius; car length, spacing, and speed; period of traffic signals; and time-dependent inflow rates. We model individual vehicles stochastically as they move through the system, resulting in more-realistic output. In addition to comparing yield and traffic-signal control, we also examine varied input rates, nonstandard roundabout configurations, and the relationships among traffic-flow volume, radius size, and average total time. However, our simulation is limited to a single-lane roundabout. This model is also compromised by the very stochasticity that enhances its realism. Since it is nondeterministic, randomness may mask the true behavior. Another drawback is that the computational cost of minimization is enormous. However, we verify that a yield sign is almost always the best form of flow control.

Geoffrey Peterson, Anil Damle, Anna Lieb, and advisor Anne Dougherty.

The text of this paper appears on pp. 281–304.

Going in Circles: A Roundabout Analysis

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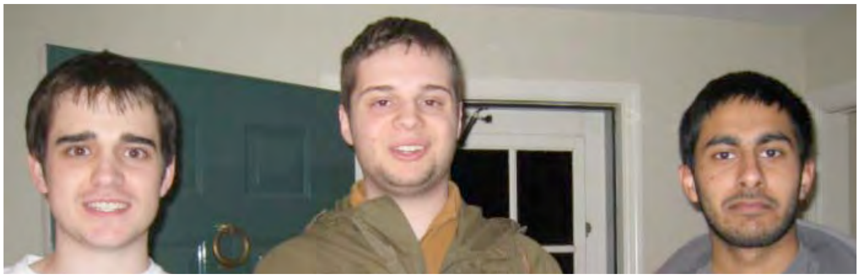
Advisor: Joe Eichholz

Summary

We present a microscopic model of traffic flow in, around, and out of traffic circles of various sizes, configurations, traffic levels, and control systems. We use artificial intelligence to ensure that cars in our simulation faithfully follow human behavior, physical laws, and traffic regulations.

We devise a measure of to compare the efficiencies of control systems on various traffic circles. We illustrate this analysis by redesigning the control systems for La Place de l'Étoile in Paris and around the Victory Monument in Bangkok. According to our model, efficiency is improved by 100% and 80%, respectively. In smaller traffic circles, efficiency can be improved by as much as 20–30% over standard control systems.

We condense our results into rules to find an efficient control system for any traffic circle.



Mark Tucker, Luke Wassink and Ameet Gohil.

[Editor's Note: This Meritorious paper won the Ben Fusaro Award for the Traffic Circle Problem. Only this summary appears in this issue.]

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Mobile to Mobil: The Primary Energy Costs for Cellular and Landline Telephones

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Summary

We determine that cellphones are the optimal communication choice from an energy perspective, using a comprehensive analysis based on multiple factors. We split phones into three categories: cellular, cordless landline, and corded landline. We average the energy used in manufacture and transportation over the life of each phone. To account for the inefficiency of production, we calculate in terms of primary energy, which is the amount of fuel supplied to a power plant per unit of energy produced. We use real-world data for population, number of mainlines, and cellphone subscriptions.

During the transition, as cellphones overtake landlines, part of the population owns both types of phone. As a result, the total energy used by telephones increases. We fit a competing-species model to past statistics; it forecasts that the net energy cost of the cellphone revolution (1995–2025) in the U.S. will be 84 TWh. At the start of this period, there were 0.1 cellphones per capita; at the end there will be 0.1 landlines per capita. Energy savings will begin in 2022. After this transition, savings will be 30 GWh/d. The competing-species model is a proven technique; we apply it to telephone lines and cellphones per capita, and also use it in conjunction with population projections to develop a closed-form solution.

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The most energy-efficient way to provide phone service in a country with no existing infrastructure is to construct a cellular network. By amortizing the fixed setup costs over the lifetime of the phone system, the energy used during construction is negligible. For a country similar to the U.S., the savings would be 12 TWh/yr. Over the next 50 years, the energy savings would equal 0.5 billion barrels of oil.

Cellphone chargers waste energy, but the total energy wasted would be almost five times as great if everyone instead used a cordless phone. Continuing advances in charger technologyare reducing charger waste. If all cellphone chargers in the future meet a 5-star Energy Star rating, they will be 10 times as efficient as now.

Our model is supported by historical data and numerous publicly available statistics. One factor not accounted for is the maintenance and operating power required for cell towers and physical telephone lines.

Jason Altieri, Katelynn Wilton, and Nevin Brackett-Rozinsky. Photo by Dominick DeSalvatore.

The text of this paper appears on pp. 313–332.

Energy Implications of Cellular Proliferation in the U.S.

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Summary

The U.S. has undergone a massive transformation in how it approaches telecommunications. In 30 years, it has gone from having an entirely landline-based phone system to one where 89% of the population uses cellphones, with 16% of households having replaced their landlines entirely. We set out to establish the key consequences and energy costs of this system.

By collecting data on wattages of cellphone chargers and modeling likely American cellphone usage, we calculate that a cellphone might waste 86% of its energy intake through its charger, the equivalent of 754,000 bbl/yr of oil. Comparing that to the energy costs of landline phones, we model two transition scenarios as cellphones replace landlines. We conclude that the faster that landlines can be phased out, the more energy will be saved.

We find that a full cell network, combined with Voice over Internet Protocol (VoIP) technology, would be the best way to provide phone service to a Pseudo U.S. completely lacking in telecommunications. Doing this would save the cost of implementation of a landline infrastructure that would be rendered mostly redundant as cellphones became more popular. Because all the cellphone chargers in this Pseudo U.S. would be brand-new models with recent energy conservation features, cellphone waste would add up to only 234,000 bbl/yr of oil. We model the increase in cellphone energy consumption in this Pseudo U.S. for the next 50 years with two models: one accounts for the growth of the population, and another also factors in a rate of technological advance. In the first model, cellphone

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energy consumption would reach 1.53 million bbl/yr of oil by 2059, while in the second it would actually decrease to 525,000 bbl/yr by then, due to increases in battery efficiency and a reduction in standby power.

Cellphone chargers are a small part of standby-power waste in America. Using extensive wattage and usage data on consumer electronics, we calculate that these devices waste 99 million bbl/yr of oil.

These models show that although a single cellphone charger may waste only a small amount of energy (one author estimates leaving a charger plugged in for a day is about equal to driving a car for one second), the sheer magnitude of cellphone users means that this loss is significant.

Advisor Michael Hitchman, with team members Benjamin Coate, Nathaniel Landis, and Zachary Kopplin.

The text of this paper appears on pp. 333–351.

Modeling Telephony Energy Consumption

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Advisor: Alexander Vladimirsky

Summary

The energy consequences of rapidly changing telecommunications technology are a significant concern. While interpersonal communication is ever more important in the modern world, the need to conserve energy has also entered the social consciousness as prices and threats of global climate change continue to rise. Only 20 years after being introduced, cellphones have become a ubiquitous part of the modern world. Simultaneously, the infrastructure for traditional telephones is well in place and the energy costs of such phones may very well be less. As a superior technology, cellphones have gradually begun to replace the landline but consumer habits and perceptions have slowed this decline from being an outright abandonment.

To evaluate the energy consequences of continued growth in cellphone use and a decline in landline use, we present a model that describes three processes—landline consumption, cellphone consumption, and landline abandonment—as economic diffusion processes. In addition, our model describes the changing energy demands of the two technologies and considers the use of companion electronics and consumer habits. Finally, we use these models to determine the energy consequences of the future uses of the two technologies, an optimal mode of delivering phone service, and the costs of wasteful consumer habits.

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Amrish Deshmukh, Matt Guay, Niko Stahl, and advisor Alexander Vladimirsky.

The text of this paper appears on pp. 353–365.

National TELEwar

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Summary

Over 89% of 303 million Americans own a cellphone, with a battery that needs to be recharged. All too often, the phone is left plugged in, constantly consuming energy. In addition, 79% of Americans are served by home landline phones.

By modeling energy consumption based on growth and decay of landlines and cell phones due to population changes, an optimized energy plan can minimize energy used by a country's communication infrastructure while still providing citizens with adequate telecommunication options.

By modeling the cell-phone growth and consequent landline decay as a logistic predator-prey model—and applying real-world energy, population, and communications-use data—we determine an optimal telecommunication system.

John Camardese, Rich Geyer, advisor Ruth Favro, and Neil Ganshorn.

[Editor's Note: This Meritorious paper won the Ben Fusaro Award for the Cellphone Problem. Only this summary appears in this issue.]

The UMAP Journal 30 (3) (2009) 222.

America's New Calling

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Summary

The ongoing cellphone revolution warrants an examination of its energy impacts—past, present, and future. Thus, our model adheres to two requirements: It can evaluate energy use since 1990, and it is flexible enough to predict future energy needs.

Mathematically speaking, our model treats households as state machines and uses actual demographic data to guide state transitions. We produce national projections by simulating multiple households. Our bottom-up approach remains flexible, allowing us to:

- model energy consumption for the current U.S.,
- determine efficient phone adoption schemes in emerging nations,
- assess the impact of wasteful practices, and
- predict future energy needs.

We show that the exclusive adoption of landlines by an emerging nation would be more than twice as efficient as the exclusive adoption of cellphones. However, we also show that the elimination of certain wasteful practices can make cellphone adoption 175% more efficient at the national level. Furthermore, we give two forecasts for the current U.S., revealing that a collaboration between cellphone users and manufacturers can result in savings of more than 3.9 billion barrels-of-oil-equivalent (BOE) over the next 50 years.

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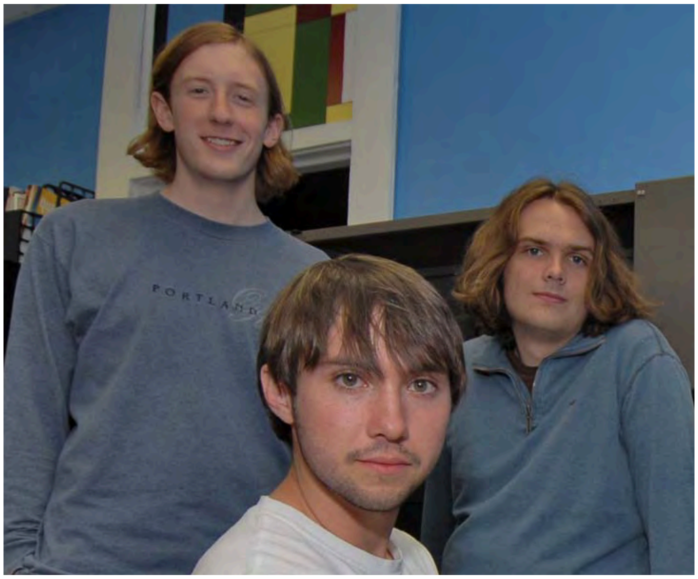

Tommy Rogers, Stephen Foster, and Bob Potter.

The text of this paper appears on pp. 367–384.

Wireless Networks: An Easy *Cell*

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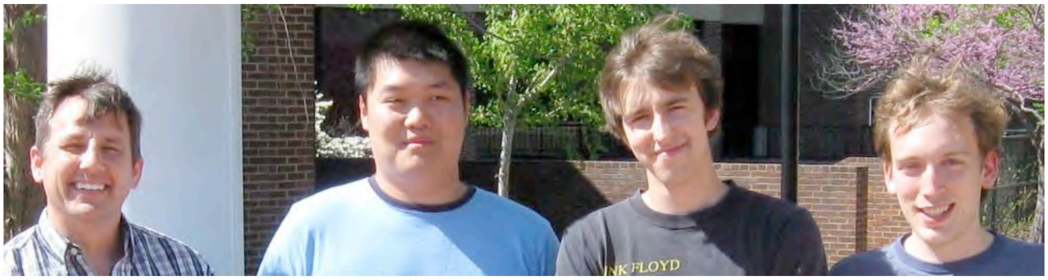
Summary

The number of cellphones worldwide raises concerns about their energy usage, even though individual usage is low (< 10 kWh/yr). We first model the change in population and population density until 2050, with an emphasis on trends in the urbanization of America. We analyze the current cellular infrastructure and distribution of cell site locations in the U.S. By relating infrastructure back to population density, we identify the number and distribution of cell sites through 2050. We then calculate the energy usage of individual cellphones calculated based on average usage patterns.

Phone-charging behavior greatly affects power consumption. The power usage of phones consumes a large part of the overall idle energy consumption of electronic devices in the U.S.

Finally, we calculate the power usage of the U.S. cellular network to the year 2050. If poor phone usage continues, the system will require 400 MW/yr, or 5.6 million bbl/yr of oil; if ideal charging behavior is adopted, this number will fall to 200 MW/yr, or 2.8 million bbl/yr of oil.



Advisor Louis Rossi with team members Bob Liu, Jeff Bosco, and Zachary Ulissi.

The text of this paper appears on pp. 385–402.

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A Simulation-Based Assessment of Traffic Circle Control

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Summary

The difficulty of evaluating the performance of a control system for a traffic circle lies largely in crucial dependence on the local interactions among individual drivers. Traffic circles are relatively small compared to highways and are therefore susceptible to blockages caused by lane changes, entrances, and exits. A complete model must account for effects of such individual car behavior. Existing models, however, do not track performance at the level of individual cars.

We propose a novel simulator-based approach to evaluating and selecting such control systems. We create a multi-agent discrete-time simulation of behavior under different control systems. The behavior of individual cars in our simulator is determined *autonomously* and *locally*, allowing us to capture the effects of local interactions. In addition, by modeling each car separately, we track the time spent in the traffic circle for each individual car, giving us a more specific measure of performance than the more-commonly-used aggregate rate of car passage.

Measuring the performance of several control strategies using both metrics, we find that the *rate of incoming traffic* and the *number of lanes* in the traffic circle are the major factors for optimal choice of a control system. Based on the simulated performance of traffic circles with varying values of these parameters, we have two different recommendations for traffic control systems based upon the rate of incoming traffic:

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- When the rate of incoming traffic is low, *entering cars should yield to cars already in the circle*.
- When the rate of incoming traffic increases beyond a certain threshold (which should be determined empirically), *traffic lights should control entering traffic and the outermost lane of the traffic circle*. These lights should be synchronized so that the time between successive lights turning green is the average time needed for a car to travel between them.

For a low rate of incoming traffic, the circle is relatively clear of cars, so entering cars can merge in without blocking the road or slowing the flow. By making entering cars yield to cars in the circle, we maximize the total throughput of cars while maintaining average speed.

When incoming traffic saturates the circle, allowing cars to merge freely into the circle impedes the flow of others. While throughput is still quite high, our simulation predicts that each car will spend an extremely long time in the circle.

Instead, we recommend that traffic lights attenuate the incoming flow of cars. While cars must wait slightly longer to enter, the number of cars in the circle is limited, allowing those cars a reasonable speed. Our simulator predicts that this policy will allow fewer cars to travel through the circle at a much higher speed.

By viewing the performance of the control system at the level of the individual cars, our simulator distinguishes between the performance of these two systems in this case and select the correct system to use.

We therefore recommend as follows: For times with high occupancies and rates of incoming traffic, implement synchronized traffic lights; for other times, require entering cars to yield to cars in the circle. Under this system, the total throughput is maximized while still maintaining an acceptable level of individual performance.

Introduction

The traffic circle is a type of circular intersection featuring traffic from multiple streets circulating around a central island, usually in one direction. An example is shown in **Figure 1**. Other examples of large traffic circles include Columbus Circle in New York City, while small, one-lane traffic circles often exist in residential neighborhoods.

Traffic circles are often notorious for frequent traffic jams due to their unconventional design, and many methods exist to control traffic in a traffic circle; we investigate their impacts.

Figure 1. An aerial view of Dupont Circle in Washington, DC. Source: U.S. Geological Survey, at `http://en.wikipedia.org/wiki/index.html?curid=1017545`.

Terms and Notation

We consider a traffic circle to be a one-way circular road with two-way roads meeting the circle at T-junctions. In particular, we do not consider circles that have separate entry and exit ramps. We assume that each road carries cars into the circle at a fixed rate and that cars have an equal probability of leaving the circle through any of the other roads. For performance, we measure two statistics:

- the average rate at which cars arrive at their desired exit location per time step, the *average throughput*; and
- the average number of time steps from a car arriving at the back of the queue to enter the circle to when it exits the circle, the *average total time*.

Problem Background

Modern traffic circles have recently been recognized as safer alternatives to traditional intersections. Research by Zein et al. [1997] and Flannery and Datta [1996] using statistical methods has demonstrated that traffic circles bring added safety to both urban and rural environments. Attempts to understand the specific safety and efficiency benefits of traffic circles have taken four primary approaches: critical-gap estimation, regression studies, continuous models, and discrete models.

- **Critical-gap models** build from how drivers empirically gauge gaps in traffic before merging or turning into a traffic stream. However, according to Brilon et al. [1997], attempts in the 1980s to model roundabout capacity based on gap-acceptance theory were not exceptionally promising; in particular, critical-gap estimation lacked valid procedures as well as general clarity [Brilon et al. 1999]. More recent research applying gap-acceptance models to understanding traffic circles has included Polus et al. [1997] and the modeling of unconventional traffic circles by Bartin et al. [2006].

Nevertheless, regression studies on empirical data

- **Regression studies** on empirical data made much progress, beginning with Kimber [1980], who studied roundabouts in England and discovered a linear relation relating entry capacity to circulating flow and constants depending on entry width, lane width, the angle of entry, and the traffic circle size. Further regression studies have built extensively on Kimber's work, such as in Polus and Shmueli [1997], which determined the importance of traffic circle diameter in small-to-medium circles.
- **Continuous models** have included fluid-dynamic models [Helbing 1995; Bellomo et al. 2002; Daganzo 1995; Klar et al. 1996]; but those papers model traffic flow in standard traffic environments, not in traffic circles.
- **Discrete models** include cellular automata models [Fouladvand et al. 2004; Klar et al. 1996] and discrete stochastic models [Schreckenberg et al. 1995]. Discrete models are suitable for small environments such as traffic circles, where individual car-to-car interactions takes priority over traffic flow as a whole. Discretized approaches have attempted to model multilane traffic flows [Nagel et al. 1998]; but to our knowledge, there has been no research on discrete models of multilane traffic circles of varying sizes.

Our Results

We approach traffic-circle control by first creating a simulator of traffic flow that treats individual cars as autonomous units, allowing us to capture local interactions, such as lane changes and traffic blockages due to cars entering and exiting. We validate this simulator against both a new stylized model of the situation and existing models of traffic-circle flow.

Using the simulator, we implement and test various control systems on different types of traffic circles. Based on the simulated results, we isolate the *rate of incoming traffic* and the *number of lanes* in the traffic circle as the driving factors behind optimal choice of a traffic-control system. We thus recommend two different systems for different circumstances:

- When the rate of incoming traffic is low, we recommend that **entering cars yield to cars already in the circle**.
- When the rate of incoming traffic increases beyond a certain threshold, we recommend **traffic lights that control entering traffic and the outermost**

lane of the traffic circle. The lights should be synchronized so that the time between successive lights turning green is the average time needed for a car to travel between them.

In subsequent sections, we

- divide the problem into two portions and define our objectives for each;
- introduce the simulator and validate its performance against a mathematical analysis and models from other sources;
- use the simulator to analyze the performance of several types of traffic circles, to produce recommendations for which control systems should be used for each type; and
- provide an overview of the advantages and disadvantages of our approach and give directions for future work.

Simulator

Our goal is a simulator that, given a set of conditions and traffic rules, can produce an accurate prediction of the behavior that will result from following these rules. To achieve this goal, we would like our simulator to fulfill the following requirements:

- *The simulator takes into account the local interactions between cars.* Because cars enter, exit, and change lanes quite frequently, interactions between cars make a major contribution to the speed and efficacy of a traffic circle.
- *The simulator can support variation in the number of cars, size of the circle, and number of lanes.*
- *The simulator can track properties of both the entire traffic circle and the individual cars passing through it.*

The real behavior of cars in a traffic circle may vary widely, but we restrict our simulated cars to idealized behavior: they follow the traffic regulations that we put in place, and no accidents happen.

Control System Evaluation

We base our recommendations for a control method on the following statistics:

1. Average throughput (the average rate at which cars pass through the traffic circle).
2. Average number of cars in the traffic circle.

3. Average total time for each car to traverse the traffic circle, including time spent waiting to enter.
4. Average time that each car spends driving through the traffic circle.

Statistics 1 and 2 measure global properties of the traffic circle, while statistics 3 and 4 are properties of each individual car. To evaluate a control system, we consider both the global performance and the differences in performance of the system for each individual. In particular, our goals are to:

1. Maximize the average throughput of the traffic circle.
2. Minimize the total time spent traversing the traffic circle (for individual cars).

We evaluate the performance of a traffic circle by **the rate of cars passing through the circle** (*average throughput*) and **the total time required to traverse the circle** (*average total time*). We choose control methods that perform best with respect to both of these metrics.

Simulator Details

Assumptions and Setup

There are two approaches to model the behavior of traffic:

1. Make a (usually continuous) abstraction away from the discrete interactions of cars and deal with a more stylized model of the entire system.
2. Model the behavior and movement of each car separately.

Continuous and fluid-like models, as in the first possibility, are suitable under a macroscopic view of traffic, for instance in the study of traffic on long roads or highways. However, for intersections and traffic circles, where car-to-car interactions occur much more frequently, such a model seems inadequate.

We follow the second approach to model traffic flow in a traffic circle using a multi-agent discrete time simulation. Our simulation is based around the following two key principles:

1. It is microscopic.
2. Behavior and information are local.

We do not use an abstract view of traffic as a flow but instead let each car in the traffic circle be its own individual agent. This allows us to account for the effects of car-to-car interaction, particularly in congested situations. From this interaction on the microscopic level, we then examine the macroscopic consequences of the simulation, instead of beginning with an arbitrary conception of what the macroscopic behavior should be.

Each car is its own independent agent, trying to enter the traffic circle and exit at the desired exit as quickly as possible; no collaboration between cars or higher-level organizational principles exists. Also, only local information, namely the cars in the immediate neighborhood, is available to each individual car.

The Simulator

The simulation operates using the following model:

- Time is modeled in discrete time steps.
- The traffic circle is a rectangular grid. The width is the number of lanes in the traffic circle, and the height represents the length of the traffic circle. The upper edge wraps around to the lower edge (so that the grid is actually a circle). At any time step, each square of the grid can either be empty or hold one car.
- Certain squares in the outermost lane are *entry squares*. A queue of cars waits at each entry square to enter the circle. (These cars are not located on the grid itself.) The queues start off empty, and for each entry square, there is a fixed probability of a car being added to its queue at each time step.
- Certain squares in the outermost lane are *exit squares*, where cars can exit the circle. When a car is added to the queue for an entry square (and thus to the system), an exit square is chosen at random for the car.
- Each car has a speed indicated by how often it gets the chance to move. For example, faster cars may move at every time step, while slower cars may move less often. This difference simulates differing levels of impatience or aggression among drivers.

In each time step, the simulation proceeds as follows:

1. Determine the subset of all the cars in the system that will move during this time step. Randomly assign the order in which these cars will move.
2. Allow each such car to move. Cars move under the following rules:
   A car that is already in the traffic circle at position (i, j) (i.e., lane i, vertical position j) will, in the following order of preference,
   (a) Exit if (i, j) is the exit square at which the car wishes to exit.
   (b) Move forward to $(i, j+1)$ if $(i, j+1)$ is unoccupied.
   (c) Move forward and right to $(i+1, j+1)$ if there is a lane to the right and locations $(i+1, j)$ and $(i+1, j+1)$ are both unoccupied.

(d) Move forward and left to $(i-1, j+1)$ if there is a lane to the left and locations $(i-1, j)$ and $(i-1, j+1)$ are both unoccupied.

(e) Stay where it is.

An exception occurs for cars that are about to exit—if the vertical distance between the car's current location and its desired exit is less than four times the horizontal distance, then items (b) and (c) above are switched. (This is to ensure that under uncongested situations, cars will be able to exit at their desired exits.)

A car that is the first car in the queue at an entrance location will, in order of preference,

(a) Move to the entry square if that square is unoccupied.

(b) Stay where it is.

All later cars in the queue cannot move for this turn.

In addition to the above rules, certain traffic control systems impose the following additional rules:

(a) **Outer-yield:** A car at the front of an entrance queue and waiting to enter can enter only when both the entry square and the square directly behind it are empty. That is, if there is a car directly behind the entry square, the entering car must yield to that car.

(b) **Inner-yield:** If a car in the circle wishes to move onto an entry square (in the rightmost lane) but the queue waiting at that entrance is nonempty, then the car cannot move to that square. If the car has no other possible moves, then it does not move for that turn. This reflects the situation in which cars in the circle need to yield to entering cars.

(c) **Traffic lights:** In this system, a traffic light controls each entry square. At any time step, the light is either green for cars in the circle and red for the waiting queue, or vice versa. If it is green for cars in the circle, then the first car in the waiting queue cannot enter the circle. If it is green for the waiting queue, then no car in the circle can move onto the entry square. In a multilane circle, this traffic light controls traffic only in the outermost lane. This behavior is inspired by the design of metering lights at highway ramps.

    We consider two methods of synchronizing the traffic lights around the circle:

    i. All lights turn green and red simultaneously.

    ii. The difference in time between each traffic light turning green and the next light turning green (and also the difference between their reds) is directly proportional to the distance between the two lights. The proportionality constant is chosen so that a car waiting at a traffic light that begins to move when that light turns green will reach the next light just as it turns green.

3. For each entry queue, add a car to the end of that queue with the fixed probability for that entry location.
4. Have the traffic lights change if it is the correct time step to do so.

Validation Against Existing Empirical Models

The two criteria on which we evaluate the various traffic control systems, average throughput and average total time, are not unrelated. In fact, our simulations indicate that increasing one comes at the cost of increasing the other. For the outer-yield system, we show in **Figure 2** average total time against reserve throughput (maximum throughput minus average throughput).

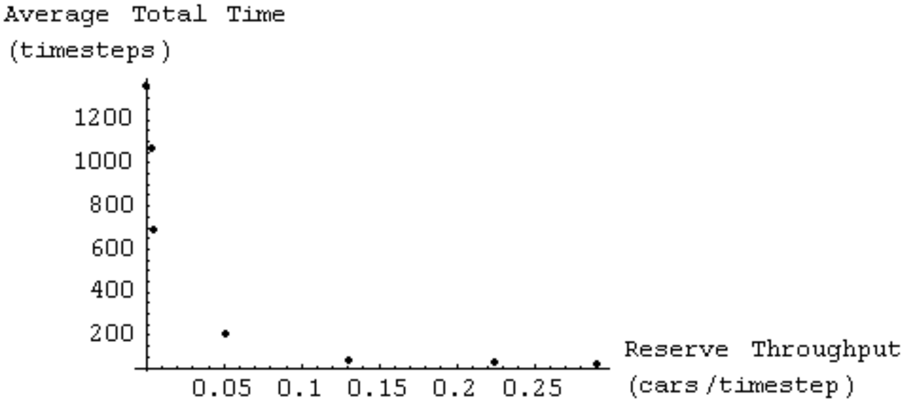


Figure 2. Average total time vs. reserve throughput.

This inverse relationship is intuitive, since greater throughput indicates a greater volume of traffic on the road and hence both slower driving speeds and longer wait times to enter the circle. This result also matches the relationship between average total time and reserve capacity given by the Kimber-Hollis delay equation in Brilon and Vandehey [1998]. This agreement indicates that the results of our simulator are reasonable.

Validation Against a Simple Model

To provide further verification of the accuracy of our simulator, we compare large-scale features of its output to a mathematical model for a simple case. In particular, we consider a single-lane traffic circle in which cars entering the circle yield to cars in the circle. For simplicity, we assume that all cars move at the same speed, one square per time step. We assume that roads at traffic circle are all two-way, so that each entry point is also an exit point. The model is given as follows.

Suppose that there are n entry/exit roads to the traffic circle, and that all cars have an equal probability of leaving through each of the n roads. For $i = 1$ to n, let r_i be the probability that a new car appears at road i at any time step. Let x_i be the volume density of traffic in the segment of the circle between roads i and $i+1$. The expected change in the number of cars between roads i and $i+1$ is given by a sum of four terms:

- The probability that a car will leave this segment through exit $i+1$ is $\frac{1}{n} \cdot x_i$, since x_i is the probability that there is a car in the exit square and the probability that this car wishes to exit is $\frac{1}{n}$.
- The probability that a car will move from this segment to the next one is $x_i \cdot \frac{n-1}{n} \cdot (1 - x_{i+1})$, since $\frac{n-1}{n}$ is the probability that the car in the exit square will not exit and $1 - x_{i+1}$ is the probability that the square after the exit square, which is the first square of the next segment, is unoccupied.
- The probability that a car will move from the previous segment to this segment is, similarly, $x_{i-1} \cdot \frac{n-1}{n} \cdot (1 - x_i)$.
- The probability that a car will enter through entrance i is the probability p of a sufficiently large space at entrance i for a car to enter, times the probability that there is a car there waiting to enter. This latter probability can be calculated as

  $$r_i + r_i(1 - r_i)(1 - p) + r_i(1 - r_i)^2(1 - p)^2 + \cdots = \frac{r_i}{r_i + p - r_i p},$$

  since there is an r_i probability of a car arriving at entrance i this time step, an $r_i(1 - r_i)(1 - p)$ probability of a car arriving at entrance i at the last time step (but not this time step) and remaining until this time step, etc. In our simulation, $p = (1 - x_{i-1})(1 - x_i)$, since a car can enter the circle if the entry square and the previous square are unoccupied.

So the expected change in the number of cars in this segment in one time step is

$$\Delta c_i = -x_i \cdot \tfrac{1}{n} - x_i \cdot \tfrac{n-1}{n} \cdot (1 - x_{i+1}) + x_{i-1} \cdot \tfrac{n-1}{n} \cdot (1 - x_i) + \frac{r_i(1-x_{i-1})(1-x_i)}{r_i+(1-x_{i-1})(1-x_i)+r_i(1-x_{i-1})(1-x_i)}.$$

In equilibrium, this change should be 0 for all segments, giving a system of equations in the x_i. If we consider the case where the roads have equal incoming traffic, i.e. r_i is the same for all i, then by symmetry the x_i are the same for all i, and we may solve the equation

$$\Delta c = -x \cdot \tfrac{1}{n} - x \cdot \tfrac{n-1}{n} \cdot (1 - x) + x \cdot \tfrac{n-1}{n} \cdot (1 - x) + \frac{r(1-x)^2}{r+(1-x)^2+r(1-x)^2} = 0$$

numerically for x in terms of r. Here, x is the traffic volume density for the circle as a function of r, the rate at which cars enter the circle through each road. The result of numerically solving for x is shown in **Figure 3** together with the corresponding plot generated by our simulator. The black data points were generated by our simulator and the curve was produced by the rudimentary model.

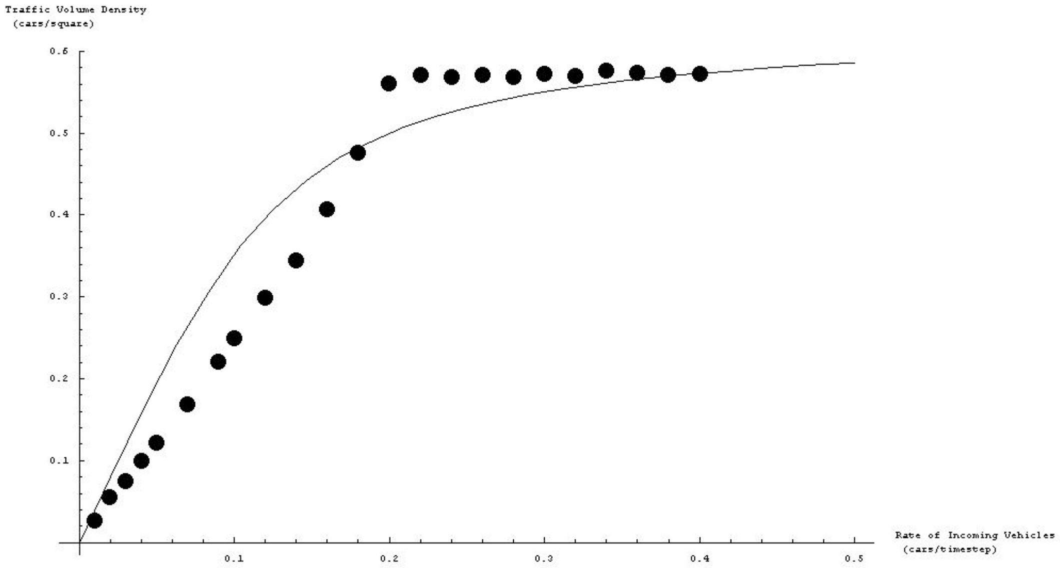


Figure 3. Traffic volume density vs. rate of incoming vehicles.

The volume density of both seems to grow in a somewhat linear fashion for low rates of incoming vehicles. When the number of incoming vehicles increases, the traffic circle appears to become saturated at a fixed density. The simulator and our mathematical model agree on these large-scale features. Disagreement about the critical rate of incoming vehicles might be explained by the fact that our mathematical model essentially considers the cars in each segment as equivalent, hence ignores the small-scale interactions that occur near gridlock.

Predictions and Analysis

We apply the simulator to analyze different types of traffic circles.

Criteria

We characterize traffic circles by the following variables:

1. **Rate of incoming vehicles:** This is a result of the amount of traffic present on the roads entering the traffic circle and will influence the total number of vehicles trying to enter the circle and hence the traffic in the circle.

2. **Length:** This affects the number of cars that can be contained in the circle at a single time, which has many implications for how the entry mechanism of the circle should be determined.
3. **Number of lanes:** This affects both the number of cars that can be in the circle at a time and their maneuverability around each other. Because cars can more easily pass one another with more lanes, increasing the number of lanes may reduce the effects of traffic blockages.
4. **Number of incoming roads:** This affects the rate at which cars need to enter and exit the traffic circle, which may influence the magnitude of traffic blockages.

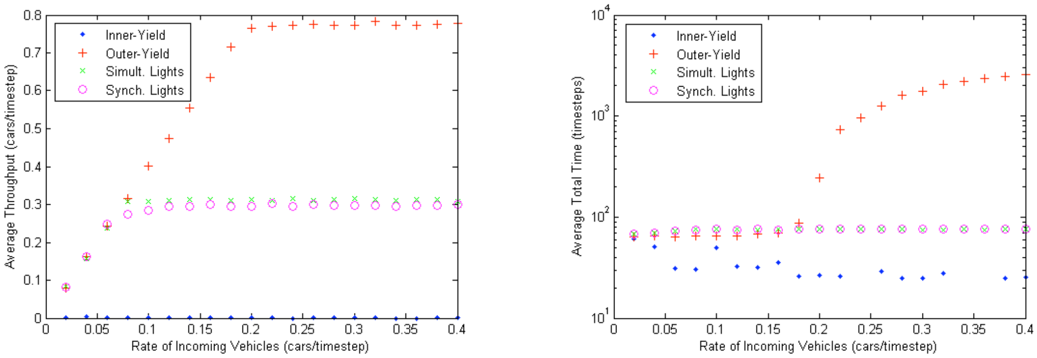
We wish to consider systems that are relatively close to conventional systems, since it would be impractical and hazardous to introduce radically different systems unfamiliar to drivers who do not encounter traffic circles frequently. Therefore, we will evaluate the performance of the following traffic control systems when we vary the parameters for our traffic circle:

1. **Outer yield:** Cars attempting to enter the circle yield to cars already in the circle at all times.
2. **Inner yield:** Cars already in the circle yield to cars attempting to enter the circle.
3. **Simultaneous lights:** The intersections between the circle and other roads are controlled by traffic lights that all turn green/red at the same time. The traffic lights apply only to the outermost lane of the traffic circle.
4. **Synchronized lights:** The intersections between the circle and other roads are controlled by traffic lights for which the time interval between a light turning green and the next light turning green is proportional to the distance between the two lights. The traffic lights apply only to the outermost lane of the traffic circle.

With the exception of the traffic lights, these control systems are all similar to existing control systems. However, there is a crucial difference between our traffic-light system and standard traffic lights: Stopping only the outer lane of the traffic circle allows traffic in the inner lane to proceed undisturbed, improving throughput. This approach is a hybrid of normal traffic lights and metering lights for congested highways.

Analysis

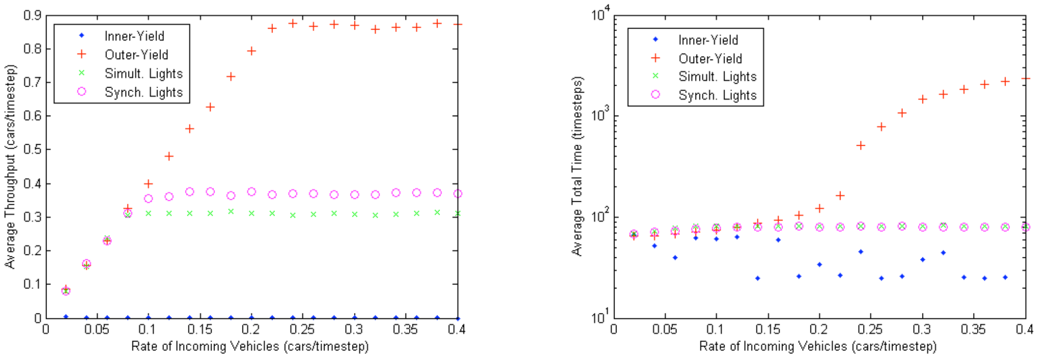
To analyze the effects of control systems, we run each on circles with varying parameters and create plots of average throughput and average total time per car for each strategy (**Figures 4–8**).



Average throughput vs.
Rate of incoming vehicles

Average total time vs.
Rate of incoming vehicles

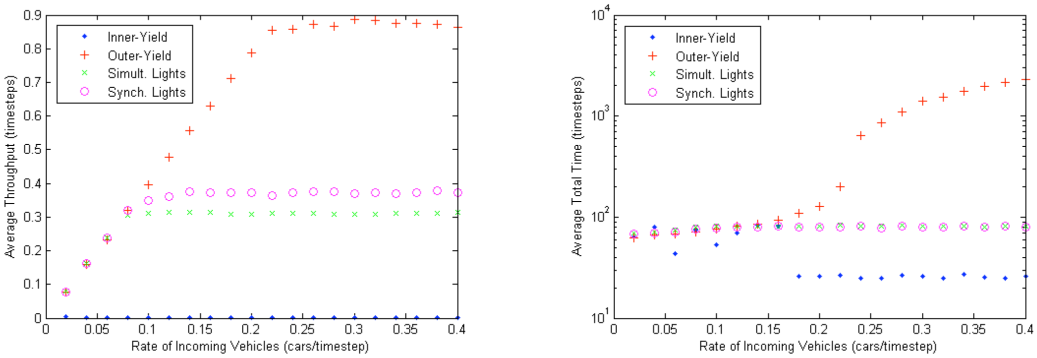
Figure 4. Performance for 1 lane, length 100, 4 roads, rate variable.



Average throughput vs.
Rate of incoming vehicles

Average total time vs.
Rate of incoming vehicles

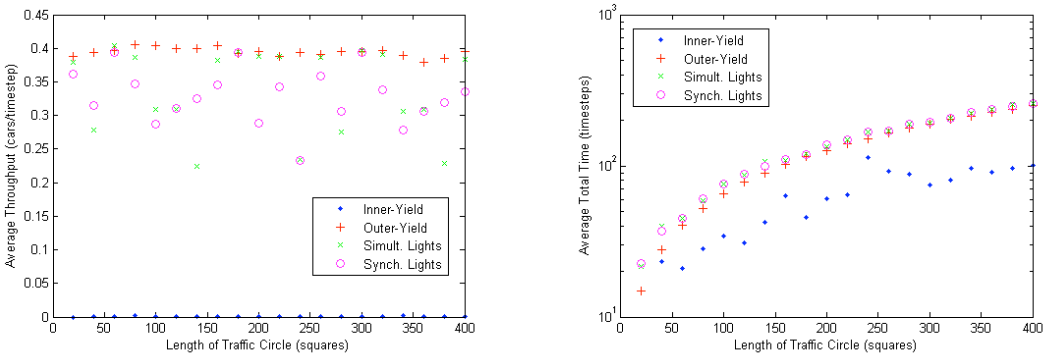
Figure 5. Performance for 3 lanes, length 100, 4 roads, rate variable.



Average throughput vs.
Rate of incoming vehicles

Average total time vs.
Rate of incoming vehicles

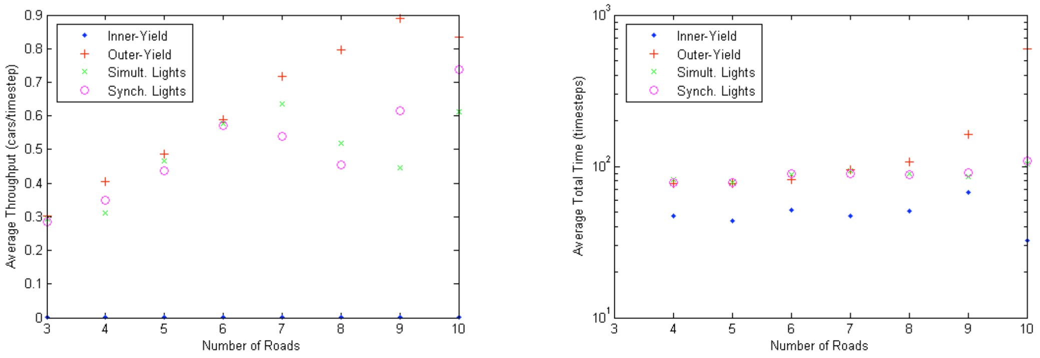
Figure 6. Performance for 5 lanes, length 100, 4 roads, rate variable.



Average throughput vs. Length

Average total time vs. Length

Figure 7. Performance for 3 lanes, rate 0.1, 4 roads, length variable.



Average throughput vs. Number of roads

Average total time vs. Number of roads

Figure 8. Performance for 3 lanes, rate 0.1, length 100, roads variable.

Our goal is to determine which parameters have the greatest effect on performance of the control systems. From the plots, we make the following observations:

- In almost all the plots, the inner-yield system has almost no throughput, since the cars in the road become gridlocked because they too often yield to incoming cars and therefore cannot exit. The low value of the average total time for this system results from the fact that the only cars that can exit do so before the road becomes entirely gridlocked. As a result, *we reject the inner-yield system.*
- As the rate is varied in **Figures 4–6**, the throughputs of the outer-yield system and the traffic-light systems correspond for small values of the rate. However, for each system, the throughput reaches a plateau beyond a certain value of the rate. At this point, the circle has been saturated, meaning that it can no longer accept more cars from the incoming roads.

- The throughput value at which saturation occurs is much higher for the outer-yield system. However, the amount of time required for each individual car to pass through the traffic circle under the outer-yield system is extremely high, almost an order of magnitude higher than needed under either traffic-light system.
- When there are either 3 or 5 lanes, the synchronized traffic-light system allows slightly greater throughput than the simultaneous traffic-light system. This might be explained by the fact that, with more lanes, cars can move in a more uniform manner, allowing them to use the synchronized lights and move through the circle more quickly.
- The number of lanes and the number of roads do not have a significant effect on either the throughput or the total time in the outer-yield system or in either of the traffic-light systems. However, traffic lights may perform worse for some values of the distance between roads, perhaps due to synchronization issues.

In general, the outer-yield and traffic-light methods have an advantage over the inner-yield method, and the correct choice of control system is largely determined by the rate of incoming vehicles on each of the entry roads.

Recommendations

Since the number of lanes and the number of incident roads do not significantly affect average throughput or average traversal time, we can restrict our attention to the rate of incoming cars and the number of lanes in the circle.

The rate of incoming cars accounts for a large part of the variation in performance, as can be seen in **Figures 4–6**.

- For values of this rate between 0 and 0.1 cars per time step , the performance of the outer-yield and traffic-light systems is identical, since in this range traffic is light and there is very little interaction between cars.
- As the rate increases to between 0.1 and 0.2 cars per time step , the traffic circle reaches its maximum throughput under the traffic-light system, while the average total time stays fixed. However, under the outer-yield system, the throughput continues to increase but at the cost of a rather dramatic increase in average total time. In this range, choosing between the outer-yield system and the traffic-light systems involves a tradeoff between throughput, the *quantity* of cars passing through, and total time, the *speed* at which cars pass through.
- Finally, as the the rate increases above 0.2 cars per time step , the circle becomes saturated with cars, meaning that the average total time for the outer-yield system increases dramatically and there is gridlock, meaning

that cars move extremely slowly and must wait a very long time to pass through. Under the traffic-light systems, however, a smaller number of cars can pass through, but the average total time required for them to pass remains similar to that with a much lower rate. Since the inner-yield system requires an extremely large total time in this range, the traffic-light systems are clearly superior for a rate of above 0.2 cars per time step.

In each of these cases, the synchronized traffic lights allow for higher throughput than simultaneous traffic lights.

We can now make the following recommendations:

- **For a low rate of entering cars, no traffic lights should be used.** Instead, cars already in the circle should be given the right of way, and cars entering the circle should yield.
- **As the rate of entering cars increases, synchronized traffic lights should be considered for the outermost lane (only),** to ensure a reasonable traversal time for most cars.
- **For large rates of entering cars, as may occur during rush hour, synchronized traffic lights should be used,** to ensure that the traffic circle does not become congested. By preserving a reasonable flow of cars within the circle, synchronized traffic lights allow a slightly smaller number of cars to pass through the circle much more quickly, which is preferable to deadlock for all cars.

For low and high rates, our recommendations agree with practice. An intersection with little traffic may have no traffic signals (alternatively, a traffic circle is installed explicitly in place of traffic lights). For highways, it is common to use metering lights during peak hours to regulate entry of vehicles, to ensure that cars already on the road can move at a reasonable speed. Our recommendations seem to be a mix of these two ideas applied to traffic circles.

Conclusions

Strengths

Our simulator takes into account the behavior and outcomes of individual cars traveling through a traffic circle. By doing so, we are able to detect interactions at a microscopic level and to track the performance of a traffic control system for each individual rather than only in aggregate. Doing this allows our model to evaluate the effects of cars changing lanes and entering and from specific lanes. We validated the simulator against both an existing empirical model and the results of a simple model for the steady-state limit.

We can simulate the performance of a widely varied spectrum of traffic control systems on a range of different traffic circles. Our results allow us to isolate the rate of incoming cars and the number of lanes in the traffic circle as the two parameters key to determining a good control system.

We recommend the either an outer-yield system or synchronized traffic lights to control the traffic circle, depending on the rate at which cars enter the circle.

Weaknesses

While our simulator attempts to model the behavior of drivers fairly accurately, it cannot completely capture the dynamics of lane-changing and braking. Further, while using a discrete-time, discrete-space model for the simulator allows us to capture the local multiagent nature of individual drivers, it forces us to make simplifications about the continuity of car movement and about simultaneous actions.

In addition, our simulation does not take into account the fact that in an actual traffic circle, the inner lanes have shorter length than the outer lanes.

We consider only traffic lights with simultaneous or synchronized light changes, and it is infeasible computationally for us to consider a wider variety of switching approaches.

Alternative Approaches and Future Work

We could evaluate the safety of a control system by counting the number of conflicting desired movements at the local level. We could then compare systems by safety as well as by performance and hence evaluate the claim that certain types of traffic circles are safer than intersections [Flannery and Datta 1996].

References

Bartin, Bekir, Kaan Ozbay, Ozlem Yanmaz-Tuzel, and George List. 2006. Modeling and simulation of unconventional traffic circles. *Transportation Research Record: Journal of the Transportation Research Board* 1965: 201–209. `http://trb.metapress.com/content/y77tu5l173658858/`.

Bellomo, N., M. Delitala, V. Coscia, and F. Brezzi. 2002. On the mathematical theory of vehicular traffic flow I: Fluid dynamic and kinematic modelling. *Mathematical Models and Methods in Applied Sciences* 12: 1217–1247.

Brilon, Werner, and Mark Vandehey. 1998. Roundabouts—The state of the art in Germany. *Institute of Transportation Engineers (ITE) Journal* 68: 48–54.

Brilon, Werner, Ning Wu, and Lothar Bondzio. 1997. Unsignalized intersections in Germany—A state of the art. In *Proceedings of the Third International Symposium on Intersections without Traffic Signals*, Portland, Oregon, 61–70. `http://www.ruhr-uni-bochum.de/verkehrswesen/vk/deutsch/Mitarbeiter/Brilon/Briwubo_2004_09_28.pdf`.

Brilon, Werner, Ralph Koenig, and Rod J. Troutbeck. 1999. Useful estimation procedures for critical gaps. *Transportation Research Part A* 33: 161–186. `http://www.sciencedirect.com/science/article/B6VG7-3VF9D7R-2/2/2b325096a3b448fd0c0a09952c091ff4`.

Daganzo, Carlos F. 1995. Requiem for second-order fluid approximations of traffic flow. *Transportation Research Part B: Methodological* 29: 277–296. `http://www.sciencedirect.com/science/article/B6V99-3YKKJ1D-F/2/f9d735df36de4048d1e62a0a20e844b0`.

Flannery, Aimee, and Tapan K. Datta. 1996. Modern roundabouts and traffic crash experience in United States. *Transportation Research Record: Journal of the Transportation Research Board* 1553: 103–109. `http://cat.inist.fr/?aModele=afficheN&cpsidt=2633749`.

Fouladvand, M. Ebrahim, Zeinab Sadjadi, and M. Reza Shaebani. 2004. Characteristics of vehicular traffic flow at a roundabout. *Physical Review E* 70: 046132. `http://prola.aps.org/abstract/PRE/v70/i4/e046132`.

Helbing, Dirk. 1995. Improved fluid-dynamic model for vehicular traffic. *Physical Review E* 51: 3164–3169.

Kimber, R.M. 1980. The traffic capacity of roundabouts. TRRL Laboratory Report 942. Crowthorne, UK: Transport and Road Research Laboratory.

Klar, Axel, Reinhart D. Kühne, and Raimund Wegener. 1996. Mathematical models for vehicular traffic. *Surveys on Mathematics for Industry* 6: 215–239. `http://citeseer.ist.psu.edu/old/518818.html`.

Nagel, Kai, Dietrich E. Wolf, Peter Wagner, and Patrice Simon. 1998. Two-lane traffic rules for cellular automata: A systematic approach. *Physical Review E* 58: 1425–1437. `http://prola.aps.org/pdf/PRE/v58/i2/p1425_1`.

Polus, Abishai, and Sitvanit Shmueli. 1997. Analysis and evaluation of the capacity of roundabouts. *Transportation Research Record: Journal of the Transportation Research Board* 1572: 99–104. `http://trb.metapress.com/content/p1j1777227757852`.

Polus, Abishai, Sitvanit Shmueli Lazar,and Moshe Livneh. 2003. Critical gap as a function of waiting time in determining roundabout capacity.

Journal of Transportation Engineering 129 (5) (September/October 2003): 504–509. `http://cedb.asce.org/cgi/WWWdisplay.cgi?0304088`.

Schreckenberg, M., A. Schadschneider, K. Nagel, and N. Ito. 1995. Discrete stochastic models for traffic flow. *Physical Review E* 51: 2939–2949.

Zein, Sany R., Erica Geddes, Suzanne Hemsing, and Mavis Johnson. 1997. Safety benefits of traffic calming. *Transportation Research Record: Journal of the Transportation Research Board* 1578: 3–10. `http://trb.metapress.com/content/875315017kux6689/`.

Zhou Fan, Christopher Chang, and Yi Sun.

One Ring to Rule Them All: The Optimization of Traffic Circles

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Summary

Our goal is a model that can account for the dynamics of vehicles in a traffic circle. We mainly focus on the rate of entry into the circle to determine the best way to regulate traffic. We assume that vehicles circulate in a single lane and that only incoming traffic can be regulated (that is, incoming traffic never has the right-of-way).

For our model, the adjustable parameters are the rate of entry into the queue, the rate of entry into the circle (service rate), the maximum capacity of the circle, and the rate of departure from the circle (departure rate). We use a compartment model with the queue and the traffic circle as compartments. Vehicles first enter the queue from the outside world, then enter the traffic circle from the queue, and lastly exit the traffic circle to the outside world. We model both the service rate and the departure rate as dependent on the number of vehicles inside the traffic circle.

In addition, we run computer simulations to have a visual representation of what happens in a traffic circle during different situations. These allow us to examine different cases, such as unequal traffic flow coming from the different queues or some intersections having a higher probability of being a vehicle destination than others. The simulation also implements several life-like effects, such as how vehicles accelerate on an empty road but decelerate when another vehicle is in front of them.

In many cases, we find that a high service rate is the optimal way to maintain traffic flow, signifying that a yield sign for incoming traffic is most effective. However, when the circle becomes more heavily trafficked,

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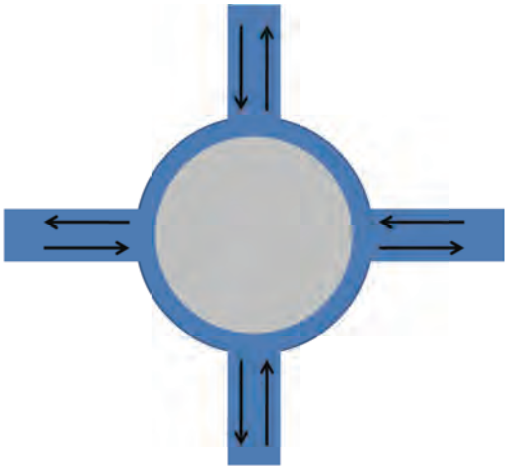

Figure 1. A simple traffic circle. Traffic circles may have more than one lane and may have a different number of intersections.

a lower service rate better accommodates traffic, indicating that a traffic light should be used. Thus, a light should be installed in most circle implementations, with variable timing depending on the expected amount of traffic.

The main advantage of our approach is that the model is simple and allows us to see clearly the dynamics of the system. Also, the computer simulations provide more in-depth information about traffic flow under conditions that the model could not easily show, as well as enabling visual observation of the traffic. Some disadvantages to our approach are that we do not analyze the effects of multiple lanes nor stop lights to control the flow of traffic within the circle. In addition, we have no way of analyzing singular situations, such as vehicles that drive faster or slower than the rest of the traffic circle, or pedestrians.

Introduction

Traffic circles, often called rotaries, are used to control vehicle flow through an intersection. Depending on the goal, a traffic circle may take different forms; **Figure 1** shows a simple model. A circle can have one or more lanes; vehicles that enter a traffic circle can be met by a stop sign, a traffic light, or a yield sign; a circle can have a large or small radius; a circle can confront roads containing different amounts of traffic. These features affect the cost of the circle to build, the congestion that a vehicle confronts as it circles, the travel time of a vehicle in the circle, and the size of the queue of vehicles waiting to enter. Each of these variables could be a metric for evaluating the efficacy a traffic circle.

Our goal is to determine how best to control traffic entering, exiting, and traversing a traffic circle. We take as given the traffic circle capacity, the arrival and departure rates at each of the roads, and the initial number of vehicles circulating in the rotary. *Our metric is the queue length, or buildup, at each of the entering roads.* We try to minimize the queue length by allowing the rate of entry from the queue into the circle to vary. For a vehicle to traverse the rotary efficiently, its time spent in the queue should be minimized.

We make the following assumptions:

- We assume a certain time of day, so that the parameters are constant.
- There is a single lane of circulating traffic (all moving in the same direction).
- Nothing impedes the exit of traffic from the rotary.
- There are no singularities, such as pedestrians trying to cross.
- The circulating speed is constant (i.e., a vehicle does not accelerate or decelerate to enter or exit the rotary).
- Any traffic light in place regulates only traffic coming into the circle.

The Models

A Simplified Model

We model the system as being continuous; our approach can be thought of as modeling the vehicle mass dynamics of a traffic circle. The simplest model assumes that the rate of arrival to the back of the entering queue and the rate of departure from the queue into the traffic circle are independent of time. Thus, the rate of change in the length of the queue is

$$\frac{dQ_i}{dt} = a_i - s_i, \tag{1}$$

where Q_i is the number of cars in the queue coming in from the ith road, a_i is the rate of arrival of vehicles into the ith queue, and s_i is the rate of removal, also called the service rate, from the ith queue into the traffic circle.

We introduce the parameter d_i, the rate at which vehicles exit the traffic circle. We let C be the number of vehicles traveling in the circle. Then we model the change in traffic in the rotary by the difference between the influx and outflux of vehicles, where the outflux of vehicles depends on the amount of traffic in the rotary:

$$\frac{dC}{dt} = \sum s_i - C \sum d_i. \tag{2}$$

An Intermediate Model

The model above simplifies the dynamics of a traffic circle. The most-glaring simplifications are that there is no way to indicate that the circle has a maximum capacity and that the flow rate into the traffic circle s_i does not depend on the amount of traffic already circulating. These are both corrected by proposing that the traffic circle has a maximum capacity C_{max}. As the number of vehicles circling approaches this maximum capacity, it should become more difficult for another vehicle to merge into the circle. At the extreme, when the traffic circle is operating at capacity, no more vehicles should be able to be added. Now, the s_i in the previous model can be represented logistically as

$$s_i = r_i \left(1 - \frac{C}{C_{\text{max}}}\right),$$

where r_i is how fast vehicles would join the circle if there were no traffic slowing them down. Thus, the equation governing the rate at which the ith queue length changes becomes

$$\frac{dQ_i}{dt} = a_i - r_i \left(1 - \frac{C}{C_{\text{max}}}\right), \tag{3}$$

and the equation for the number of vehicles in the traffic circle becomes

$$\frac{dC}{dt} = \sum r_i \left(1 - \frac{C}{C_{\text{max}}}\right) - \sum d_i C. \tag{4}$$

A Congestion Model

The previous two models still fail to take into account congestion, which alters the circulation speed, which in turn affects the departure rate d_i of vehicles from the circle. Equation **(3)** still holds, but we need to vary d_i. The vehicles will travel faster if there is no congestion, so they will be able to depart at their fastest rate $d_{i,\text{max}}$. When the circle is operating at maximum capacity, the departure rate will decrease to be $d_{i,\text{min}}$. Thus, the number of vehicles present in the circle is affected positively in the same manner as in **(4)**, but the lessening factor changes to the weighted average of the $d_{i,\text{max}}$ and $d_{i,\text{min}}$:

$$\begin{aligned}\frac{dC}{dt} = {}& \sum r_i \left(1 - \frac{C}{C_{\text{max}}}\right) \\ & - C\left[\sum d_{i,\text{max}} \left(1 - \frac{C}{C_{\text{max}}}\right) + \sum d_{i,\text{min}} \left(\frac{C}{C_{\text{max}}}\right)\right].\end{aligned} \tag{5}$$

Extending the Model Using Computer Simulation

We create a computer simulation in Matlab to account for variables that would be too complicated to use in the mathematical model. The mathematical model does not address the vehicles' speeds while inside the traffic circle, so the computer simulation focuses mostly on areas related to vehicle speed:

- enabling drivers to accelerate to fill gaps in the traffic (with a maximum speed),
- forcing drivers to decelerate to maintain distance between vehicles,
- requiring that drivers accelerate and decelerate when entering and exiting the circle,
- giving probabilistic weights to the different directions of travel,
- keeping track of time spent within the traffic circle for each vehicle, and
- giving each intersection a different vehicle introduction rate.

Figure 2 on p. 250 shows an outline of the program flow and design.

Simulation Assumptions

This model makes several key assumptions about the vehicles and the circle:

- All vehicles are the same size, have the same top speed, and accelerate and decelerate at the same rate.
- The circle has four intersections and a single lane of traffic.
- All drivers have the same spatial tolerance.
- There are no pedestrians trying to cross the circle.

Limitations

The assumption of one lane is not a key factor, because we assume that vehicles travel at the same speed. Hence, we do not need to put the slow vehicles in one lane and vehicles passing them in another lane. However, in reality there will indeed be slower vehicles, and vehicles decelerating to exit would offer opportunities for other vehicles to use a different lane to maintain a faster speed. Additionally, we cannot let emergency vehicles through the circle if there is only one lane; for a more detailed discussion of emergency vehicles and traffic circles, see Mundell [n.d.].

By not allowing control devices inside the circle, we restrict possible configurations. We also limit the effectiveness of our stoplight model; it prevents vehicles from entering the circle but does not inhibit the movement of vehicles within in the circle.

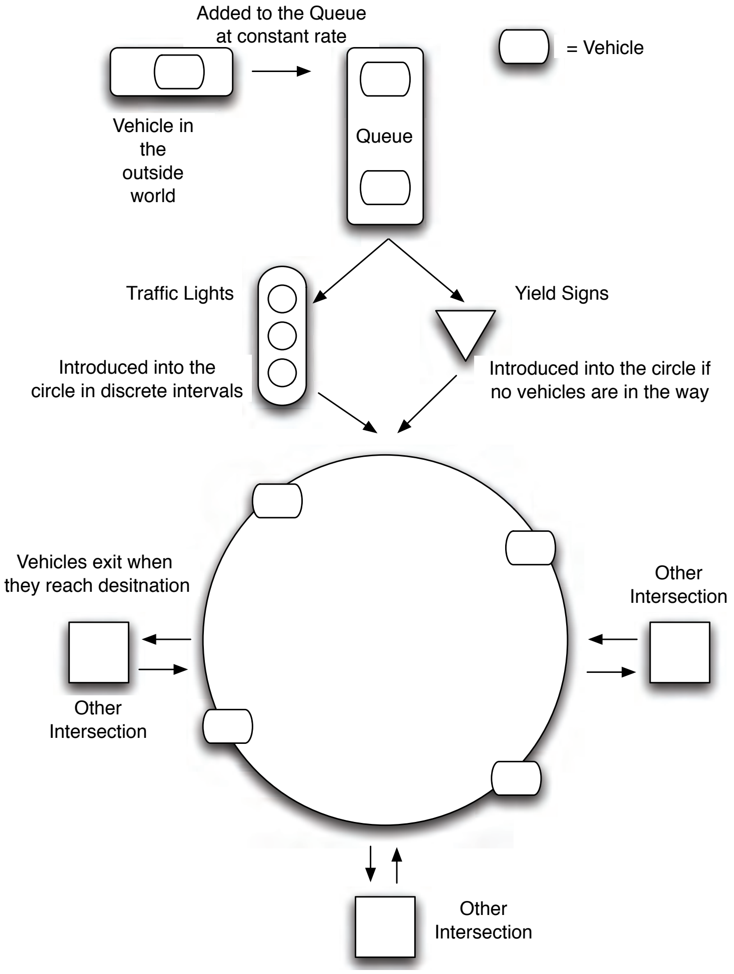

Figure 2. Program flow. Each intersection is modeled as a queue of vehicles with a traffic control device. Vehicles are added to the queue at a constant rate. For a vehicle to leave the queue and enter the traffic circle, the area in the circle must be clear of other vehicles. Additionally, if the queue has a traffic light, the light must be active.

Since we do not allow for different vehicle properties (size, acceleration, top speed, etc.), we cannot model the effects of large trucks, motorcycles, or other nonstandard vehicles (such as large and unwieldy emergency vehicles) on the flow of traffic.

Giving all of the vehicles the same acceleration and top speed, along with forcing all drivers to have the same spatial tolerance, prevents modeling aggressive drivers and their interaction with timid ones. Additionally, since cars in the simulation decelerate before exiting, even if they are already moving slowly, we generate a small proportion of false traffic backups.

Limiting the size and number of intersections of the circle does not really limit our ability to model real-world traffic circles. Since we are mostly looking at driver behavior with the computer simulation, we should see the same behaviors as we scale up the circle and its corresponding traffic.

Analyzing the Models

The Simplest Model

In all of the above models, the rate r_i is indicative of the regulation imposed at the ith intersection. A near-zero r_i indicates that a traffic light is in use; a larger r_i indicates that a yield sign, regulating only the incoming traffic, is in place.

For the simplest model, we can use **(1)** and **(2)** to find explicit formulae for the queue length and the number of vehicles in the rotary by integrating with respect to time:

$$Q_i = [a_i - s_i]\,t + Q_{i0}, \qquad C = \frac{\sum s_i}{\sum d_i} + \left(C_0 - \frac{\sum s_i}{\sum d_i}\right) e^{-\sum d_i t}.$$

Therefore, given the inputs of the system, we can predict the queue length. To minimize the queue length, we solve **(1)** for when the queue length is decreasing ($dQ_i/dt < 0$) and find that the s_i term should be maximized.

Intermediate Model

For the model with a carrying capacity, again we find explicit formulae for the queue length and the number of vehicles in the rotary:

$$Q_i = \left[a_i - r_i\left(1 - \frac{C}{C_{\max}}\right)\right] t + Q_{i0},$$

$$C = \frac{\sum r_i}{\frac{\sum r_i}{C_{\max}} + \sum d_i} + \left(C_0 - \frac{\sum r_i}{\frac{\sum r_i}{C_{\max}} + \sum d_i}\right) e^{-\left(\frac{\sum r_i}{C_{\max}} + \sum d_i\right)t}.$$

We can also solve for where **(3)** is less than zero to find the service rates for which the queue lengths are decreasing:

$$r_i > \frac{a_i}{1 - \frac{C}{C_{\max}}}.$$

Congestion Model

In modeling congestion, the model is too complex to intuit what conditions would minimize the queue length. The differential equation **(5)** is quadratic:

$$\frac{dC}{dt} = AC^2 + BC + D,$$

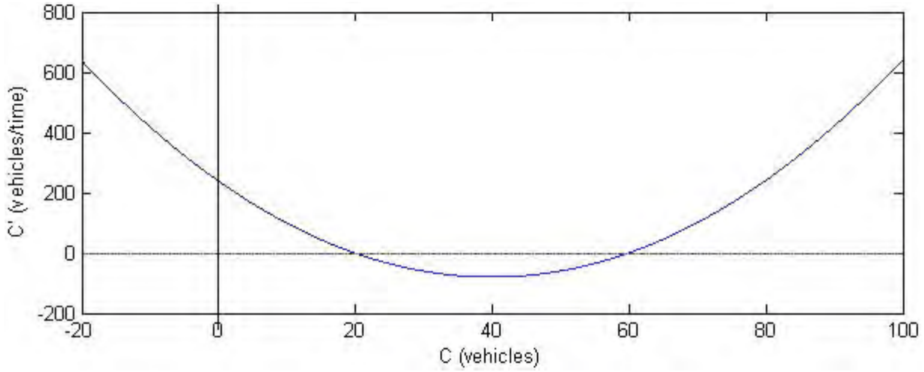

Figure 3. The relationship between dC/dt and C for the congestion model using sample parameters values $r_1 = r_2 = r_3 = r_4 = 60$, $d_{1,\max} = d_{2,\max} = d_{3,\max} = d_{4,\max} = 2$, $d_{1,\min} = d_{2,\min} = d_{3,\min} = d_{4,\min} = 0.5$, and $C_{\max} = 30$.

where

$$A = \frac{\sum d_{i,\max}}{C_{\max}} - \frac{\sum d_{i,\min}}{C_{\max}}, \qquad B = -\left(\frac{\sum r_i}{C_{\max}} + \sum d_{i,\max}\right), \qquad D = \sum r_i.$$

Since $\sum d_{i,\max} > \sum d_{i,\min}$, it will always be the case that $A > 0$. In addition, $B < 0$ and $D > 0$. This means that the curve for dC/dt is a concave-up quadratic curve with a positive y-intercept and a global minimum at some $C > 0$. Furthermore, for $C = C_{\max}$, we have

$$\frac{dC}{dt} = -\frac{d_{i,\min}}{C_{\max}},$$

which is always negative for $d_{i,\min} > 0$. Thus, the global minimum for the curve must be in the fourth quadrant. **Figure 3** shows an example of such a curve, using sample parameters.

We notice from **Figure 3** that there are two equilibrium points for the differential equation:

$C = \frac{-B-\sqrt{B^2-4AD}}{2A}$ is a stable equilibrium point, and

$C = \frac{-B+\sqrt{B^2-4AD}}{2A}$ is an unstable equilibrium point.

Also, since for $C = C_{\max}$, we have $dC/dt < 0$, the number of vehicles will eventually decrease to an equilibrium value less than $C_{\text{limit}} < C_{\max}$.

Since our metric for how well a traffic circle operates depends on how many vehicles are in the queues, we would like the queue flow $(a_i - s_i)$ to be as small as possible. In other words, we would like s_i to be as large as possible. In the congestion model, the queue flow is given by **(3)**.

Without loss of generality, we analyze queue 1. The equations for each queue differ only by their a_i and r_i, and we keep these the same for each queues in the simulations. Since the only changing variable in **(3)** is C, when $C = C_{\text{limit}}$ the queue length Q_1 will also be at its equilibrium.

Using this fact, we can evaluate whether to use a traffic light or not and how long the light should be red. We compare different values for the service rate constant r_1 and the value of dQ_1/dt at $C = C_{\text{limit}}$. The results can be seen in **Figure 4**, which shows that when r_1 increases, dQ_1/dt decreases.

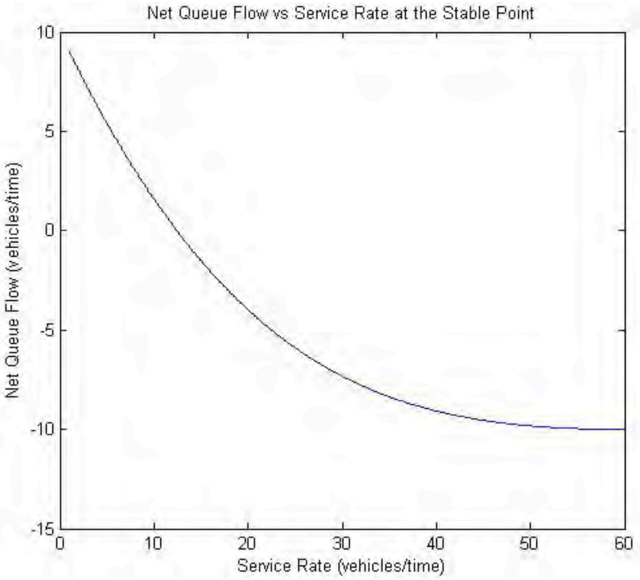

Figure 4. The relationship between r_1 and dC/dt for the congestion model with $C = C_{\text{limit}}$. The parameter values are $d_{1,\max} = d_{2,\max} = d_{3,\max} = d_{4,\max} = 2$, $d_{1,\min} = d_{2,\min} = d_{3,\min} = d_{4,\min} = 0.5$, $C_{\max} = 30$, and r_1 changed from 1 to 60.

A real-life situation is congestion of the traffic circle. Decreasing $d_{1,\min}$ would cause vehicles to exit the circle more slowly when there is more congestion. Using lower departure rates to approximate slower vehicle speeds inside the circle, we can examine what happens for decreasing values of $d_{1,\min}$. The results are shown in **Figure 5**. For values of $d_{1,\min} < 0.5$, the smallest value for dQ_1/dt is not at $r_1 = 60$ but at a smaller value.

Another situation that the congestion model can approximate is additional lanes. A crude approximation is that each lane adds $C_{\max}$ to the capacity. **Figure 6** shows the results of plotting r_1 versus the $C_{\max}$ for different numbers of lanes. As in the previous plots, the correlation is negative.

Simulation Results

An interesting effect that we see in our simulation is the buildup of vehicles in front of each exit. As vehicles decelerate to exit, they force vehicles behind them to decelerate to maintain a safe distance. This buildup creates a longer queue at the intersection before the exit, since the buildup prevents those vehicles from entering the circle. In **Figure 7**, we see a large number of vehicles in the fourth queue and a buildup in the fourth quadrant.

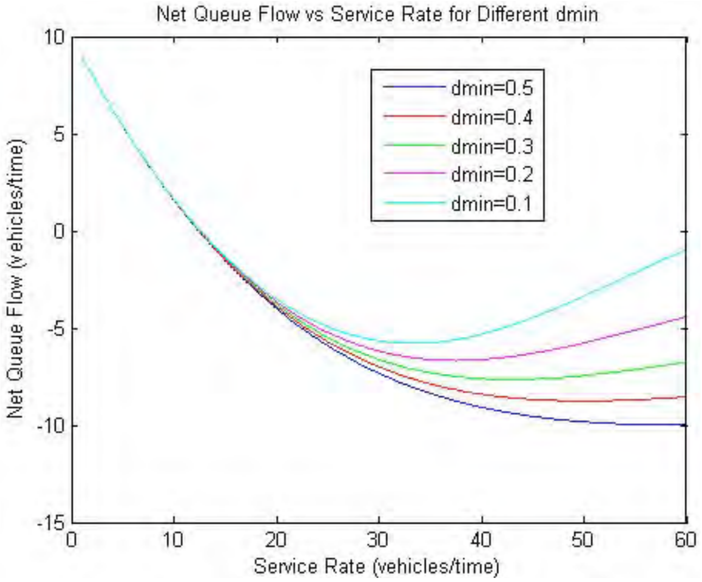

Figure 5. The relationship between r_1 and dC/dt for the congestion model with $C = C_{\text{limit}}$, with parameter values $d_{1,\max} = d_{2,\max} = d_{3,\max} = d_{4,\max} = 2$, and $C_{\max} = 30$. The values of r_1 range from 1 to 60 for different values of $d_{1,\min}$.

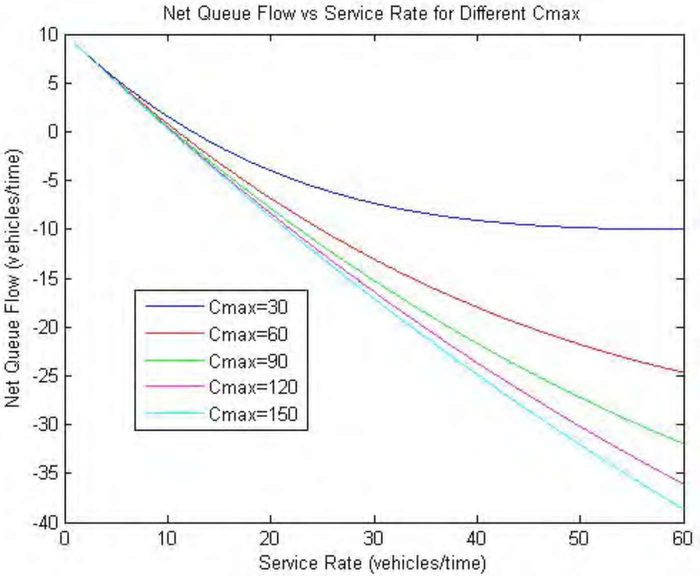

Figure 6. The relationship between r_1 and dC/dt for the congestion model with $C = C_{\text{limit}}$. The parameter values are $d_{1,\max} = d_{2,\max} = d_{3,\max} = d_{4,\max} = 2$, $d_{1,\min} = d_{2,\min} = d_{3,\min} = d_{4,\min} = 0.5$, $C_{\max} = 30$, and r_1 changed from 1 to 60.

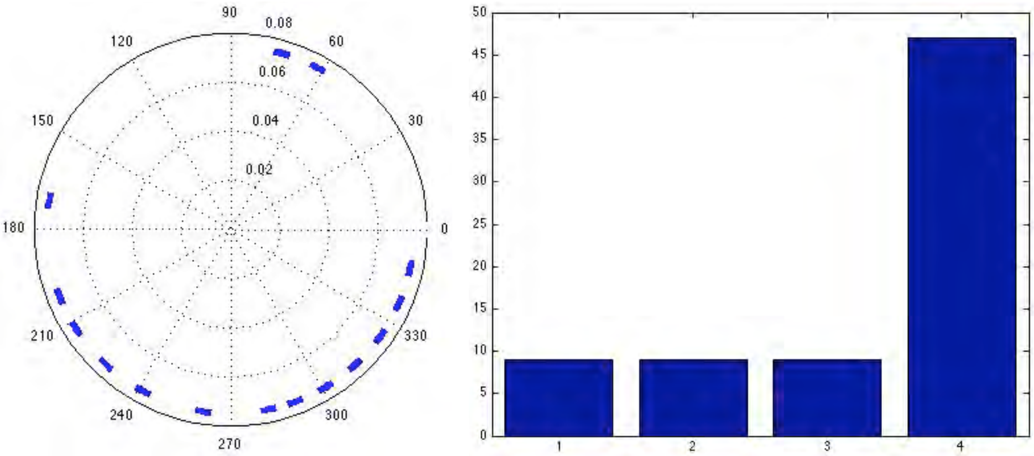

Figure 7. Vehicles build up before the first intersection as vehicles slow down to exit. Additionally, the queue at the fourth intersection is quite long, because vehicles cannot enter the traffic circle.

Another interesting element of real life that the simulation shows is the bunching and expanding effect that vehicles experience. Because vehicles can decelerate more quickly than they accelerate, the vehicles bunch up behind a slow moving vehicle, then expand again as that vehicle accelerates into the free space ahead. **Figure 8** shows an example of this compaction.

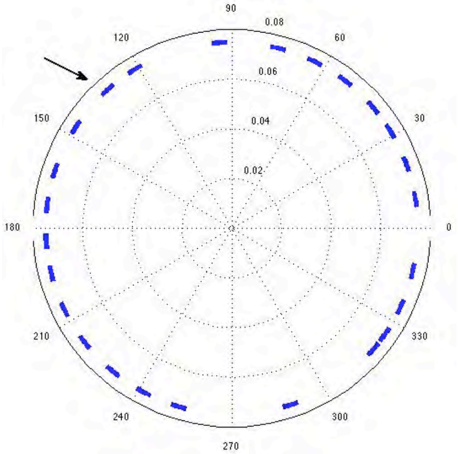

Figure 8. The arrow in the second quadrant points out a real-life effect, bunching, which happens because drivers decelerate faster than they accelerate.

We test several rotary and vehicle setups to explore optimal circle design:

- A single intersection with high arrival and service rates creates a large traffic buildup in the quadrant immediately following it, even though the vehicles have random destinations. **Figure 9** shows the buildup in quadrant 1 when the first intersection (at angle 0) has a high arrival and service rate. However, queue 1 is not appreciably longer than the others.

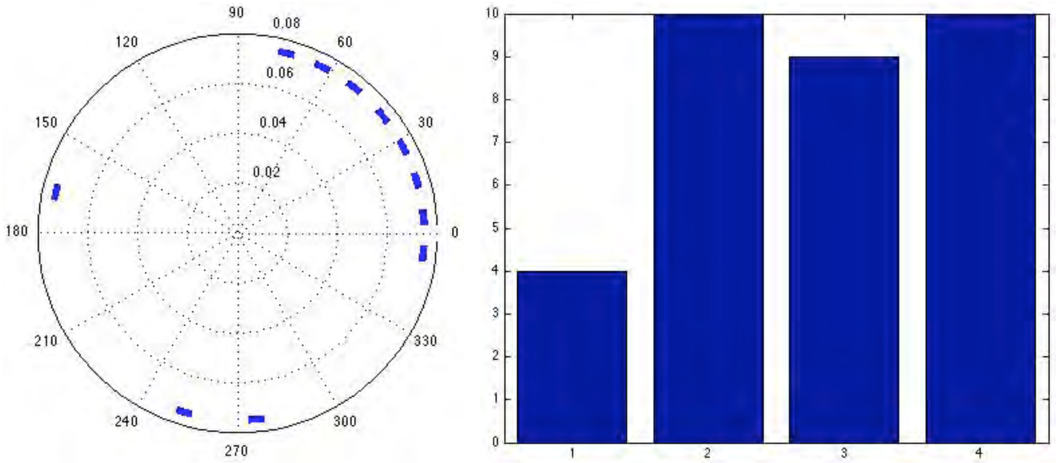

Figure 9. The first intersection has both high arrival and service rates, which creates a traffic buildup before the next intersection. However, the queue for the first intersection does not increase, since there is limited traffic coming from the intersection behind it.

- One intersection having a much higher chance of being a destination creates the expected buildup in front of the likely exit (**Figure 10**). However, it also creates a substantial buildup in front of the previous exit and a severe increase in that intersection's queue as vehicles are prevented from entering the circle. The buildup in the adjacent road must be taken into account when constructing a traffic circle at a high-volume intersection.
- If one intersection has a high service rate and the standard arrival rate, and another intersection has a high arrival rate and standard service rate, the traffic distribution is mostly random, with a slight tendency towards backups in the quadrant following the intersection with high service rate. We expect this result, since the intersection with high service rate can add only as many vehicles as in its queue, which is limited by its low arrival rate. Also, the intersection with high arrival rate and low service rate has a much longer queue than the other intersections, entirely as expected.

Conclusion

We model the dynamics of a traffic circle to determine how best to regulate traffic into the circle. As shown in **Figure 6** on p. 256, increased capacity decreases the queue flow, which leads to a decrease in queue length. This result indicates that a multiple-lane traffic circle might better accommodate more cars by decreasing the length of the queue in which they wait. However, as shown in the same figure, the marginal utility of increasing the maximum capacity does decrease. When applying a cost function (with cost proportional to the space that the circle occupies), there would exist an

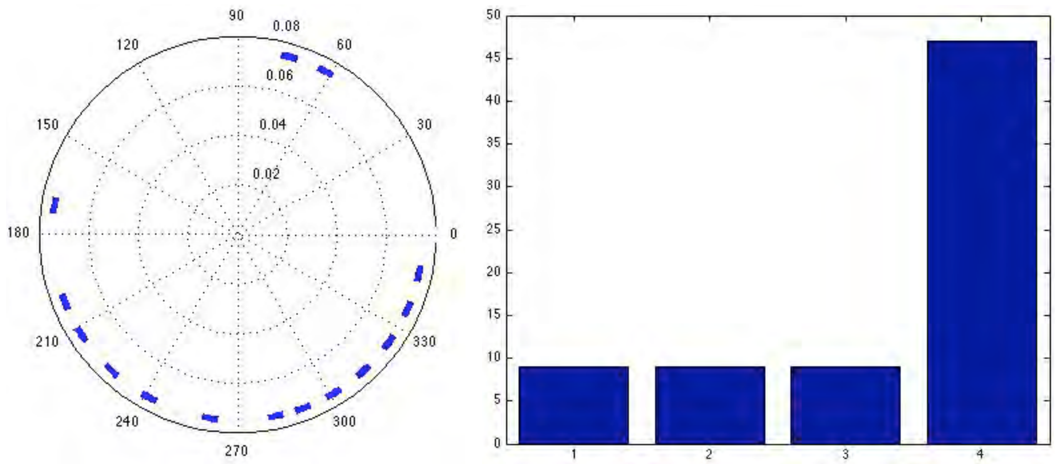

Figure 10. The first intersection has a higher probability of being chosen as a destination. This creates a buildup in front of that intersection and a smaller buildup in front of the previous intersection. It also creates a very large increase in the queue of the previous intersection since those vehicles cannot enter the full circle.

optimum size of the traffic circle.

Although the simpler models indicate that letting vehicles into the rotary as fast as possible would be optimum, analysis of the congestion model shows that if $d_{i,\min}$ is sufficiently small, then the highest service rate is no longer optimal. The implication of this result is that traffic lights could make travel through the rotary more efficient. When many vehicles use the traffic circle, such as during the morning and evening commutes, there could be enough vehicles so that the C_{limit} is reached. In this case, using traffic lights would help ease congestion. However, the duration of the red light should be adjusted according to the $d_{i,\min}$ for the specific traffic circle.

In addition to the mathematical models, we create a computer simulation that tracks individual vehicles' progress through the traffic circle, and their effect on other vehicles. Our simulation shows several traffic effects that can be observed in real life, namely a buildup of vehicles in front of the exits and vehicles bunching together and expanding apart as drivers brake and accelerate. We also test several traffic circle configurations.

Recommendations

Based on both our mathematical and computer models, we recommend:

- **Yield signs should be the standard traffic control device.** Most of the time, letting vehicles enter the circle as quickly as possible is optimal.
- **For a high-traffic rotary, traffic lights should be used.** With high traffic, slowing the rate of entry into the circle helps prevent congestion.

- **If any single road has high traffic, its vehicles should be given preference in entering the circle.** Doing so helps prevent a large queue.
- **Introduce separate exit lanes.** Traffic can build up in front of each intersection as cars exit, so a separate exit lane could help keep traffic moving.

References

Mundell, Jim. n.d. Constructing and maintaining traffic calming devices. Seattle Department of Transportation. `http://www.seattle.gov/Transportation/docs/TORONTO4.pdf`.

Aaron Abromowitz, Andrea Levy, and Russell Melick.

Three Steps to Make the Traffic Circle Go Round

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Summary

With growing traffic, control devices at traffic circles are needed: signals, stop/yield signs, and *orientation signs*—a special sign that we designed.

We create two models—one macroscopic, one microscopic—to simulate transport at traffic circles. The first models the problem as Markov chain, and the second simulates traffic by individual vehicles—a "cellular-automata-like" model.

We introduce a multi-objective function to evaluate the control. We combine saturated capacity, average delay, equity degree, accident rate and device cost. We analyze how best to control the traffic circle, in terms of:

- placement of basic devices, such as lights and signs;
- installation of orientation signs, to lead vehicles into the proper lanes; and
- self-adaptivity, to allow the traffic to auto-adjust according to different traffic demands.

We examine the 6-arm-3-lane Sheriffhall Roundabout in Scotland and give detailed suggestions for control of its traffic: We assign lights with a 68-s period, and we offer a sample orientation sign.

We also test smaller and larger dummy circles to verify strength and sensitivity of our model, as well as emergency cases to judge its flexibility.

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Introduction

We develop two models to simulate traffic flow in a traffic circle. The macroscopic model uses a Markov process to move vehicles between junctions, while the microscopic model concentrates on the behavior of each vehicle, using a modified cellular-automata algorithm. The outcomes of these two approaches show great consistency when applied to a real scenario in Scotland.

We characterize a "good" traffic control method in terms of five main objectives and combine them with an overall measure.

We employ a genetic algorithm to generate the final control method, in particular to determine the green-light period. We also consider the ability to deal with unexpected affairs such as accidents or breakdowns.

General Assumptions

- The geometric design of the traffic circle cannot be changed.
- The traffic circle is a standard one (at grade) with all lanes on the ground, that is, no grade separation structure.
- The flow of incoming vehicles is known.
- People drive on the left (since the example later is from the UK).
- Pedestrians are ignored.
- Motorcycles move freely even in a traffic jam.

Terminology and Basic Analysis

Terminology

- **Junction:** an intersection where vehicles flow in and out of the traffic circle.
- **Lane:** part of the road for the movement of a single line of vehicles. The number of lanes directly affects flow through the circle by limiting entrance and exit of vehicles. However, since both the conventional design and real-time photos suggest that vehicles exit easily, our model ignores restrictions on outward flow.
- l_0: the number of lanes in the traffic circle.
- **Section:** part of the traffic circle between two adjacent arms.
- **Yield/stop signs:** A yield sign asks drivers to slow down and give right of way; a stop sign asks drivers to come to a full stop before merging.

- **Orientation sign:** a sign indicating the lane for vehicles to take according to their destination.
- **Traffic light:** a signaling device using different colors of light to indicate when to stop or move. A traffic light with direction arrows performs much better [Hubacher and Allenbach 2002], so we are inclined to use such a light. However, compared to yield/stop signs, traffic lights slow vehicle movement. At the same time, however, even at a remote motorway traffic circle with few pedestrians, a a traffic-light malfunction will probably lead to an accident [Picken 2008].
- **Cycle period:** the time in which a traffic light experiences exact stages of all three colors. An optimal cycle period is critical whenever traffic lights are employed. The method we use is called the *Webster equation* [Garber and Hoel 2002]. The value that we use in our model is calculated as 68 s.
- **Green-light period:** the time that a traffic light keeps green in one cycle.
- **Timestamps:** a sequence of characters denoting the start/end time of red/green lights.

A Glance at Sheriffhall Roundabout

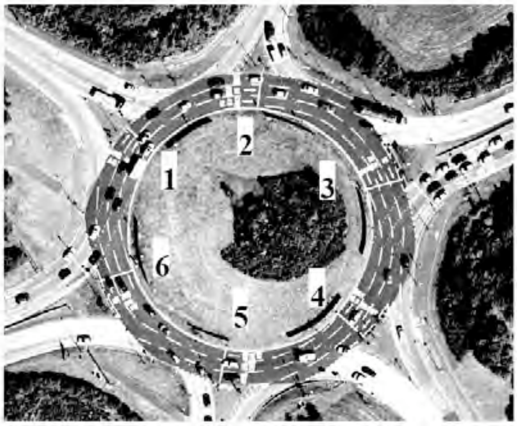
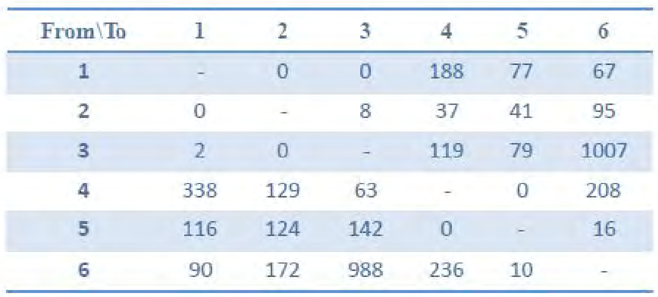


Figure 1. The Sheriffhall Roundabout. Source: Google Earth.

One characteristic of this traffic circle (**Figure 1**) is that the arms in the southwest (6) and northeast (3) directions have larger flow than the others. The arms in the north (2) and south (5) directions have two lanes, while the other four arms and the circle have three lanes. We model the traffic circle as a ring with an inner radius of 38.9 m and an outer radius of 50.4 m.

We use the origin-destination flow (**Table 1**) given by Yang et al. [Maher 2008]. Since the traffic demand is far from saturated, we experiment on different scalings of this inflow matrix, specifically, multiples by 1.2, 1.4, 1.6, and 1.8.

Table 1.
Origin-destination flows (vehicles/hr) at Sheriffhall Roundabout.
Source: Wang et al. in Maher [2008].



From\To	1	2	3	4	5	6
1	-	0	0	188	77	67
2	0	-	8	37	41	95
3	2	0	-	119	79	1007
4	338	129	63	-	0	208
5	116	124	142	0	-	16
6	90	172	988	236	10	-

Simulation Models

Model I: The Macroscopic Simulation

Usually, we do not know where each vehicle enters and exits the circle; we know only the numbers of vehicles coming in and out of each arm, so we adopt a macroscopic simulation.

We first combine the lanes in the sections and arms together and regard them as one-lane roads. We then explain how the multilane simulation works.

Assumptions

- Vehicles in the same section of the circle are distributed uniformly in the section.
- The arrival rate at each arm is constant in the period that we simulate.
- For simplicity, we consider an ideal round traffic circle (**Figure 2**). The macroscopic simulation itself does not depend on the shape of the circle.

Sections and Arms

We divide the traffic area into sections and take vehicles in the same section as a whole. We label the sections and the arms as in **Figure 2**. Associated to section i are the quantities:

1. Number num_i^t of vehicles in the section at time t.
2. Number arm_i^t of vehicles waiting to enter through one arm at time t.
3. The maximum number cap_i^t of vehicles that can enter the traffic section through one arm per unit time.

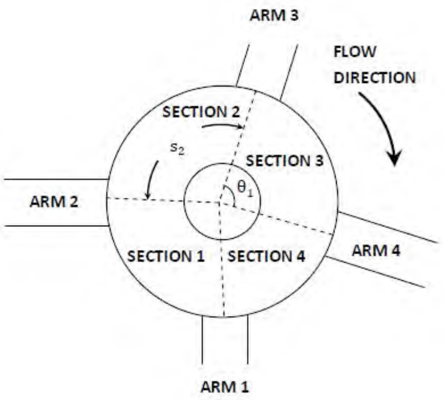


Figure 2. Sample traffic circle.

A Markov Process

The traffic state at time $t+1$ depends only on the traffic state at time t, so traffic is a Markov process. To describe the state of the whole system, only the quantities num_i^t and arm_i^t are needed. To implement the simulation, we must determine num_i^{t+1} and arm_i^{t+1}, for $i = 1, 2, \ldots, n$.

In principle, we can calculate the transition probability matrix; but not in our problem. For a traffic circle with four arms/sections and each holding up to 10 vehicles, the number of traffic states is 10^8.

Considering this sobering fact, we use the expectations $\overline{\text{num}}_i^t$ and $\overline{\text{arm}}_i^t$, instead of the actual distribution of cars, to denote a state.

The Simulation Process

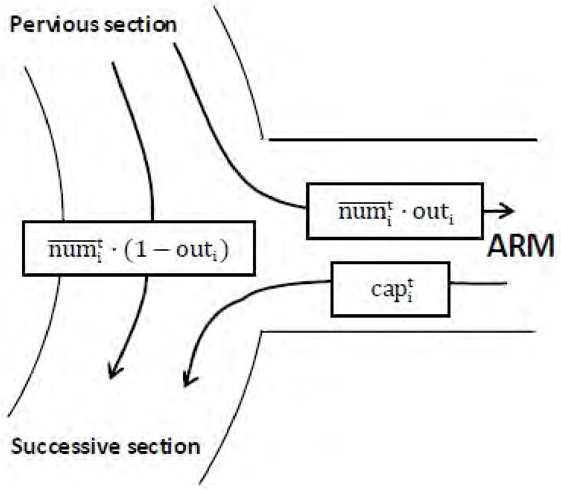


Figure 3. Flows at a junction.

- $\overline{\text{num}}_i^t \times \text{out}_i^t$ vehicles leave the circle from section i. The ratio out_i^t drops when $\overline{\text{num}}_i^t$ approaches its capacity.
- To deal with the junction, there are two streams $\overline{\text{num}}_i^t \cdot (1 - \text{out}_i)$ and cap_i^t trying to flow into the next section. If there is a traffic light, only one of them is allowed. If stop/yield sign is used (at the arm side, for example), then only a small fraction of cap_i^t can flow in. This fraction is denoted by the *disobey rate* α_{stop} or α_{yield}.
- An inflow of in_i newly-arrived vehicles runs into arm i.

Multilane Traffic Circle

We assume that vehicles do not change lanes within arms or sections, which means that they can change lanes only at junctions.

To treat lanes differently, we need to know what proportion of vehicles pass through each lane. At each junction, the outflow for a given lane is distributed into successive lanes according to their popularity.

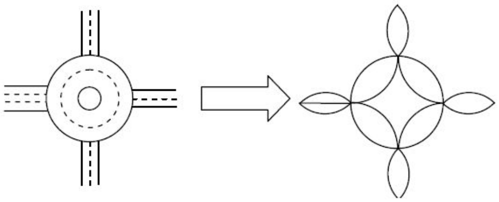


Figure 4. A two-lane circle divided into lanes. Each arc on the right denotes a single lane.

Model II: The Microscopic Simulation

Partially inspired by sequential cellular automata, we adopt a microscopic model. The traffic circle is divided into l_0 lanes. Vehicles are points with polar coordinates but with discrete radius values. We model the behavior of each individual vehicle, with the help of some general principles:

- **Traffic coming in:** As described in **Table 1**, the number of vehicles per hour is given in a matrix $(a_{i,j})_{n\times n}$. We use a Poisson distribution with mean $a_{i,j}/T$ to describe the incoming vehicles from arm i to arm j.
- **Lane choosing and changing:** For a specific vehicle from arm i to arm j, the driver has a desired ideal lane to be in. The hidden principle is [SetupWeasel 1999]: The more sections the vehicle has to pass before its exit, the more likely the driver will wish to take an inner lane, both in the arm and in the circle. We adopt this rule.

- **Vehicle speed:** We define a maximum speed and a maximum acceleration for vehicles, and record the speed individually. The principles for a vehicle to accelerate or decelerate are:
  - When a vehicle faces a red light or other vehicles, its speed decreases to zero.
  - When a vehicle changes lanes, it decelerates.
  - Otherwise, a vehicle attempts to accelerate up to maximum speed.
- **The function of a yield sign:** When a vehicle faces a yield sign, it checks whether the lane is empty enough for it to enter the junction. If not, the vehicle waits until it is empty enough—but with a disobey rate α_{yield}, it ignores the sign and scrambles. Naturally, this reaction affects the accident rate.
- **The function of a stop sign.** When a vehicle faces a stop sign, it should stop instantaneously. At the next time step, it functions as if at a yield sign; the only difference is that it will accelerate from a zero speed. The disobey rate is α_{stop}.
- **The effect of traffic lights:** Just like normal.

We discretize time and follow the rules above for each vehicle after it comes to the circle. We calculate the average traversal time for a vehicle, as well as the accident rate (by the total number of touches of vehicles). A vivid view of the simulation result is presented in **Figure 5**.

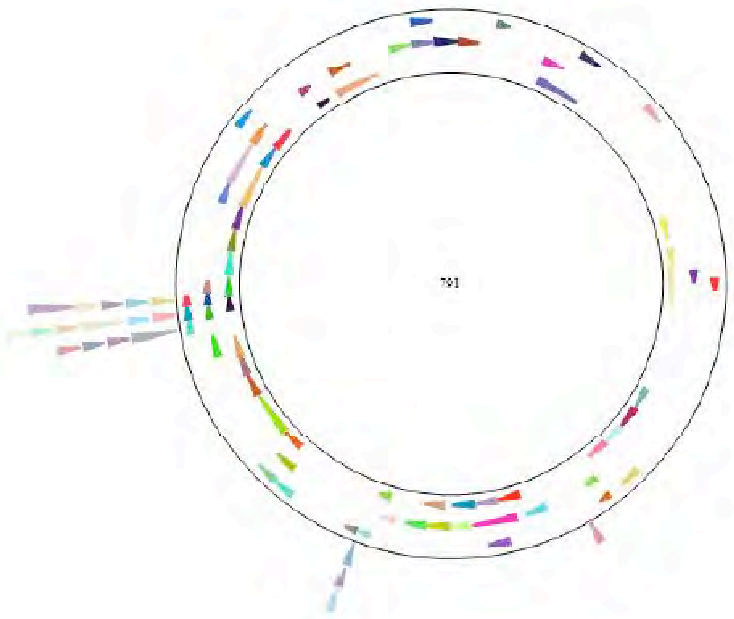


Figure 5. The vehicles around the traffic circle.

Comparison and Sensitivity Analysis

Results

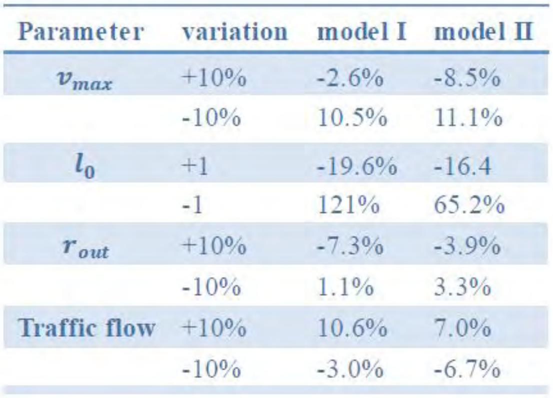
We use the two different models to simulate a real traffic circle: The Sheriffhall Roundabout in Scotland. We use the traffic-light configuration in Maher [2008]. For simplicity, we consider only the average time needed for a vehicle to traverse the traffic circle. This value is 42.7 s for Model I and 41.6 s for Model II. The two results are close, so we can believe that the actual traversal time is around 42 s.

Sensitivity

We analyze sensitivity by running the program with modified parameters (see **Table 2**).

Table 2.

Sensitivity test of simulation mode.



Parameter	variation	model I	model II
v_{max}	+10%	-2.6%	-8.5%
	-10%	10.5%	11.1%
l_0	+1	-19.6%	-16.4
	-1	121%	65.2%
r_{out}	+10%	-7.3%	-3.9%
	-10%	1.1%	3.3%
Traffic flow	+10%	10.6%	7.0%
	-10%	-3.0%	-6.7%

The two models give similar sensitivity results. The average passing time is relatively insensitive to all the parameters except l_0. This is reasonable, since the number of traffic lanes in the circle affects the passing time significantly.

Model II is a random simulation, which enables us to calculate the standard deviation of the traversal time, which is no larger than 3% of the mean.

Complexity

The time complexity of the algorithms for the two models is proportional to the maximum number of vehicles that the circle can hold and the number of iterations. In practice, 1,000 iterations suffice.

Model I is a little simpler than Model II, since we do not need to trace each individual. Conversely, Model II needs more a priori information than

Model I. Since the two models are consistent and give similar results, we adopt Model II for further study.

The Multi-Objective Function

Basic Standards

We want to include both subjective evaluations (such as the feelings of drivers) and objective measures (such as the expense of devices). Also, the standards should be calculated from available data. We choose five evaluation standards:

- **Saturated flow capacity:** The threshold flux to avoid backing up traffic on the arms.
- **Average delay:** The difference between the average time to traverse the traffic circle and the time to traverse an empty one.
- **Equity degree:** A multi-arm traffic circle may distribute the incoming flow inequitably, to the annoyance of drivers. The relative difference in average delay is the equity degree.
- **Accident expectation:** The average number of accidents per vehicle.
- **Device cost:** The total expense of traffic signs and lights.

How the Objectives Are Affected

Saturated Flow Capacity

A yield sign is likely to work effectively, since it seldom causes unnecessary stops for vehicles. A stop sign, however, at least adds the acceleration/deceleration delay to every vehicle rushing inside. The efficiency of a traffic light is highly related to its green-light period. Fixed-period lights sometimes block vehicles from entering an empty circle, while adaptive ones can work according to conditions.

In fact, a traffic circle with yield signs at all junctions bears the heaviest traffic in the simulations above, and traffic lights are left with great potential to improve in optimization.

Average Delay

The average delay is controlled by the incoming flow. The delay time will increase rapidly when traffic starts to congest. In our model, the delay time of a vehicle is calculated when it exits the traffic circle. When this delay time is considered in the overall objective, there should be penalties on congestion, which is calculated from the current flow and the saturated flow capacity.

Equity Degree

Equity degree is calculated directly from the delay time distribution. Not only the flow distribution but also the total flux contributes to the equity degree, since high flux may lead to unexpected distribution failures.

Accident Expectation

We assume that each kind of signal reduces accidents by a specific percentage; we use data from Hubacher and Allenbach [2002], Transport for London. . . [2005], and Fitzpatrick [2000].

Device Cost

This expense is based on the numbers of each kind of signal.

The Combined Objective: The Money Lost

Now we come to a combined objective, the *combined expense (CE)*, that takes into account expense and economic loss, which we attempt to minimize.

The prices of traffic-control devices are easy to find [Traffic Light Wizard n.d.; TuffRhino.com n.d.]. Apart from the expense of maintenance and operation, we calculate the average operating cost per hour for each kind of device. Since traffic lights consume much electricity, we ignore the money spent on other types of devices. A traffic light is expected to cost \$0.23/hr [Wang 2005; Ye 2001].

For accident expense losses, we take data from an annual report of a local traffic office on average loss per accident [Hangzhou Public Security Bureau. . . 2006] and set

$$\text{Accident loss} = \$630 \times \text{Flux}.$$

The average delay time must be accompanied by a cost of delay. According to the Federal Highway Administration [2008], about \$1.20 per vehicle is lost in a delay of 1 hr:

$$\text{Delay expense} = \$1.20 \times \text{Flux} \times \text{Average delay time}.$$

The unused part of saturated capacity takes care of any extra incoming traffic; we set its value as

$$\begin{aligned}\text{Capacity bonus} = 5\% \times \$1.2 \times (\text{Saturated capacity} - \text{flux}) \times \\ (\text{Average delay time}),\end{aligned}$$

in which 5% is the probability of an unexpected vehicle coming.

Equity degree (ED) is a tricky component in the determination.The most annoying situation is to keep two “main arms” open to traffic by sacrificing all other arms. Equity degree is estimated to be a function of the number of arms n:

$$\text{Reference equity degree (RED)} = \sqrt{\frac{n(n-2)}{2(n-1}}.$$

The equity degree will be normalized by this reference and appear in a penalty on delay expense:

$$\text{Corrected delay expense} = \text{Delay expense} \times \left(1 + \frac{\text{ED}}{\text{RED}}\right).$$

The combined index is then calculated as

$$\text{CE} = \text{Corrected delay expense} - \text{Capacity bonus} + \text{Accident loss} + \text{Device cost},$$

which serves as the final objective function that we use in the following optimization.

Application: Evaluate Typical Arrangements

We take a glance at three general control methods: pure traffic light, stop sign only, or yield sign only.

We first normalize the five objectives, converting values to an interval between 0 and 1, from worst to best. A superficial look at the radar chart of **Figure 6** raises doubt about the expensive traffic lights. However, traffic lights are superior in controlling the accident rate, while the two signs may be hazardous by accelerating the flow. The convoluted relationship is clear when we compare their CE values, in **Table 3**.

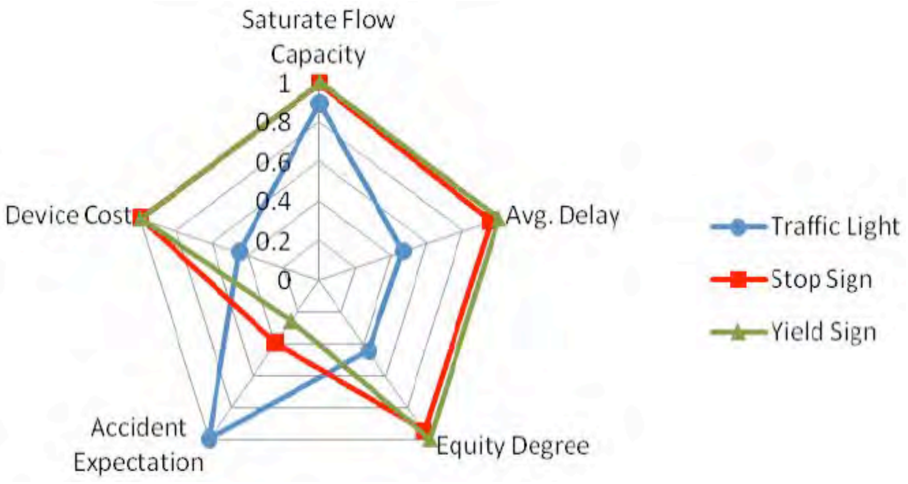
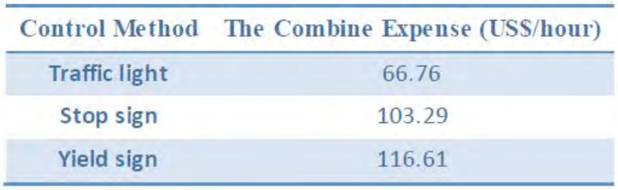


Figure 6. A view of 5 objectives of 3 general control methods.

The results above suggest that traffic lights are worthwhile for heavy traffic. Optimization, however, needs more insight.

Table 3.

Combined expense for 3 typical control methods.



Control Method	The Combine Expense (US$/hour)
Traffic light	66.76
Stop sign	103.29
Yield sign	116.61

Optimization Model

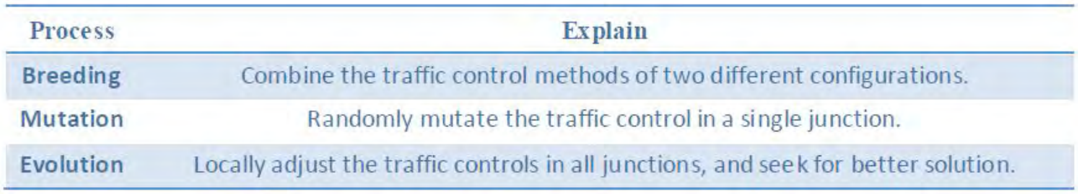
The All-Purpose Solution

Because the objective function is calculated in our simulation model, an analytical form for it is difficult to obtain. In such a situation, a quasi-optimal solution is welcome, and approximation algorithms become candidates.

In this problem, a normal approximation algorithm can fall into local maxima. However, some high-level technique can be used such as simulated annealing or—what we use—a genetic algorithm. Specifically, the traffic controls in different junctions are used as genes. The configuration of a traffic circle is a vector of genes, containing all the devices used in different junctions. **Table 4** gives details.

Table 4.

Explanation of the genetic algorithm used for optimization.



Process	Explain
Breeding	Combine the traffic control methods of two different configurations.
Mutation	Randomly mutate the traffic control in a single junction.
Evolution	Locally adjust the traffic controls in all junctions, and seek for better solution.

We consider three kinds of traffic control devices: 1) traffic lights, 2) yield/stop signs, and 3) *orientation signs*, a special kind of traffic sign that we designed ourselves. We call the first two basic devices.

Step 1: Basic Device and Timestamp Choice

A traffic junction can be equipped with any one of the following five devices: 1) traffic light, 2) yield sign in the circle, 3) yield sign at the entrance, 4) stop sign in the circle, and 5) stop sign at the entrance. Besides, the timestamps of red/green lights for traffic lights are also changeable.

Sheriffhall Roundabout

Considering all potential variables above, we run our program against the Sheriffhall Roundabout, using the origin-destination flow data in **Table 1**, assuming that this flow matrix remains fixed over a one-hour period. The solution of our program shows that *traffic lights should be used rather than stop/yield signs*; otherwise, the accident rate will be dramatically higher.

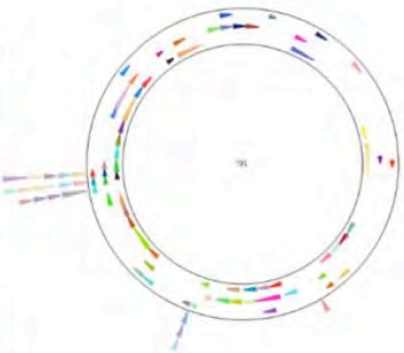
In **Figure 7**, green (light) represents right of way for vehicles from the incoming road, and red (dark) indicates right of way for vehicles in the circle. The optimal configuration creates a long period of red light for all junctions and allows digestion of vehicles quickly during the interval. This configuration accelerates the flows but has a lower saturated flow capacity, as **Table 5** summarizes.

Arm	G	R
1	0	14
2	7	14
3	38	18
4	3	19
5	3	13
6	38	14

Figure 7. The traffic light timestamps in 6 junctions (green (light) vs. red (dark)). Period = 68 s (calculated in assumption). Original flow information is used.

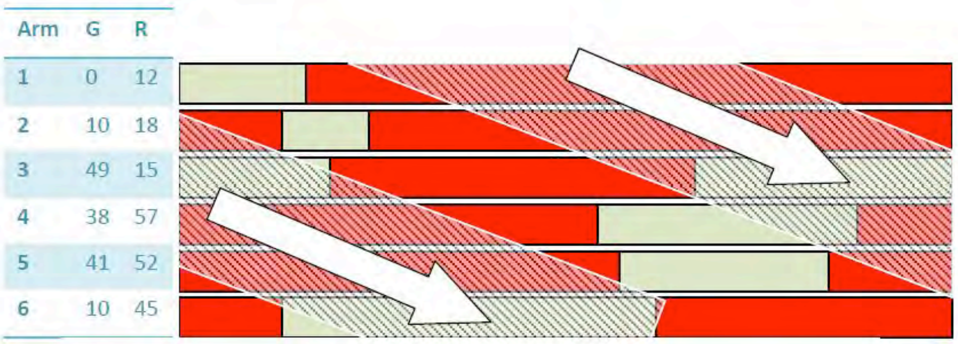
Table 5.
The multi-objectives of the optimal configuration of Sheriffhall, original flow.

Objective	Value
Saturated Flow Capacity	6904 vehicles / hour
Average Delay	42.763 seconds / vehicle · hour = 62.04$ / hour
Equity Degree	0.3187
Accident Expectation	4.63$ / hour
Device Cost	1.38$ / hour
Combined expense	**78.98$ / hour**



Sheriffhall Roundabout with 1.8 × Original Inflow

When the incoming flow density increases to 1.8 times as much, the optimal configuration shows a significant difference—see **Figure 8**.



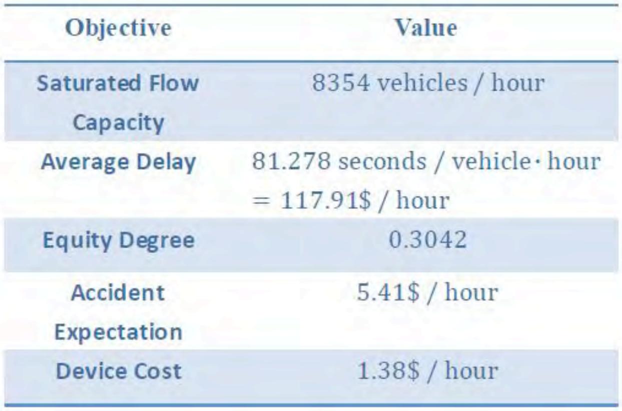
Arm	G	R
1	0	12
2	10	18
3	49	15
4	38	57
5	41	52
6	10	45

Figure 8. The traffic light timestamps in 6 junctions. Period = 68 s (calculated in assumption). Original flow × 1.8 is used.

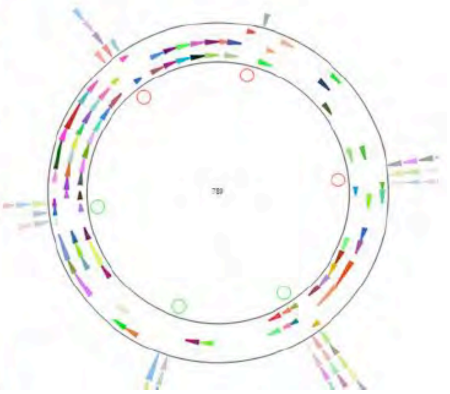
In **Figure 8**, the green-light periods for all junctions are shortened to let the circle digest the greater number of incoming vehicles. There is no longer a long period with all junctions having a red light. As an alternative, there is free passage between Junction 3 and Junction 6 (shadowed stripe), which greatly increases the saturated flow capacity but reduces the traversal speed (see **Table 6**). To see why, one needs to look at the origin-destination flow **Table 1**, in which the flow between Junction 3 and 6 constitutes a significant portion of all the inflows. The white stripe in **Figure 8** actually gives a good opportunity for vehicles to travel between them.

Table 6.

The multi-objectives of the optimal configuration of Sheriffhall, original flow × 1.8.



Objective	Value
Saturated Flow Capacity	8354 vehicles / hour
Average Delay	81.278 seconds / vehicle· hour = 117.91$ / hour
Equity Degree	0.3042
Accident Expectation	5.41$ / hour
Device Cost	1.38$ / hour



Step 2: Orientation-Sign Placement

Normally, the number of lanes in a traffic circle and the number of junctions are not equal. In some countries, a hidden rule [SetupWeasel 1999] is: *the vehicle nearer its exit should stay left* (Remark: we are driving on the left!) We refine this rule.

Let there be n arms. Suppose that a vehicle is at Junction a $(1 \leq a \leq n)$, and its destination is b $(1 \leq b < n)$ junctions farther on. We manage two variables lower_a^b and upper_a^b so that such a vehicle is suggested to stay in the range $[\mathrm{lower}_a^b, \mathrm{upper}_a^b]$. Our aim is to distribute vehicles into lanes to minimize congestion. To optimize these intervals $[\mathrm{lower}_a^b, \mathrm{upper}_a^b]$, we use a genetic algorithm again.

Figure 10 demonstrates the effect of the orientation sign of **Figure 9** in reducing the average delay, for different amounts of inflow. As the number of incoming vehicles increases, the positive effect of our orientation sign becomes evident. The configuration without orientation sign has saturated flow capacity 8354 (**Table 6**), and this number has increased to 8812 with the help of this newly-introduced sign. In short, *the very last potential capacity has been extracted in our model.*

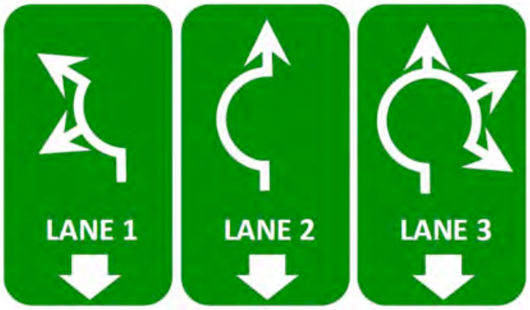


Figure 9. The orientation sign over the junction entrance. (At junction 3, with 1.8 × original inflow.)

Step 3: Time Variance and Self-Adaptivity

Origin-destination flows vary from morning to evening. The easiest way to handle this is to run our previous program with different traffic demand information for different time periods. Actually, we can go further, and make the traffic control self-adaptive by using traffic lights.

Given the traffic light timestamps calculated in Step 1, and assume that in the following hour the traffic demands change to new values. We select the original configuration as our seed, and carry out the genetic algorithm to gain a similar but better solution. **Figure 11** gives an example.

One may find that the timestamps change little and hence will not significantly affect vehicles already in the circle. As night falls, traffic demands

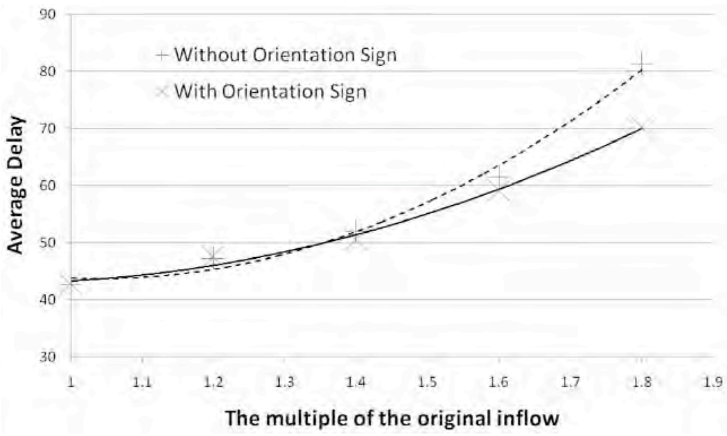
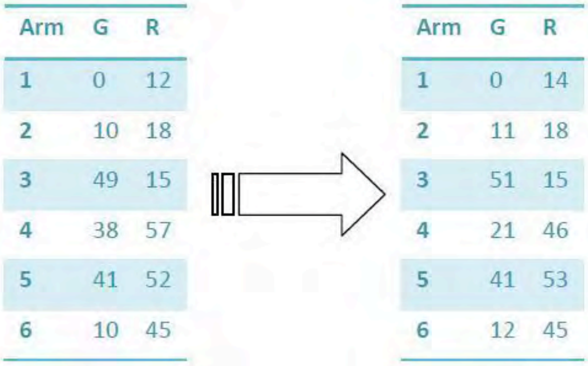


Figure 10. The magic effect of the orientation sign.



Arm	G	R
1	0	12
2	10	18
3	49	15
4	38	57
5	41	52
6	10	45

Arm	G	R
1	0	14
2	11	18
3	51	15
4	21	46
5	41	53
6	12	45

Figure 11. Self-adaptivity as inflow drops in 1 hr from 1.8 to 1.4 × original inflow.

fall off, and the traffic lights could be replaced in effect by yield signs by switching the lights to flashing yellow, an international signal [Wikipedia 2009] to remind drivers to be careful.

Verification of the Optimization Model

The Circle at Work

Figure 12 shows that when the inflow is 1.8 times as high as in **Table 1**, the traffic circle still works.

Accuracy

As a follow-up study to verify the optimization model, we need to test it on different traffic circles. For lack of data, we create our own dummy

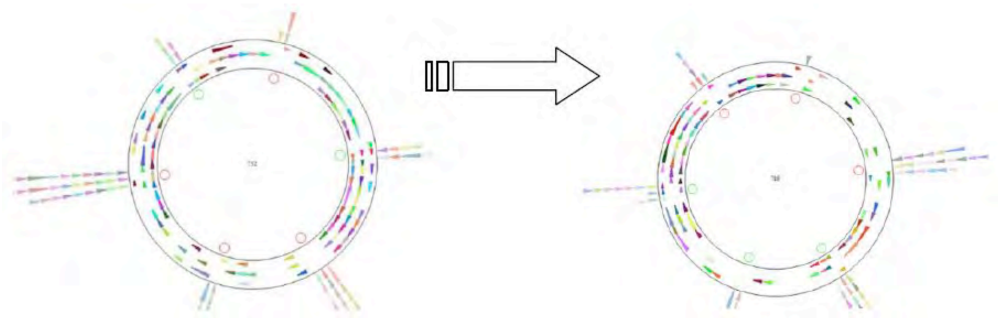


Figure 12. 39 seconds later, most of the vehicles waiting at Junction 6 move in.

traffic circles. In particular, we test a large traffic circle with 12 arms and 6 lanes, and the result shows that our model can deal with such large cases.

Testing on a dummy suburban circle with 4 arms and relatively lower traffic demand, we find as optimal solutions either in-circle stop signs or else a mixture of stop signs and traffic lights (**Figure 13**). In this example, the origin-destination flow between Junction 1 to Junction 3 is remarkably greater than all other pairwise flows.

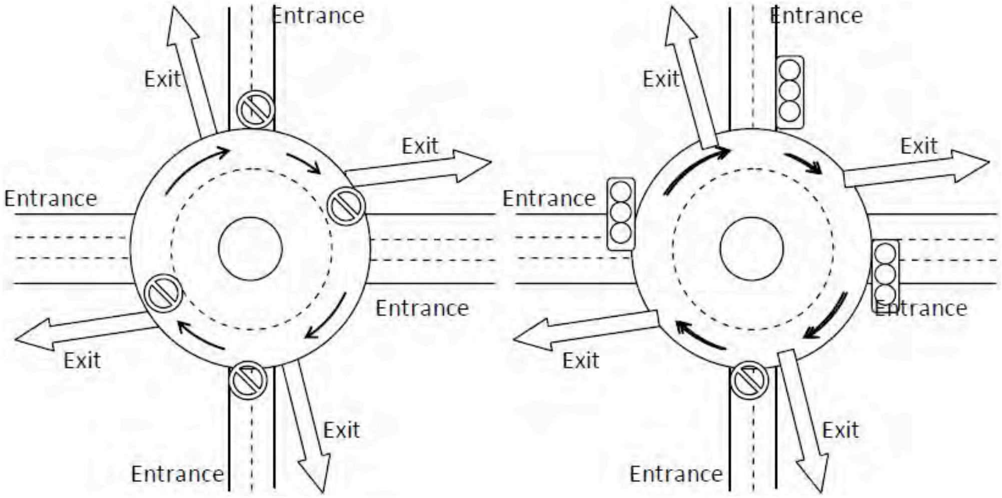


Figure 13. Two intuitive configurations generated by our model. The one on the left has two in-circle stop signs and guarantees fast pass from left to right; the one on the right has a mixture of traffic lights and stop signs.

Sensitivity

We tested sensitivity of our model by running it 50 times. **Table 7** shows the mean and standard deviations for runs against various level of inflow.

Table 7.

Sensitivity test of the optimization model.

The multiple of income flow	Average Delay	Standard Deviation
1.0 times	42.76 seconds / vehicle · hour	0.95 seconds / vehicle · hour
1.2 times	47.22 seconds / vehicle · hour	1.56seconds / vehicle · hour
1.4 times	51.99 seconds / vehicle · hour	2.54 seconds / vehicle · hour
1.6 times	61.54 seconds / vehicle · hour	3.81 seconds / vehicle · hour
1.8 times	81.28 seconds / vehicle · hour	8.30 seconds / vehicle · hour

Emergency Case

Our model can simulate an emergency. In **Figure 14**, one of the cars breaks down and block an entire lane. However, the traffic circle still works, but the average delay time has increased by 10 s.

The self-adaptivity of our model lets us adjust the light timestamps and reduce the traffic jam in an emergency case. However, because of limited time, we cannot describe the adaptivity here.

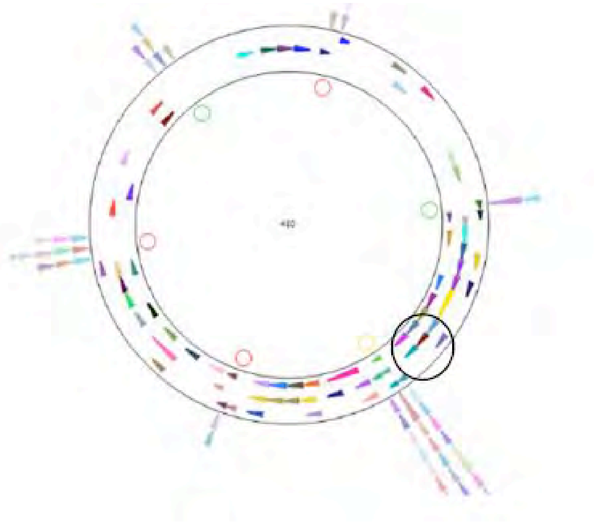


Figure 14. Breakdown of a vehicle slows the traffic, but traffic still circulates.

Conclusion

To estimate the overall performance of a traffic circle with a specific vehicle flow, we develop two simulation models. The first uses a Markov process to consider the entire flow, the second devotes its attention to the individual behavior of each vehicle.

We choose five objectives to evaluate the control method and convert them to a combined expense. We apply this standard to a real-life traffic circle with typical traffic control device setups.

We offer an optimization model to select traffic devices and determine the green-light period when traffic lights are used. In addition, we introduce orientation signs as a thoroughly new measure to bring efficiency. The flexibility of these solutions is proved when confronted with accidents.

References

Federal Highway Administration, U.S. Department of Transportation. 2008. The importance of transit. Chapter 14 in *2004 Status of the Nation's Highways, Bridges, and Transit: Conditions and Performance*. `http://www.fhwa.dot.gov/policy/2004cpr/chap14.htm`.

Fitzpatrick, Kay. 2000. *Accident Mitigation Guide for Congested Rural Two-lane Highways*. Transportation Research Board, National Research Council.

Garber, N.J., and L.A. Hoel. 2002. *Traffic and Highway Engineering*. 3rd ed. Pacific Grove, CA: Brooks / Cole.

Hangzhou Public Security Bureau Traffic Police Detachment. 2006. March on the city's briefing on road traffic accidents. `www.hzpolice.gov.cn/tabid/67/InfoID/9343/Default.aspx`.

Hubacher, Markus, and Roland Allenbach. 2002. Safety-related aspects of traffic lights. `http://www.bfu.ch/PDFLib/676_68.pdf`, `http://www.bfu.ch/English/Forschung/Forschungsergebnisse/pdfResults/r48e.pdf`.

Maher, Mike. 2008. The optimization of signal settings on a signalized roundabout using the cross-entropy method. *Computer-Aided Civil and Infrastructure Engineering*. 23: 76–85.

Picken, Andrew. 2008. Businesses want Sheriffhall flyover. *Edinburgh Evening News* (26 June 2008). `http://edinburghnews.scotsman.com/edinburgh/Businesses-want-Sheriffhall--flyover.4225078.jp`.

SetupWeasel. 1999. Traffic circles. `http://www.bbc.co.uk/dna/h2g2/A199451`.

Traffic Light Wizard. n.d. `http://trafficlightwizard.com`.

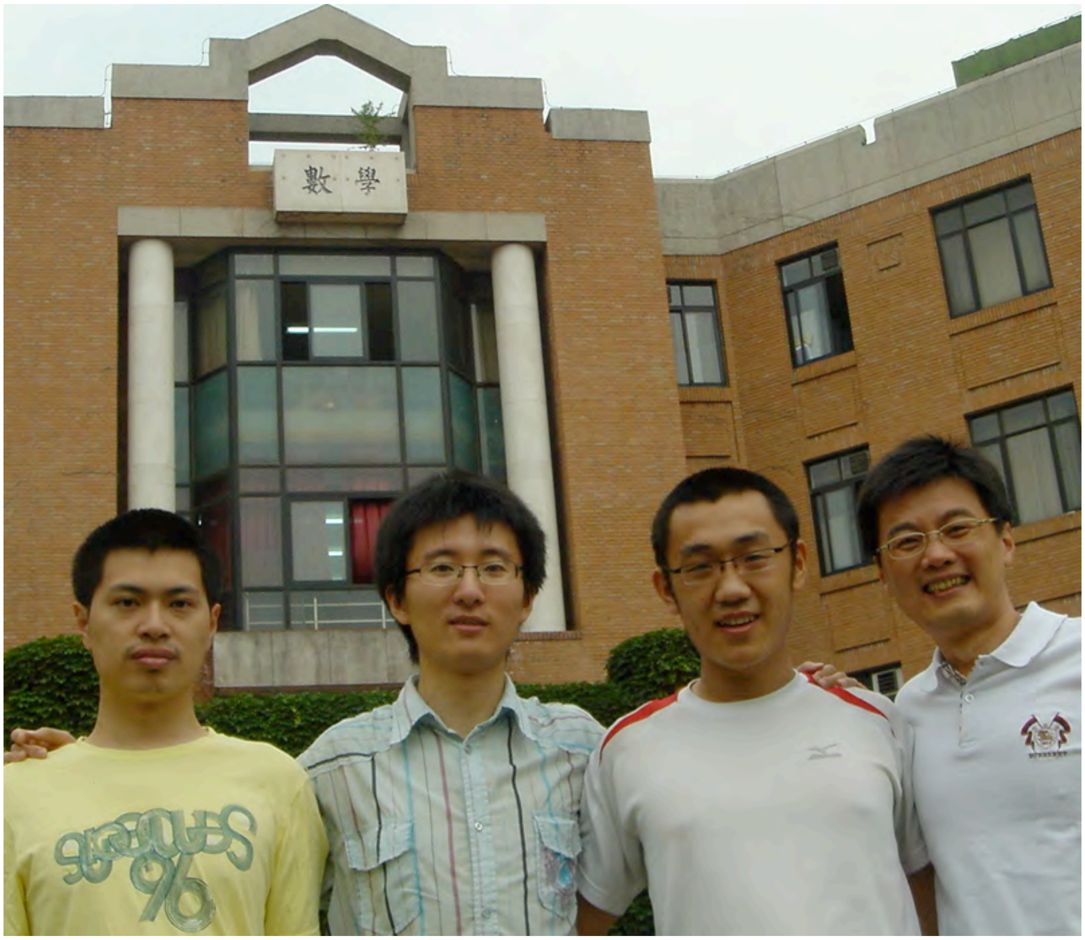
Transport for London Street Management—London Road Safety Unit 5. Do traffic signals at roundabouts save lives? *IHT Transportation Professional* (April 2005). `http://www.tfl.gov.uk/assets/downloads/SignalsatRoundabouts-TransportationProfessiona-Article.pdf`.

TuffRhino.com. n.d. `www.tuffrhino.com/Yield_Sign_p/sts-r1-2.htm`.

Wang, Shen Lin. 2005. People look forward to the traffic lights at an early date, "induction." (Chinese). People's Government of Kaifeng. `http://www.kaifeng.gov.cn/html/4028817b1d926237011d9332cfe300bb/3214.html`.

Wikipedia. 2009. List of variations in traffic light signalling and operation. `http://en.wikipedia.org/wiki/unusual_uses_of_traffic_lights`.

Ye, Ran. 2001. California power-saving traffic lights out of new tactics to use energy-saving foam. (Chinese). news.eastday.com (15 August 2001). `http://www.shjubao.cn/epublish/gb/paper148/20010815/class014800004/hwz462812.htm`.



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Pseudo-Finite Jackson Networks and Simulation: A Roundabout Approach to Traffic Control

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Summary

Roundabouts, a foreign concept a generation ago, are an increasingly common sight in the U.S. In principle, they reduce accidents and delays. A natural question is, "What is the best method to control traffic flow within a roundabout?" Using mathematics, we distill the essential features of a roundabout into a system that can be analyzed, manipulated, and optimized for a wide variety of situations. As the metric of effective flow, we choose time spent in the system.

We use Jackson networks to create an analytic model. A roundabout can be thought of as a network of queues, where the entry queues receive external arrivals that move into the roundabout queue before exiting the system. We assume that arrival rates are constant and that there is an equilibrium state. If certain conditions are met, a closed-form stationary distribution can be found. The parameters values can be obtained empirically: how often cars arrive at an entrance (external arrival rate), how quickly they enter the roundabout (internal arrival rate), and how quickly they exit (departure rate). We control traffic by thinning the internal arrival process with a "signal" parameter that represents the fraction of time that a signal light is green.

A pitfall of this formulation is that restricting the capacity of the round-

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about queue to a finite limit destroys the useful analytic properties. So we utilize a "pseudo-finite" capacity formulation, where we allow the roundabout queue to receive a theoretically infinite number of cars, but we optimize over the signal parameter to create a steady state in which a minimal number of waiting cars is overwhelmingly likely. Using lower bound calculations, we prove that *a yield sign produces the optimal behavior for all admissible parameter values*. The analytic solution, however, sacrifices important aspects of a real roundabout, such as time-dependent flow.

To test the theoretical conclusions, we develop a computer simulation that incorporates more parameters: roundabout radius; car length, spacing, and speed; period of traffic signals; and time-dependent inflow rates. We model individual vehicles stochastically as they move through the system, resulting in more-realistic output. In addition to comparing yield and traffic-signal control, we also examine varied input rates, nonstandard roundabout configurations, and the relationships among traffic-flow volume, radius size, and average total time. However, our simulation is limited to a single-lane roundabout. This model is also compromised by the very stochasticity that enhances its realism. Since it is nondeterministic, randomness may mask the true behavior. Another drawback is that the computational cost of minimization is enormous. However, we verify that a yield sign is almost always the best form of flow control.

Introduction

A report from the Wisconsin Dept. of Transportation notes that "to many, the idea of replacing four-way signaling with a roundabout seems like replacing hot dogs with crepes at the ballpark" [McLawhorn 2002]. For many Americans, the roundabout (traffic circle, rotary) is a foreign idea, even though the first one was built in New York in 1903. Roundabouts fell out of favor in the U.S.; but since midcentury, as studies showed how much safer and more efficient they can be, there has been a resurgence in their construction [National Cooperative Highway Research Program 1998]. Half of the states in the U.S. now have roundabouts, more than 1,000 installations. One study indicated that, on average, fatal crashes decreased 90% after traditional traffic lights were replaced by roundabouts [Arizona Department of Transportation n.d.].

A crucial aspect of efficiency and safety is entry. Until the 1920s, "yield-to-right" rules gave right of way to incoming cars, which tended to cause "locking" and delays at high traffic volumes. British studies indicated that adopting "priority-to-the-circle" rules allows more cars to to move through the circle more quickly and diminishes accident rates. The deflection of entering traffic serves to prevent excessive speed within the roundabout and to reduce further the incidence of accidents [National Cooperative Highway Research Program 1998]. So in modern roundabouts, incoming traffic yields to traffic in the circle (and changes direction to some extent). With that rule, entry may be governed in various ways. The simplest and most common is a yield sign at each entry point. The U.S. Dept. of Transportation advises that roundabouts "should never be planned for metering or signalization" [Robinson et al. 2000].

We develop a mathematical model for flow in roundabouts. We introduce assumptions used in determining the key parameter inputs and developing a metric for "effectiveness." We subsequently formulate and solve a simple analytic model of networked queues in an equilibrium state. After discussing limitations of the analytic model, we adapt it into a computer simulation that allows for detailed analysis and can be used to optimize the flow-control method.

Assumptions

Exponential arrivals/departures: Arrivals and departures follow a Poisson process, with exponentially-distributed interarrival times.

Local variable selection: External forces such as weather, special events, or acts of God may alter the system, but we do not address these factors.

Unbounded output: Although blockages would affect the roundabout, we assume that cars can always leave the roundabout at their exits.

Yield and stop sign equivalence: In terms of efficiency, a stop sign performs only as well as (or worse) than a yield sign, so a yield sign is preferable. Stop signs may, however, may be appropriate for safety, for example, with high pedestrian traffic.

Effectiveness

The most effective roundabout design minimizes delay.

Analytic Formulation

Our analytic model consists of a network of M/M/s queues (queues with Markovian arrivals, Markovian departures, and s servers), known as a *Jackson network*. (**Figure 1**). We give the model's parameters in **Table 1**. We assume that there is a steady state and no explicit time dependence. Effectiveness is quantified by the probability of few cars waiting, which can be calculated from the stationary distribution of the Jackson network. For the most effective roundabout, the most likely stationary states will be those with the fewest total cars, implying that delay is minimized.

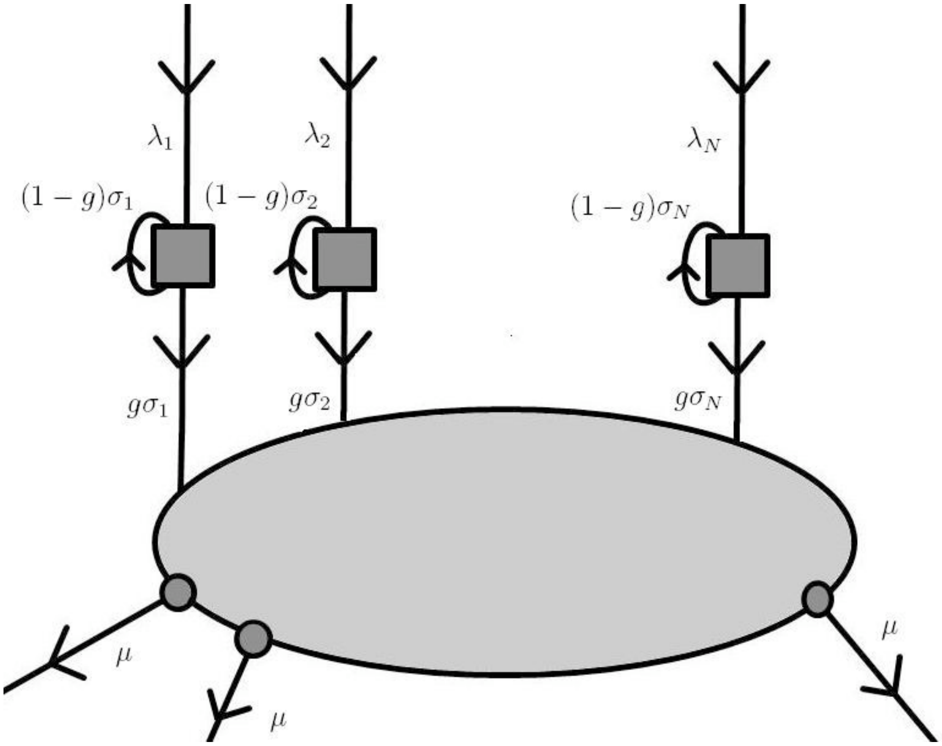


Figure 1. Visual schematic of queuing network.

Table 1.
Summary of parameters to the analytic model.

N	Number of streets which connect to roundabout
λ_i	External arrival rate of cars to entrance i
σ_i	Rate at which cars may enter roundabout from entrance i
μ	Rate at which cars may exit roundabout
$\pi(n_1, \ldots n_{N+1})$	Stationary distribution for the network

Additional Assumptions

Constant arrival rates: Constant arrival rates produce a system for which we can both derive a stationary distribution analytically and understand asymptotic behavior. The equilibrium behavior serves as a basis to build a more-complex and realistic simulation.

Perfect driver behavior: We do not allow drivers to miss their exits or break the rules (swerve wildly while talking on cellphones, scrape tires on the curb, mow down pedestrians, or cut in front of other drivers). These behaviors are infrequent enough that we can neglect them.

Description of Simple Queuing Network

The basic idea behind our Jackson network is to break the system up into queues. We assume that the roundabout is the intersection of N streets, which yields a network of $N + 1$ queues. Each street contributes an input stream, modeled as an M/M/1 queue with its own arrival rate λ_i. An input queue releases cars at rate σ_i from the incoming street into the roundabout. The presence of traffic lights or yield signs is represented by a thinning parameter g, the percentage of time when a traffic light at the intersection is green; setting $g = 1$ corresponds to a yield sign. Thus, cars enter at a thinned rate $g\lambda_i$. The queue representing the roundabout itself is an M/M/N queue, where the N servers represent the N exits.

The stationary distribution $\pi(n_1, \ldots n_{N+1})$ (if it exists) for this system of queues gives the asymptotic fraction of time that the system spends in the state with n_i cars in queue i, for all i. We are interested in a network for which the stationary distribution can be found. Then we choose g such that the system spends a larger fraction of time in a state where the total number of cars in the system is minimal.

With a limited-capacity queue for the roundabout, the input queues would input cars into it only if there is space. However, finite-capacity queuing networks do not generally yield closed-form solutions for stationary distributions [Bouchouch et al. 1992].

To ensure an analytic solution, we allow infinite capacity and get a stationary distribution, which can tell us the probability that a certain *total* number of cars is "stuck" in the system. In the original model, cars wait in the street from which they are to enter; in our model, they wait inside the (infinitely large) roundabout. We are not actually concerned with where they wait but rather with how many wait and how likely it is that that many cars will be waiting.

If the roundabout is not full, finite- and an infinite-capacity roundabout queues are equivalent because an incoming car can always enter. Now suppose that the roundabout is full:

- **(i)** If a car waits in its street, three events must occur before it exits the system: another car in the circle must leave it (with departure rate μ), the new car must enter it (rate σ), and the new car must exit (rate μ).
- **(ii)** If a car "waits" in the roundabout and then exits, three events will also have occurred: the car will have entered (rate σ) and joined the interior queue. Since there are only N servers, another car must exit that queue (rate μ) before the car in question is served (rate μ).

Either treatment is the superposition of three Poisson processes with the same set of rates, and order is unimportant. Thus, the queue exhibits "pseudo-finite capacity," since it mimics the qualitative behavior of a network where one queue size is bounded and the others are infinite.

Formulation of Stationary Distribution

In an equilibrium state, the input of each queue equals its output. Let r_i be the asymptotic departure rate from queue i, equal to the sum of the arrival rates to queue i. Defining $p(i, j)$ as the probability that a car leaving queue i enters queue j, we can write an expression for the asymptotic departure rate:

$$r_j = \lambda_j + \sum_{i=1}^{N+1} r_i p(i, j),$$

in matrix form expressed as

$$\mathbf{r} = \mathbf{\Lambda} + \mathbf{r}\mathbf{p},$$

where **r** is a row vector of departure rates, $\mathbf{\Lambda}$ is a row vector of arrival rates, and **p** is the matrix with elements $p(i, j)$.

Following Durrett [1999], we define two conditions on the system:

(A) For each queue i, there exists a path of positive probability along which it is possible to exit the system.

(B) Define $\varphi_i(n)$ as the departure rate from queue i when that queue contains n cars and let

$$\psi_i(n) = \begin{cases} \prod_{m=1}^{n} \varphi_i(m), & n \geq 1; \\ 1, & n = 0. \end{cases}$$

Then there exists a positive constant c_j such that

$$\sum_{n=0}^{\infty} \frac{c_j r_j^n}{\psi_j(n)} < \infty.$$

Durrett [1999] shows that if condition **(A)** is met, then the matrix $(\mathbf{I} - \mathbf{p})$ is invertible; and if condition **(B)** is also met, then a stationary distribution π exists with form

$$\pi(n_1, \ldots, n_{N+1}) = \prod_{j=1}^{N+1} \frac{c_j r_j^{n_j}}{\psi_j(n_j)}.$$

We apply this approach to our system, where $\mathbf{\Lambda} = (\lambda_1, \ldots, \lambda_N, 0)$. The $(N + 1)$st queue has zero external arrival rate because it is the queue for the roundabout.

The $(N+1)\times(N+1)$ matrix for $\mathbf{p}$ has the form

$$\begin{pmatrix} 1-g & 0 & \dots & g \\ 0 & \ddots & \dots & \vdots \\ 0 & \ddots & 1-g & g \\ 0 & 0 & \dots & 0 \end{pmatrix}.$$

From any location in the network, there is a nonzero probability of exiting the system. Thus, condition **(A)** is satisfied, the matrix $\mathbf{I}-\mathbf{p}$ is invertible, and we can write the vector of asymptotic release rates as

$$\mathbf{r} = \mathbf{\Lambda}(\mathbf{I}-\mathbf{p})^{-1}.$$

The simplicity of our system allows us to solve directly for $(\mathbf{I}-\mathbf{p})^{-1}$ via Gauss-Jordan elimination:

$$\begin{pmatrix} \frac{1}{g} & 0 & \dots & 1 \\ 0 & \ddots & \dots & \vdots \\ 0 & \ddots & \frac{1}{g} & 1 \\ 0 & 0 & \dots & 1 \end{pmatrix}.$$

Thus, the asymptotic departure rates have the form

$$\mathbf{r_j} = \begin{cases} \lambda_j/g, & 1 \le j \le N; \\ \sum_{i=1}^{N}, \lambda_i & j = N+1. \end{cases}$$

Now we formulate the parameters required by condition **(B)** to solve for a stationary state. For the entry queues, we have

$$\varphi_j(n) = \sigma_j, \qquad 1 \le j \le N;$$

and for the roundabout queue, we have

$$\varphi_{N+1}(n) = \begin{cases} n\mu, & 1 \le n \le N; \\ N\mu, & n > N. \end{cases}$$

Also, we formulate for the entry queue:

$$\psi_j(n) = \sigma_j^n \qquad 1 \le j \le N;$$

and for the roundabout queue:

$$\psi_{N+1}(n) = \begin{cases} n!\mu^n, & 1 \le n \le N; \\ N!(N\mu)^n, & n > N. \end{cases}$$

We investigate under what circumstances condition **(B)** is met. For the entry queues ($1 \leq j \leq N$), we need to find a positive constant c_j such that

$$\sum_{n=0}^{\infty} c_j \left(\frac{\lambda_j}{g\sigma_j}\right)^n < \infty.$$

We can choose a nonzero c_j only if this geometric series converges, which occurs when

$$\frac{\lambda_j}{g\sigma_j} < 1.$$

For the roundabout queue, we examine the convergence of

$$\sum_{n=0}^{N} \left(\frac{r_{N+1}}{\mu}\right)^n \frac{1}{n!} + \sum_{n=N+1}^{\infty} \frac{1}{N!} \left(\frac{r_{N+1}}{N\mu}\right)^n.$$

For fixed N, the first term is finite and does not affect convergence. The second sum is a geometric series that converges if

$$\frac{r_{N+1}}{N\mu} < 1.$$

Thus, the two conditions necessary for the existence of equilibrium and a stationary distribution for our queuing network are

$$\textbf{(i)} \quad \lambda_j < g\sigma_j,$$

$$\textbf{(ii)} \quad \sum_{i=1}^{N} \lambda_i < N\mu.$$

If these conditions are met, we can solve for the stationary distribution. First, we choose the constant c_j such that

$$\sum_{n=0}^{\infty} \frac{c_j r_j^n}{\psi_j(n)} = 1.$$

We find:

$$\frac{1}{c_j} = \frac{1}{1 - \frac{\lambda_j}{g\sigma_j}}, \qquad q \leq j \leq N;$$

$$\frac{1}{c_{N+1}} = \sum_{n=0}^{N} \left(\frac{r_{N+1}}{\mu}\right)^n \frac{1}{n!} + \frac{1}{N!} \left[\frac{1}{1 - \frac{r_{N+1}}{N\mu}} - \sum_{n=0}^{N} \left(\frac{r_{N+1}}{N\mu}\right)^n\right].$$

These can be used in the closed form of the stationary distribution presented in [Durrett 1999]:

$$\pi(n_1,\ldots,n_{N+1}) = \left(1-\frac{\lambda_1}{g\sigma_1}\right)\left(\frac{\lambda_1}{g\sigma_1}\right)^{n_1} \times \cdots$$
$$\times \left(1-\frac{\lambda_N}{g\sigma_N}\right)\left(\frac{\lambda_N}{g\sigma_N}\right)^{n_N}\left(\frac{c_{N+1}}{N!}\right)\left(\frac{r_{N+1}}{\mu N}\right)^{n_{N+1}}.$$

Optimization of Stationary State

The parameters μ, λ_j, and σ_j are fixed by the physical location of the roundabout and the number of cars that use it. Hence, the stationary state π is a function of g that can be optimized over g. The idea is to maximize the amount of time spent in a state in which the total number of cars in the system is less than or equal to the capacity of the roundabout. Define

$$\mathcal{K} \equiv \left\{\text{all } \{n_i\}_{i=1}^{N+1} \text{ such that } n_1 + \cdots + n_{N+1} = k\right\}$$

and define

$$\pi(k) = \underbrace{\sum}_{\text{all } \{n_i\} \in\ \mathcal{K}} \pi(n_1,\ldots,n_{N+1}).$$

We analyze how $\pi(k)$ depends on g for small k. For a given k, the number of terms in the sum is the number of nonnegative integer solutions to

$$n_1 + \cdots + n_{N+1} = k,$$

which is given by the well-established formula [Ross 2006]

$$\frac{(N+k)!}{N!k!}.$$

The number of terms in the sum grows exceptionally quickly, so directly examining g-dependence is impossible. Instead, we establish a lower bound for $\pi(k)$ in terms of $\pi(0,0,\ldots,0)$, the fraction of time in which no cars remain in the system. For this case, denoted $\pi(0)$, we have:

$$\pi(0) = \prod_{i=1}^{N}\left(1-\frac{\lambda_i}{g\sigma_i}\right)\left(\frac{c_{N+1}}{N!}\right).$$

Neither c_{N+1} nor $N!$ depends on the choice of g. Therefore, $\pi(0)$ is maximized over g if the product

$$\prod_{i=1}^{N}\left(1-\frac{\lambda_i}{g\sigma_i}\right)$$

is maximized over g. The conditions under which this stationary distribution was constructed include

$$\frac{\lambda_i}{g\sigma_i} < 1,$$

ensuring that all terms of the product are between 0 and 1. Therefore, for a fixed set of constraints $\{\lambda_i/\sigma_i\}$, the optimal choice of g minimizes each $\lambda_i/g\sigma_i$ so as to maximize the quantity $1 - (\lambda_i/g\sigma_i)$. Therefore, the largest g will maximize $\pi(0)$. Given the constraint $0 < g \leq 1$, the optimal choice is $g = 1$.

Every other stationary state can be written in terms of $\pi(0)$:

$$\pi(k) = \pi(n_1, \ldots, n_{N+1}) = \pi(0)\left(\frac{c_{N+1}}{N!}\right)\left(\frac{\lambda_1}{g\sigma_1}\right)^{n_1} \cdots \left(\frac{r_{N+1}}{N\mu}\right)^{n_{N+1}}.$$

We establish a lower bound for $\pi(k)$ by defining

$$\frac{\epsilon}{g} \equiv \min\left\{\frac{\lambda_i}{g\sigma_i}, \frac{r_{N+1}}{N\mu}\right\}, \qquad C \equiv \frac{c_{N+1}}{N!}.$$

We assert that since each term in the product is less than or equal to 1, the sum of the powers of these terms is k; since there are $(N+k)!/N!k!$ distinct elements of $\mathcal{K}$, we have

$$\pi(k) \geq \frac{(N+k)!}{N!k!}\, C\left(\frac{\epsilon}{g}\right)^k \pi(0).$$

In the event that

$$\min\left\{\frac{\lambda_i}{g\sigma_i}, \frac{r_{N+1}}{N\mu}\right\} = \frac{r_{N+1}}{N\mu},$$

all g-dependence comes from $\pi(0)$, which is maximized for $g = 1$. In the event that for some index j we have

$$\min\left\{\frac{\lambda_i}{g\sigma_i}, \frac{r_{N+1}}{N\mu}\right\} = \frac{\lambda_j}{g\sigma_j},$$

we first define

$$\max\left\{\frac{\lambda_i}{g\sigma_i}\right\} = \frac{\delta}{g},$$

which allows us to assert

$$\pi(0) \geq \left(1 - \frac{\delta}{g}\right)^N,$$

which in turn implies that

$$\pi(k) \geq \frac{(N+k)!}{N!k!} C \left(\frac{\epsilon}{g}\right)^k \left(1-\frac{\delta}{g}\right)^N .$$

We turn our attention to the behavior of the part that governs the g-dependence of the lower bound of $\pi(k)$:

$$f(g) = \left(\frac{\epsilon}{g}\right)^k \left(1-\frac{\delta}{g}\right)^N .$$

We differentiate with respect to g and find that

$$\frac{\partial f}{\partial g} = \frac{\epsilon^k (g-\delta)^{N-1}[(N-k)(g-\delta)+Ng]}{g^{2(k-N)}}.$$

Since $\epsilon > 0$, and $g - \delta > 0$ according to the assumptions with which we set up the system, the sign of $\partial f/\partial g$ is determined by the expression

$$(N-k)(g-\delta)+Ng,$$

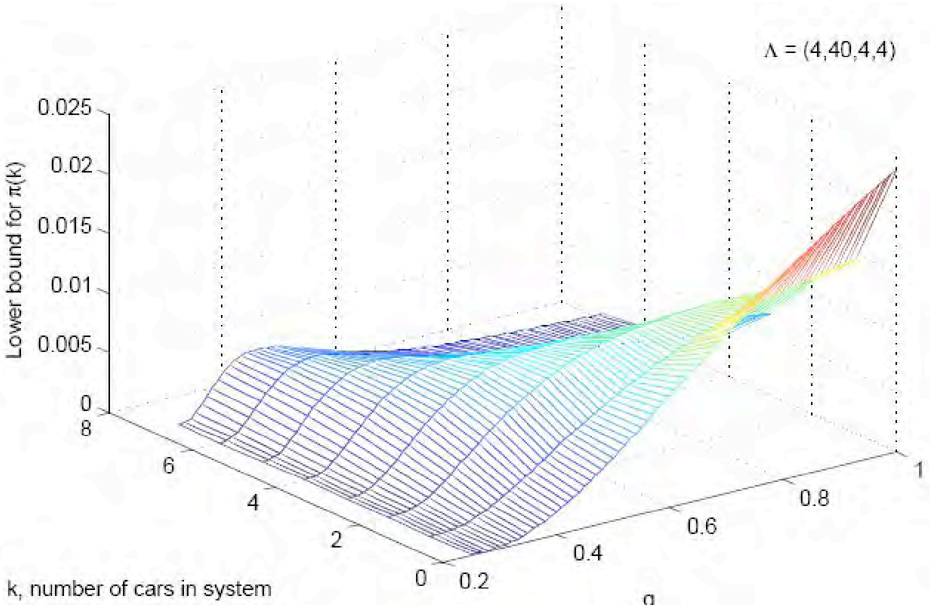
which is guaranteed positive for

$$k < N + \frac{Ng}{g-\delta}.$$

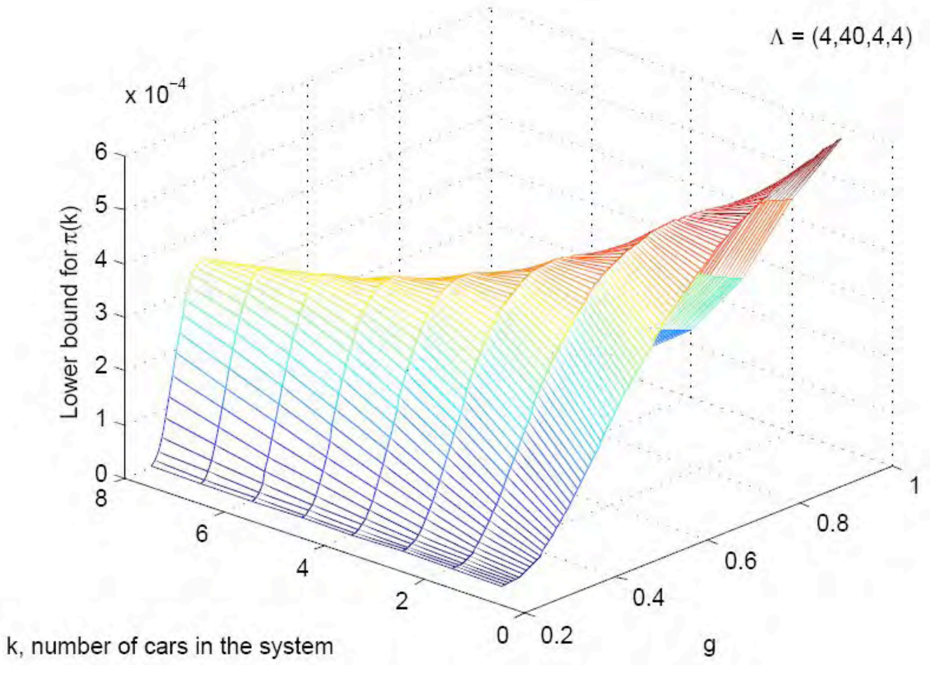
So, for small k, the slope is positive for all g in our domain, implying that increasing g increases the lower bound on $\pi(k)$, which ensures that the stationary distribution is larger. For our analytic model, the value of g that guarantees the largest lower bound on $\pi(k)$ for small k is $g = 1$, regardless of other parameters. *Our analytic model always recommends a yield sign.*

To examine the actual stationary state behavior, we implement a computer program that calculates $\pi(k)$ for each value of k, summed over all the stationary states for which the total number of cars in the system equals k. We examine this quantity for a wide range of values of λ, σ, and μ. In all cases, the stationary distribution for lower k values is highest for $g = 1$. In **Figures 2** and **3**, we compare the lower-bound behavior and the actual behavior for a four-entrance roundabout. We examine both the case where all input rates are equal and the case where they are not. Our lower-bound-estimate curves and our calculated curves have very similar shapes. Thus, a choice of g that maximizes the area under the lower-bound curve for small k also maximizes the area under the actual curve. This fact validates our use of the lower-bound estimate as a basis for the optimal choice of g.

Our analytic formulation always finds the optimal entrance rule to be a yield sign at every intersection. Although this is in part a result of the limitations of the model, such as lack of time-dependence, it is mostly consistent with both the results of our computer simulation and our research into real-world practices.

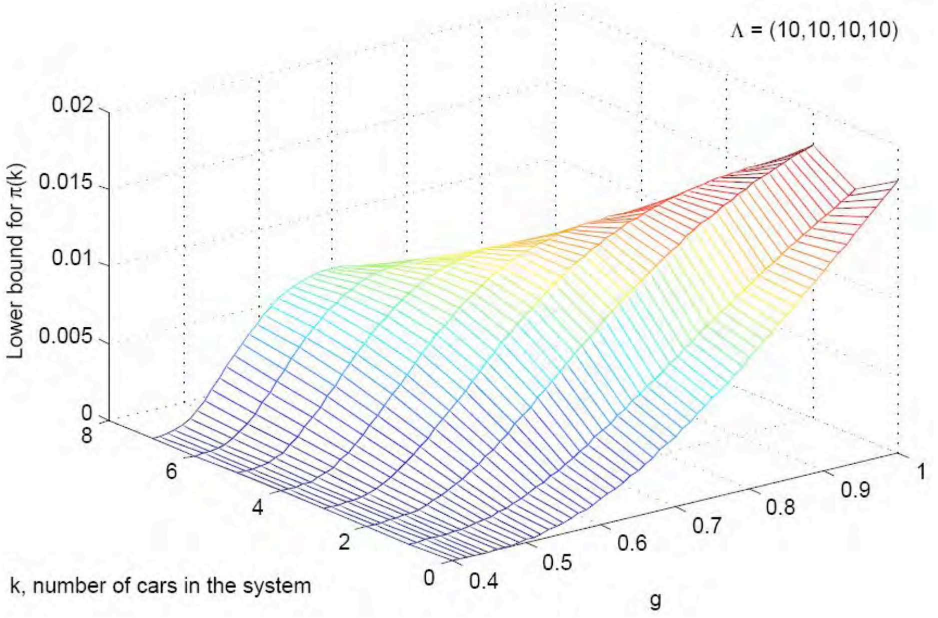


(a) Actual value.

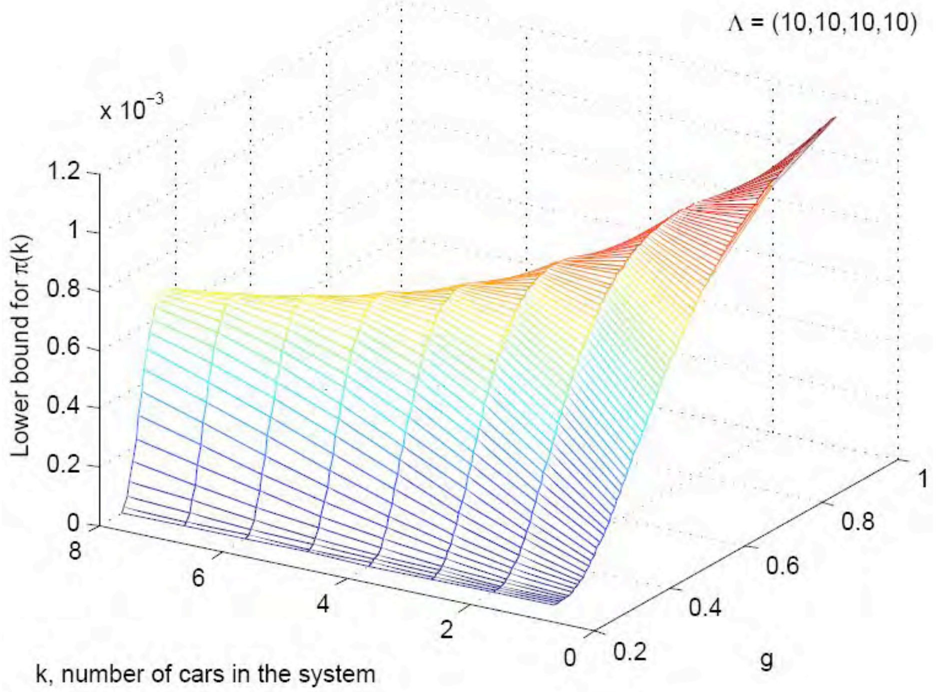


(b) Lower bound estimate.

Figure 2. Comparison of actual stationary distribution and lower bound estimate for unequal input rates.



(a) Actual value.



(b) Lower bound estimate.

Figure 3. Comparison of actual stationary distribution and lower bound estimate for equal input rates.

Computer Simulation

Given the weaknesses of the analytic model, we adapt it to create a computer simulation with the freedom to change some assumptions in order to create a more realistic model.

Assumption Modifications

Independent arrival processes: The probability of a car approaching the circle from one street does not depend on the probability of a car approaching the circle from a different street, nor does it depend on the probability distribution of how cars enter or leave the traffic circle.

Drivers' intentions: Every driver wants to leave the traffic circle through a specific exit in the least amount of time possible. However, since a driver may be confused or unaware of surroundings, we define a fixed probability for a car to leave the circle successfully. While this feature allows for the possibility of getting stuck in the circle forever (reminiscent of Chevy Chase in National Lampoon's *European Vacation*), the probability of continually missing the exit is vanishingly low.

Constant car length and speed: Vehicles all have the same length and speed. Adding variation would introduce unnecessary complexity into the model.

Yield sign is optimal for low traffic volume: According to both literature and common sense, a periodic traffic light in a roundabout with few cars only hampers flow.

Computer Simulation of One-Lane Roundabout

We want to compare our analytical results to a more-realistic simulation. We simulate cars arriving to a theoretical traffic circle, entering the circle, moving through it toward, and exiting as desired.

We fix the length of the car at 5 m and vary the speed inside the circle from 8 to 13 m/s, based on the ranges presented in Robinson et al. [2000]. The capacity of the roundabout (the number of cars that can be inside at any one time) is determined by vehicle length, vehicle speed, and roundabout radius. At full capacity, cars inside the roundabout are spaced by 1 s of driving, ensuring sufficient space to maneuver.

Description of Simulation Process

Our simulation determines when cars arrive to the circle from each entrance street, considered independently. For a random variable $U \sim$

Uniform$[0, 1]$, the variable $-\ln(U)/\lambda$ is exponentially distributed with parameter λ; we use the latter random variable to determine the interarrival times for each entrance road.

We vary arrival rates by time of day; fewer cars should arrive at night than in the middle of the day or during rush hours. To account for this behavior, we scale the peak arrival rates. The scaling function $f(t)$ consists of narrow Gaussians centered at each rush-hour time and a smaller-amplitude slowly-varying Gaussian centered at midday. This function is plotted in **Figure 4** for rush periods of 1 hr each at 8:00 A.M. and at 5:00 P.M.

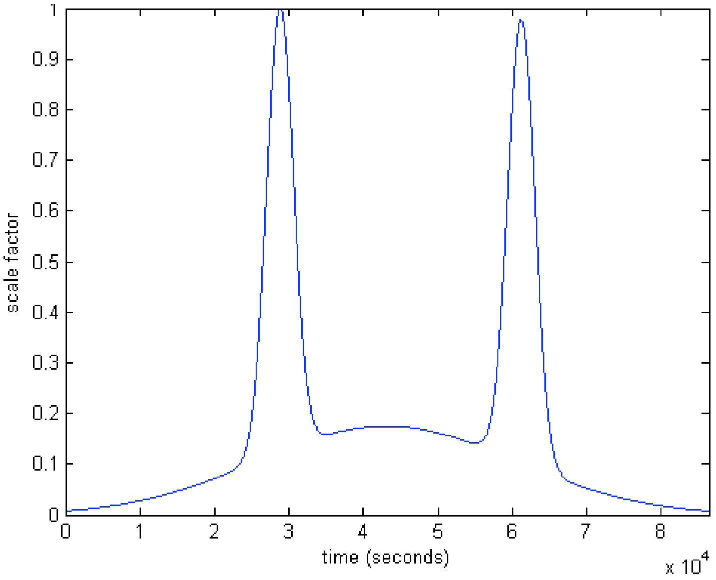


Figure 4. Time-dependent arrival rate multiplier.

The arrival times for each entrance queue are recorded and computed prior to simulation. At each simulated arrival time, we add a vector to the right end of a dynamic matrix that represents the entry queue. The vector, corresponding to a car, contains parameters that govern the car's behavior: arrival time, destination, and probability of missing the exit. The matrix columns represent the order of cars waiting to enter traffic circle; and because we treat the entrance queue as a first-in/first-out buffer, only the car of the leftmost column can enter the circle.

A car's destination is determined by relative exit popularity. When a car arrives at its exit, a random variable $U \sim$ Uniform$[0, 1]$ is simulated; if the value is less than 0.05, the car misses its exit and stays in the traffic circle.

To simulate traffic moving through the circle, we divide the circle into discrete positions based on the circumference and the length of a typical car. We number these positions in the same direction as the flow of traffic. Vectors from the leftmost position in an entry-queue matrix are placed into the traffic circle if the entry position and the position immediately behind the entrance are both vacant. Thus, we obtain a "circle matrix" where each column pertains to a position. Moving an entire column of the matrix simulates an individual's movement through the circle.

At regular time intervals, based on the speed of the cars and the size of the circle, we rotate the columns of the matrix. After each rotation, cars check to see if they have reached their destination and, if so, determine whether they exit. Once a car exits, it calculates time spent in the circle by subtracting arrival time from exit time. The simulation then erases the values of the vector representing the car's current position from the circle matrix to indicate that that car has left the circle. After exiting cars leave, cars waiting to enter the circle make the following two checks, both of which must be satisfied in order to enter the circle:

Check traffic signal: The car checks a *signal matrix* whose rows are indexed by the entrance locations and whose columns represent a fraction of time in a traffic-light cycle. Thus, each entry (i, j) of the signal matrix indicates whether the ith light is red or green during the jth signal interval, where each signal interval is

$$\frac{20}{\text{round}\left(\dfrac{\text{carlength + speed}}{\text{speed}}\right)}$$

seconds long. At the start of the simulation, $j = 1$; once one signal interval has elapsed, $j = j + 1$. Iteration continues until we reach the end of the signal matrix, signifying the end of the traffic-light cycle; at that point, j is set to 1. The time t of the simulation step determines which value of j is used. If the entry of the matrix is 0, the light is red and the car cannot enter the circle; if the entry is 1, the light is green and the car can enter if there is space.

For each run of the simulation, three signal matrices are used: one each for late night/early morning, rush hours, and midday. A signal matrix whose entries are all identically 1 is referred to as a *yield matrix* because it acts like a yield sign; the late night/early morning signal matrix is always a yield matrix.

Check for cars in the circle: Cars permitted to enter the circle by the signal matrix must nonetheless yield to traffic in the circle. A car checks the circle matrix to see if both the entrance position and the position before it are unoccupied, so that it does not hit a car in the circle nor cut one off.

If both conditions are satisfied, the simulation puts the car into the circle by removing the leftmost column of its entry matrix and copying it into the entrance position on the circle matrix.

A Metric to Measure Traffic Flow

We can use the average time spent in the system per car over one day as a good estimator of how the simulation behaves. However, cars during rush hour should be waiting longer than cars at midday or at night. As a result, the *maximum* time spent should give us a sense of the worst-case scenario. A good flow-control system (or signal matrix) should produce lower values for both the average time and the maximum time.

Justification of Experimental Methodology

Literature search and our analytic model reveal that yield control is by far the most common and effective form of roundabout flow control, so we use our simulation to test the effectiveness of a yield sign vs. a traffic light.

First, we assume that late at night and early in the morning, when traffic flow is minimal, a yield sign (or a perpetually-green traffic light) would be optimal. We ran three simulations on each of 100 combinations of matrices, 98 of which were randomly generated; we always compared the random signal matrix results to the yield signal matrix and a fixed non-yield signal matrix. Every matrix set was run on the same roundabout.

We want to eliminate matrices that represent unrealistic periods of red light. We force our midday signal matrix to satisfy the following (where $\mathbf{g_{yield}}$ represents the matrix of all ones and $\mathbf{g_{mid}}$ is our midday matrix):

$$\|\mathbf{g_{yield}} - \mathbf{g_{mid}}\|_\infty \leq 2. \tag{1}$$

For our rush-hour signal matrix $\mathbf{g_{rush}}$, we enforce the following condition:

$$\|\mathbf{g_{yield}} - \mathbf{g_{rush}}\|_\infty \leq 3. \tag{2}$$

These conditions force a sufficient number of 1s in each row of the matrix. We enforce slightly different conditions during rush hour vs. midday because of the decreased traffic volume at midday; as traffic volume decreases, necessity for control decreases.

Simulation Results, Part 1: Flow-Control Considerations

From the analytic model, we conclude that the most effective control is for entering cars to yield to cars in the circle. We wish to see if this result holds for the more-complicated simulation; in terms of simulation variables, we want to know if the yield matrix is the optimal choice of signal matrix.

Using the yield matrix, we ran simulations using different relative distributions for input rates from four entrance streets, with the ratio of smallest input rate to largest ranging from 1:1 to 1:8. We plot the number of cars in each part of the system against time. Traffic congestion appears in the plots as extreme peaks in the density.

In **Figure 5(a)**, all streets have the same entrance rate. As one can see in the second row from the back, the majority of cars enter the circle almost immediately.

We observe similar behavior when some streets have higher input rates than others; we consider a street to have the "major" input rate if its rate is as high or higher than the others. In **Figure 6(a)**, two streets have major inputs, but the plots appear almost exactly the same as in **Figure 5(a)**, with a discrepancy only in the peak total density. Furthermore, with only one major street (**Figure 6(b)**), we see even better performance—the peaks are significantly decreased. This shows that yield signs are self-regulating enough to behave well under both high input and low input.

We now turn our attention to systems with traffic lights at each entrance. This means that the traffic-signal matrices contain both 1s and 0s, although we enforce the condition that no row contain all 0s (stopping all traffic). Also, we use these non-yield methods only during the rush-hour and midday periods and use the standard yield matrix at night.

Using the same input rates as in **Figure 5(a)**, we obtain the plots in **Figure 5(b)**. The shapes of the plots appear similar but the scaling is different. The maximum peak in **Figure 5(a)** barely reaches 50 cars, but in **Figure 5(b)** the peak reaches 70 cars.

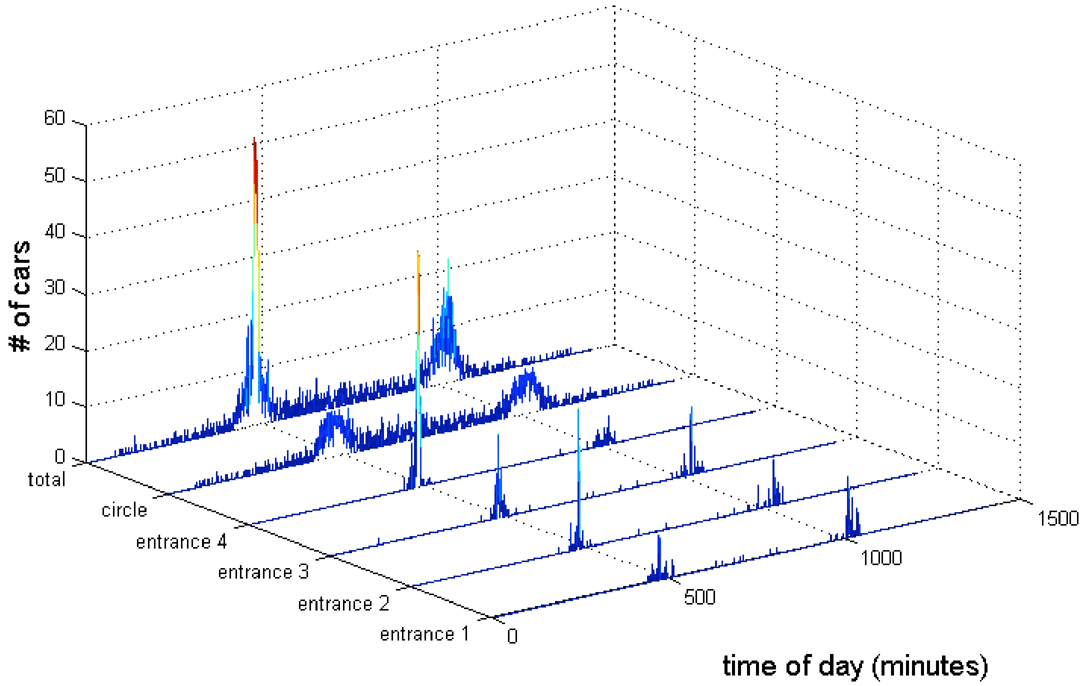
Non-yield control may not be optimal, but we should not jump to that conclusion. We ran 100 such trials with different random traffic-signal matrices and compared them with a trial that used the yield matrix. The results are in **Figure 7**. The horizontal lines indicate the mean (396 s), median (232 s), and minimum (23 s) values. The yield matrix, with an average of 32 s, was *not* the best trial (although the granularity of the plot partially hides this fact). In fact, 4 of the 100 trials with random non-yield traffic matrices beat the yield matrix by a margin of about 9 s.

Nonetheless, these few results do not shatter the conclusions from our analytic model. The matrices that seem to improve flow are extremely similar to the yield matrix, with only one or two 0s in the entire matrix and no row containing more than one 0, and we used these matrices only during peak traffic hours (less than 1/12 of the day); so that these matrices showed better performance than the yield matrix can be attributed to chance. The overall experimental result is telling: 96% of the trials were significantly worse than the yield matrix, and the "better" matrices improved the process by only a small margin, too small to warrant the cost of traffic lights.

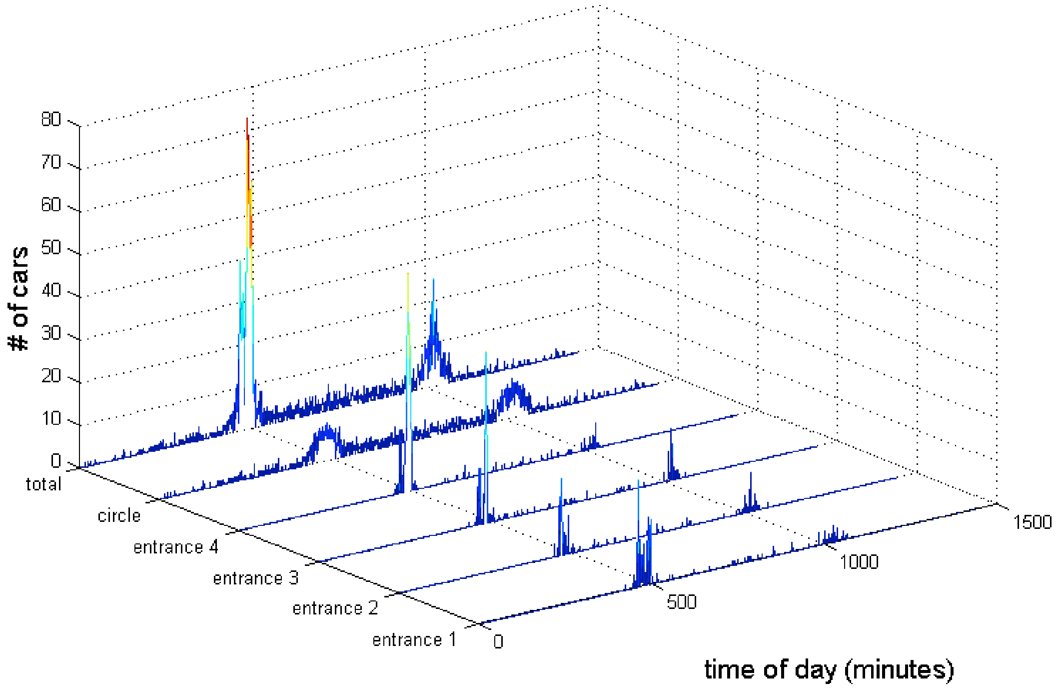
We also tested our simulation on various roundabouts. For five other parameter sets, varying size, speed, and input flow, we ran the same experiment with 19 random matrices per parameter set. In each case, the yield matrix performed as well as, or better than, any of the signal matrices. Thus, we base our recommendation on consistent results over 200 trials across various roundabout designs.

Simulation Results, Part 2: Size Considerations

Because larger traffic circles have higher capacities, we investigate the effects of the circle radius on traffic flow. As the radius of the circle increases,

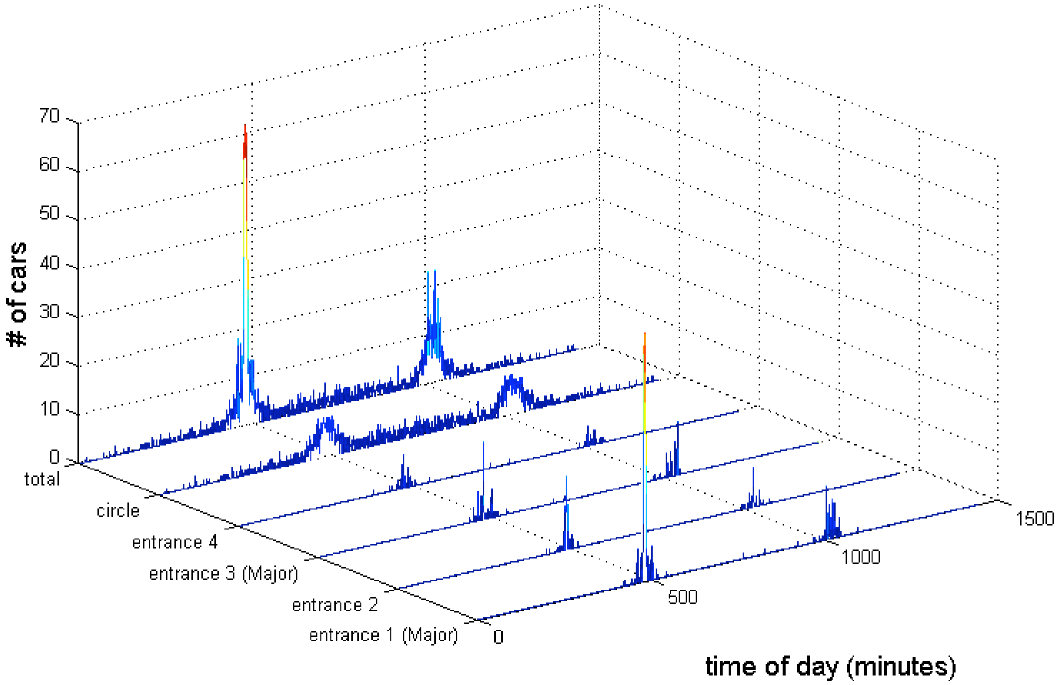


(a) Yield flow control.

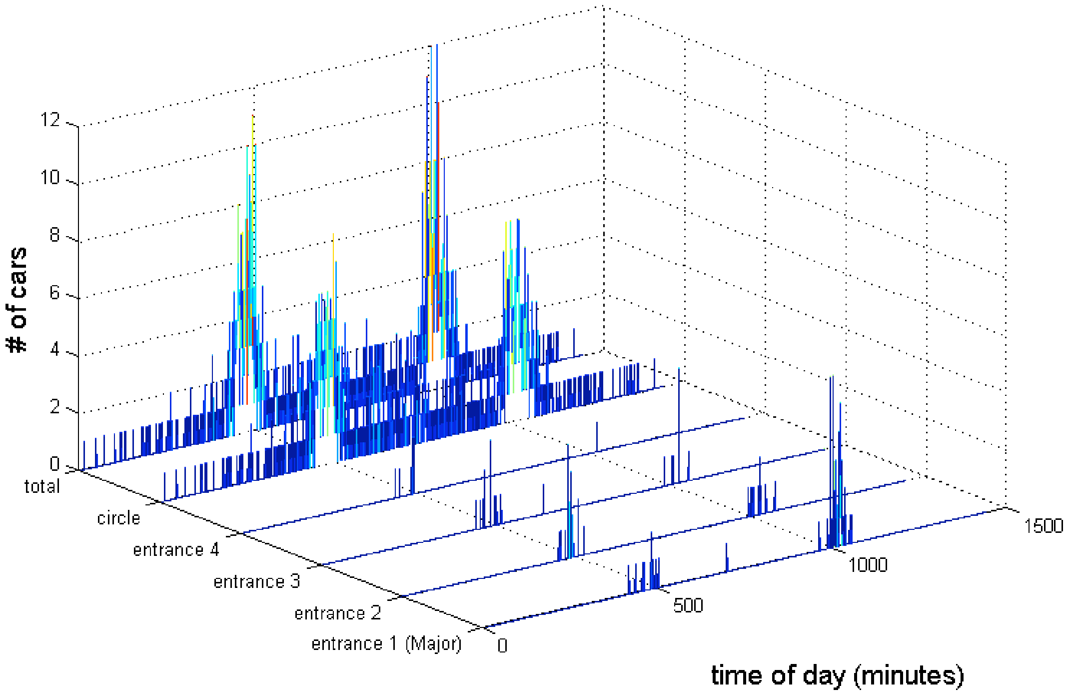


(b) Non-yield flow control.

Figure 5. Car density comparison of yield vs. non-yield when all entrances have similar input rates.

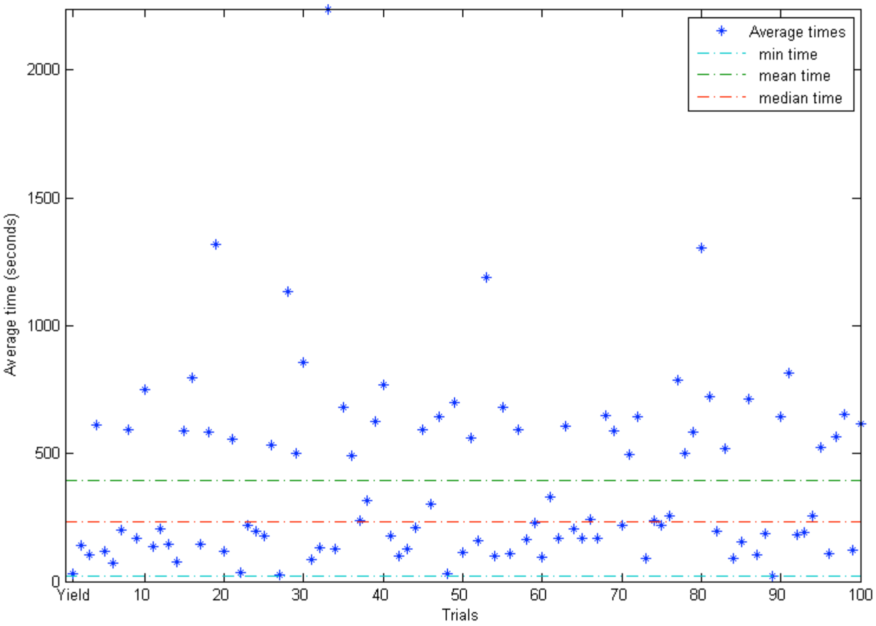


(a) Two major input rates.



(b) One major input rate.

Figure 6. Car density for yield flow-control.



(a) 100 trials.

Figure 7. Average time spent in system with random traffic-signal matrices, for 100 trials.

the number of cars that can fit in the circle also increases, so fewer cars should be waiting in the entrance queues. Thus, for large total input, a larger circle should perform better. Of course, as we increase the radius, the circle becomes like a very long one-way street that curves; cars take a long time to pass through simply because they drive farther. Larger circles cost more and demand more space, so we wish to find an optimum radius.

However, we cannot use our simulation to find an exact relation between total input rate and optimum radius. Simulation results may demonstrate typical behaviors, yet the nature of random simulation prevents establishing an exact function of optimum radius in terms of radius.

Using the yield matrix and two major streets with two minor streets, we ran a set of trials varying total input rate and circle radius. In **Figure 8**, the flat plane represents "well-behaved" systems where larger radii produce lower average time in a one-lane roundabout. Also, for fixed radius, the average time spent in the system increases with total input.

What is most interesting in the plot is the rapid change in behavior after the total input rate goes above 3,000 cars/hr (just under one car per second). We expect more delays as more cars try to enter the system, but we also expect larger radii to decrease the delays. With total input 4,000 cars/hr, a circle with radius 35 m performs better than one of 30 m, as expected; but one of radius 40 m performs worse than both, which is entirely unexpected. Thus, we conclude that for total peak flow of less than 3,000

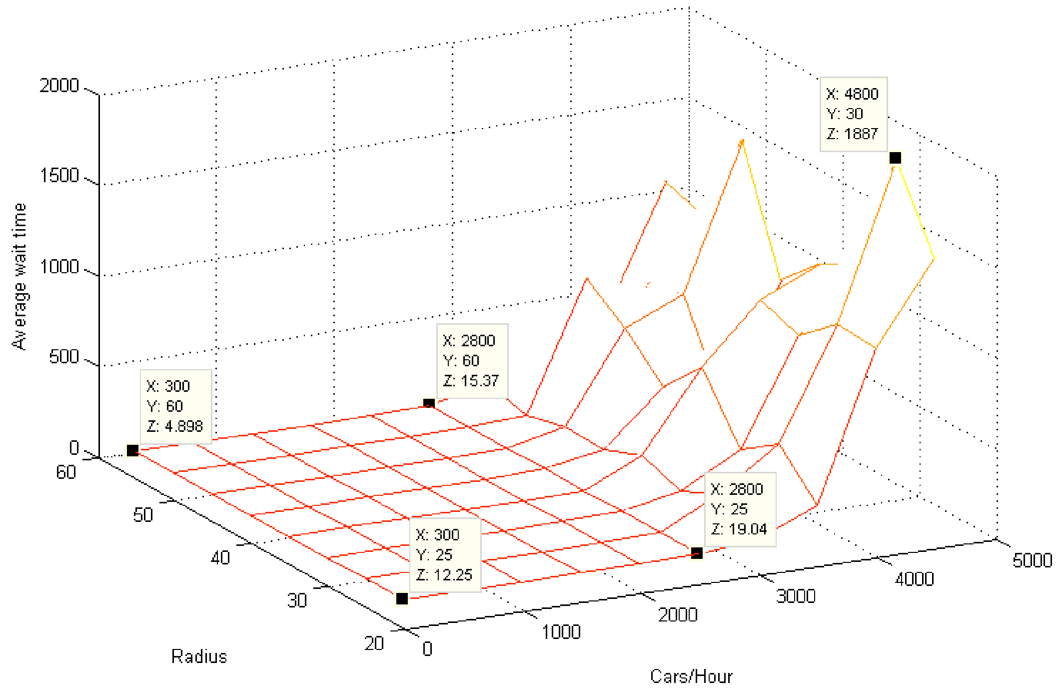


Figure 8. Average time spent in system for various input rates and circle radii.

cars/hr, increasing radius is directly correlated with decreasing average total time, but at higher flow rates, the correspondence becomes erratic.

This unexpected behavior reveals the limitations of our model. A single-lane roundabout with four entrances cannot handle grossly inflated input, regardless of size.

Strengths and Weaknesses

Analytic Model

The analytic model is limited in many ways. We compromise many kinds of complexity to formulate a closed-form stationary distribution; but in the end, the sheer variety of equivalent states that our system could take thwarts analysis. Our lower-bound calculations for the stationary distribution are pretty but provide a bound that is an order of magnitude less than the function itself. We can show that the lower bound grows with g for small k, but we do not prove that the overall shape of the lower bound always emulates the actual function. We do show that the two functions are behaviorally similar in two specific cases.

This model is useful as a basis for our computer simulation and narrowing our search for effective control systems.

Computer Simulation

The computer simulation copes with many of the limitations of the analytic model. It introduces time-dependent flow, limits the capacity of the roundabout, and more directly simulates the action of a traffic light as a discrete system rather than as a time-averaged parameter. This formulation allows us to explore a wide range of parameters beyond the convergence constraints of the analytic model.

The computer simulation is limited by the vastness of the parameter space. We could not implement an optimal signal-matrix search because determining the functional value of a signal matrix is computationally intensive and because the dimensionality of the variable space is so large. The independent variable space for a signal matrix for a 4-entrance roundabout has 16 dimensions, and the simulation uses 3 different signal matrices in every run.

The analytic model was useful, therefore, in restricting our search to signal matrices close to the yield matrix. We ran hundreds of trials with randomly-generated signal matrices containing no more than three 0s per row. Within this search space, the yield matrix performed better in the vast majority of cases. Thus, our simulation confirms that compared to yield signs, traffic signals have at best comparable efficacy.

The simulation is limited in scope. It does not account for pedestrian traffic, driver mistakes or accidents , weather conditions, or other factors. It is also limited to one-lane roundabouts. As **Figure 8** shows, flow rates in excess of 2,500 cars/hr clog the roundabout, regardless of input control. To some extent, increasing flow can be mitigated by increasing roundabout radius; however, for flow rates in excess of 3,000 cars/hr, a two-lane roundabout is necessary. A simple case of this would be a roundabout with outer "express" lanes from which a vehicle can travel only from one entrance to the next exit. In this case, traffic signals would always impair flow, because the "express" lanes are always vacant for an entering vehicle.

Conclusion

Our search through literature, parameter space, and computer-generated experimental results bring us to a conclusion validated in intersections across the U.S.: *yield-sign control is nearly always the best way to regulate roundabout entry*.

References

Arizona Department of Transportation. n.d. Modern roundabouts. `http://www.azdot.gov/CCPartnerships/Roundabouts/index.asp`.

Bouchouch, A., Y. Frein, and Y. Dallery. 1992. A decomposition method for the analysis of tandem queing networks with blocking before service. In *Queueing Networks with Finite Capacity: Proceedings of the Second International Conference on Queueing Networks with Finite Capacity, May 28–29, 1992*, edited by by Raif O. Onvural and Ian F. Akyildiz, 97–112. Amsterdam, Holland: North-Holland.

Durrett, Rick. 1999. *Essentials of Stochastic Processes*. New York: Springer.

McLawhorn, Nina. 2002. Roundabouts and public acceptance. `http://on.dot.wi.gov/wisdotresearch/database/tsrs/tsrroundabouts.pdf` .

Medhi, J. 2003. *Stochastic Models in Queueing Theory*. San Diego, CA: Academic Press.

National Cooperative Highway Research Program. 1998. *Modern Roundabout Practice in the United States*. Washington, DC: National Academy Press.

Robinson, Bruce W., et al. 2000. *Roundabouts: An Informational Guide*. Washington, DC: Federal Highway Administration, U.S. Dept. of Transportation. `http://www.tfhrc.gov/safety/00068.htm`, June 2000.

Ross, Sheldon. 2006. *A First Course in Probability*. 7th ed. Englewood Cliffs, NJ: Prentice Hall.

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Judge's Commentary: The Outstanding Traffic Circle Papers

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Overview of the Problem

Teams who decided to explore the "A" problem in this year's Mathematical Contest in Modeling examined ways to control the movement of vehicles in a traffic circle. A broad overview of the criteria developed by the judges and the experiences of the judges is given.

In the following section, a brief overview of the problem statement is explored. Next, an overview of the judging itself is given. In the subsequent section, a list of some of the common approaches adopted by the teams is given. Finally, a list of some of the common themes and more detailed points that emerged as the judging proceeded is given.

Traffic Circles

The focus on the "A" problem is to control the movement of vehicles in a traffic circle. A number of controls are explicitly given in the problem statement. The teams who submitted papers for this problem mainly focused on the given controls and very few examined other types of controls.

The problem statement includes two requirements. First, the teams were asked to find a way to control the flow of traffic in an optimal way. Second, the teams were asked to write a summary of their findings. These two aspects are explored individually in the subsections that follow.

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The Goal

The goal for this problem is to find a way to move vehicles through a traffic circle in an optimal way. This was stated in the second paragraph of the problem statement :

> The goal of this problem is to use a model to determine how best control traffic flow in, around, and out of a circle.

It is not clear what "best" means. It was left open for the teams to decide what "best" means. The teams were required to make it clear in their report how they interpreted this part of the problem:

> State clearly the objective(s) you use in your model for making the optimal choice as well as the factors that affect this choice.

The judges expected the teams to clearly describe the objectives, and we expected that the subsequent evaluation of the model be consistent with the stated objectives. This can be difficult for the teams to achieve given the dynamic of writing as a team, the nature of how approaches evolve as the problem is explored, and the intense time pressure. Teams that managed to maintain a high level of consistency tended to elicit a more-positive response from the judges.

Technical Summary

An essential requirement was to write a technical summary. The requirements for the technical summary were given in the problem statement. This was a difficult aspect to the problem. The teams were expected to provide a broad set of guidelines for a traffic engineer in a brief note.

The traffic engineer should be able to read the summary and have a strong sense of the different methods available. Additionally, the different circumstances that impact the decision should also be included. Examples of important parameters are the radius or geometry of the circle, the rate of flow of traffic coming into the circle, and the density of traffic coming into the circle. Very few teams considered the traffic capacity of roads leaving the circle, and most assumed that the incoming traffic was a primary limiting factor.

The traffic engineer is also expected to obtain a broad understanding of the conditions for which the model is applicable. This implies that the engineer should be able to read the summary and obtain a basic understanding of how the model was developed and an understanding of the potential pitfalls.

Writing the summary was a difficult task for the teams. The teams had a diverse amount of information to convey in two pages. The teams that managed to convey a sense of the basic models, the underlying assumptions, and the limitations of their models tended to make a stronger impression.

Grading Process

First, a brief overview of the evaluation process is given. The papers are evaluated in three stages. There is an initial round where the focus is on which papers to remove from the pool. The second, or screening round, focuses on which papers meet the minimal requirements for an advanced score. In the final round, the judges focus on which papers meet the highest standards.

Initial Grading

The initial round is designed to remove papers from the pool that are not likely to meet the standards in the following round. Each paper is read by at least two people. Papers that receive consistent low scores are not passed on to the next round. Papers with mixed reviews are read by more people. When the reviewers are unsure, they try to err on the side of caution and pass the paper on to the next round.

It is absolutely essential that a paper be well-written and have a clear, concise summary to make it past the initial round. A paper that does not provide a clear overview including results and a synopsis of the techniques used will not make a strong impression on the judges. The summary and the rest of the paper must also be consistent. Differences between the summary and the following pages can be immediately apparent and do not make a positive impression of the paper.

Screening Rounds

As the judges examine papers in the next set of rounds, they try to decide if the paper meets the minimal requirements to do well in the following rounds. The number of times that a paper is read in these rounds varies from year to year. Again the judges try to err on the side of caution; but as the rounds proceed, the criteria for doing well becomes increasingly stringent.

It is still important to have a strong summary, but the need for consistency across the whole paper is more important. The need for proper citations and correct grammar is also important. This year, a large body of literature was available for the teams. It was even more important than usual to include proper citations and make clear what work was done by the team and what work was found in the literature search.

Final Rounds

In the final rounds of judging, the focus is on finding the best submission. At this point, each paper is read many times, and more time is available for each reading. The judges are able to focus more on each individual step and focus on consistency across the whole paper. The papers that remain in these final stages must maintain high scores to move forward.

Approaches

The flow of traffic in roundabouts is an active research area. The available literature influenced many of the teams. Most teams used either a deterministic approach or a stochastic approach. Here we examine each of these approaches separately.

Deterministic

The teams that adopted a deterministic approach tended to make greater use of models based on partial differential equations. There are a variety of different conservation laws that have been derived to model traffic flow. Such models tend to focus on relatively simple traffic geometries and require considerable adaptations to model a traffic circle.

At first glance, a conservation law for a traffic circle seems to avoid the issues associated with boundary conditions because it is a periodic geometry. Unfortunately, the exits and entrances of the feeder roads create other difficulties. Adapting models to include the exits and entrances occupied the majority of the modeling efforts.

The second difficulty with this approach is to find an approximation to the solution. The equilibrium solutions to the equations are piecewise-constant functions, and the conversation law gives rise to shocks. Given the complex boundaries, the method of characteristics is complicated, and the numerical approximations can be daunting since the techniques must account for upwinding.

Stochastic

The majority of teams used a stochastic approach. In general, they examined either queues or networks, and a common approach was to use a hybrid model combining the two. A typical paper included an overview of the model, some theoretical results for a simple situation, and results for a computational model.

Teams adopting this approach were expected to use proper citations because of the wide body of work available. The judges also paid more attention to the consistency across the whole paper. The summary, model, results, and discussion had to be consistent.

Another issue that emerged with some papers is the disconnect between the section in the paper discussing the theory and that with the numerical simulations. Many of the top-rated papers provided some theoretical results for simplistic geometries or simulations. The majority of these went on to include the results of numerical simulations for the more complicated cases. The few teams that provided a confirmation of the numerical model on a simple geometry made an immediate positive impression.

The other issue is how to report the results of simulations in a coherent manner. The development of the model requires a probabilistic approach. The analysis of the numerical trials requires a shift to a statistical approach. The majority of teams simply reported means and sometimes standard deviations. Few teams reported results using qualitative methods such as boxplots or histograms, and even fewer teams made use of appropriate quantitative statistical methods.

Finally, when designing the numerical trials, few teams examined a range of values for the parameters in their models. Every year, the judges rate this aspect of the problem as a crucial part of the problem. We expect to see an exploration of the results given small changes in parameters or assumptions. The few teams that did examine this aspect immediately caught the judges attention.

Common Themes

In the previous section, some observations specific to this year's competition are given. Some general observations that come up every year are explored here.

Summary

The summary is an important part of the team's entry. It is the first thing that a judge will read. The summary is the first impression. It is vital that a paper have a complete and well-written summary to make it past the initial rounds. It is also vital that the details in the summary be consistent with the rest of the paper.

Writing a one-page summary of the team's efforts is a difficult task. The teams are expected to provide a brief overview of the problem. They are then expected to let the reader know their specific conclusions and recommendations. Finally, the teams are expected to provide the reader with an overview of the approach that they used.

It is difficult to include all three of these parts within the one-page summary. Many teams find it tempting to include a large amount of background information or provide clever narratives motivating the problem. Unfortunately, such material in the summary can drastically reduce the amount of space available to discuss the team's results and discussion of the approach that they adopted.

Grammar, Punctuation, and Equations

The presentation of the team's model and results cannot be separated from the model itself. A team must have a reasonable model including a basic analysis of the model. The teams are expected to then share their results in a clear and concise discussion.

Teams that do not make use of proper grammar and punctuation are not likely to make it past the initial rounds of the competition. Teams must know how to include equations in their writing and use proper punctuation. Advisers should not take it for granted that their students know how to do these things.

Proper Citations

The judges expect every entry to include proper citations. Many teams are comfortable exploring the resources available to them, and it is unusual to come across an entry with a unique approach. The different types of approaches can be easily categorized, and the judges quickly figure out the sources available for each approach.

Sensitivity and Stability

Sensitivity and stability are always important. The few teams that make a concerted effort to explore this aspect of their model will almost always stand out. Exploration of the sensitivity of a model can be as simple as testing what happens for a different range of values in a parameter, and it can include the use of more sophisticated methodologies such as an exploration of a sensitivity matrix.

Every year, teams are able to implement nontrivial numerical simulations. The teams must make decisions about what numerical trials to examine. It is extremely rare for teams to scale a problem as a way to decide the combination of parameters that are important.

Figures and Tables

The integration of graphs and tables into a paper is a challenge for many teams. It is not uncommon to see entries in which figures and tables are included with no detailed discussion of them. The teams need to integrate the figures and tables into their discussion.

Given the increased use of simulations and numerical results it is vital that the teams find a way to include descriptions of their figures and tables into their narrative. The teams need to make sure to let their readers know the key aspects of their figures and tables and inform their readers how to look at the figures and tables.

Consistency Across the Paper

The teams have a limited time to understand the problem, derive a mathematical description of the problem, perform the requisite analysis of their model, and then come back and interpret their work with respect to the original context. Over the course of the weekend, teams make decisions and explore a variety of different approaches. The time constraints make it extremely difficult to complete a paper in which the wide array of assumptions and analyses are consistent across the whole paper.

Conclusions

A team's submission must satisfy a wide array of criteria to be successful and proceed through each stage of the judging. The presentation and grammar are vital aspects of a submission. The team's results are given through the filter of the team's writing.

The team must provide a strong analysis. The teams only have four days, and the judges do not expect extensive and sophisticated models. A careful analysis of the resulting model is required, though.

Each year, the expectations are different, but there are a few constants. For example, a clear discussion of the basic assumptions—with some justification, citations, and a discussion of the implications—is necessary. Additionally, judges always expect a focused discussion on stability and sensitivity.

In this year's competition, the use of simulation was a part of the majority of entries. Incorporating an analysis of simulations is a difficult task, and the top entries did a remarkable job of integrating the development and analysis of their model with the discussion of the results of their numerical trials.

Teams that were able to tie together the theoretical analysis of their model along with their numerical trials received immediate positive recognition. The best entries were able to develop multiple models of varying complexity and verify their numerical models with the theoretical results of the simpler models.

About the Author

Kelly Black is a faculty member in the Dept. of Mathematics and Computer Science at Clarkson University. He received his undergraduate degree in Mathematics and Computer Science from Rose-Hulman Institute of Technology and his Master's and a Ph.D. from the Applied Mathematics program at Brown University. He has wide-ranging research interests, including laser simulations, ecology, and spectral methods for the approximation of partial differential equations.

Mobile to Mobil: The Primary Energy Costs for Cellular and Landline Telephones

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Summary

We determine that cellphones are the optimal communication choice from an energy perspective, using a comprehensive analysis based on multiple factors. We split phones into three categories: cellular, cordless landline, and corded landline. We average the energy used in manufacture and transportation over the life of each phone. To account for the inefficiency of production, we calculate in terms of primary energy, which is the amount of fuel supplied to a power plant per unit of energy produced for consumption. We use real-world data for population, number of mainlines, and cellphone subscriptions.

During the transition, as cellphones overtake landlines, part of the population owns both types of phone. As a result, the total energy used by telephones increases. We fit a competing-species model to past statistics; it forecasts that the net energy cost of the cellphone revolution (1995–2025) in the U.S. will be 84 TWh. At the start of this period, there were 0.1 cellphones per capita; at the end there will be 0.1 landlines per capita. Energy savings will begin in 2022. After this transition, savings will be 30 GWh/d. The competing-species model is a proven technique; we apply it to telephone lines and cellphones per capita, and also use it in conjunction with population projections to develop a closed-form solution.

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The most energy-efficient way to provide phone service in a country with no existing infrastructure is to construct a cellular network. By amortizing the fixed setup costs over the lifetime of the phone system, the energy used during construction is negligible. For a country similar to the U.S., the annual savings would be 12 TWh. Over the next 50 years, the energy savings would equal 0.5 billion barrels of oil.

Cellphone chargers waste energy, but the total energy wasted would be almost five times as great if everyone instead used a cordless phone. Continuing advances in charger technology are reducing charger waste. If all cellphone chargers in the future meet a 5-star Energy Star rating, they will be 10 times as efficient as now.

Our model is supported by historical data and numerous publicly available statistics. One factor not accounted for is the maintenance and operating power required for cell towers and physical telephone lines.

Introduction

Over the past 15 years, cellphone subscriptions in the U.S. have increased dramatically. At the same time, growing concerns over oil supplies have increased public consciousness of energy efficiency. We compare the energy use of cellphones to that of traditional landlines. Major factors include:

- power used while charging,
- power used while idle,
- time charging each day,
- time idle each day,
- energy to manufacture and transport the phone,
- lifespan of the phone, and
- total number of phones.

These values, many of which depend on the type of telephone, allow for a comprehensive analysis of the energy consequences of the cellphone revolution. Our model quantifies the effects of cellular and landline telephones on power consumption.

Assumptions

- Cellphones and landline phones compete for the same market.
- Residential, commercial, nonprofit, and government telephones are included in the total number of phones.

- The total number of phones is averaged by household.
- Every cellphone comes with a charger and lithium-ion battery [1].
- A cellphone's battery will not be replaced but discarded with the phone.
- Overcharging or undercharging a lithium-ion battery does not affect its life or performance [6].
- Nickel-hydride batteries are used in cordless phones [7].
- The total energy used in manufacturing a landline phone is half that of manufacturing a cellphone.
- A person may own more than one telephone.
- In a household with cellphones, each of its m members has their own cellphone.
- Every person within the population is part of a household.
- A charger is any item used to recharge batteries, including those within electronic devices such as laptop computers, cellphones, and cordless phones. Appliances such as televisions, refrigerators, and microwaves are not included, since they are not rechargeable devices.
- The fixed energy required to construct telephone infrastructure, when averaged over the duration of the phone system, is negligible.

Important Variables

- H, the number of households in the country;
- Z_{cell}, the number of cellphones per hundred people;
- Z_{landline}, the number of landlines per hundred people;
- N_{cell}, the number of cellphones;
- N_{landline}, the number of landline phones;
- population;
- power drawn by each type of phone when idle;
- power drawn by each type of phone when charging or active;
- $W_p = 3.0128$, ratio of primary energy input at a power plant to energy drawn off the grid [9];
- on average, a cellphone's battery must charge for one hour a day [2];
- 75% of landline phones are cordless and 25% are corded;
- the average lifespan of a corded landline phone is 20 years;

- the average lifespan of a cordless landline phone is 10 years;
- cellphones last 1.5 years [3,4], whereas lithium-ion batteries and chargers last 3–4 years [5];
- each landline connection has an average of m phones connected to it; and
- $m = 2.37$ members in the average household [8].

Part 1: Existing Infrastructure

Transition

The U.S. has a mixture of cellphones and traditional landline phones. Currently, 84% of the population has a cellphone subscription, with 16% of U.S. households owning only cellphones [10]. The U.S. is currently in transition from exclusive use of landlines to exclusive use of cellphones. During this transition, cellphones and landline phones compete for consumers. The target market is the entire population, which grows over time. As such, the number of cellphones and landlines per hundred people is time-dependent, as seen in **Figure 1**.

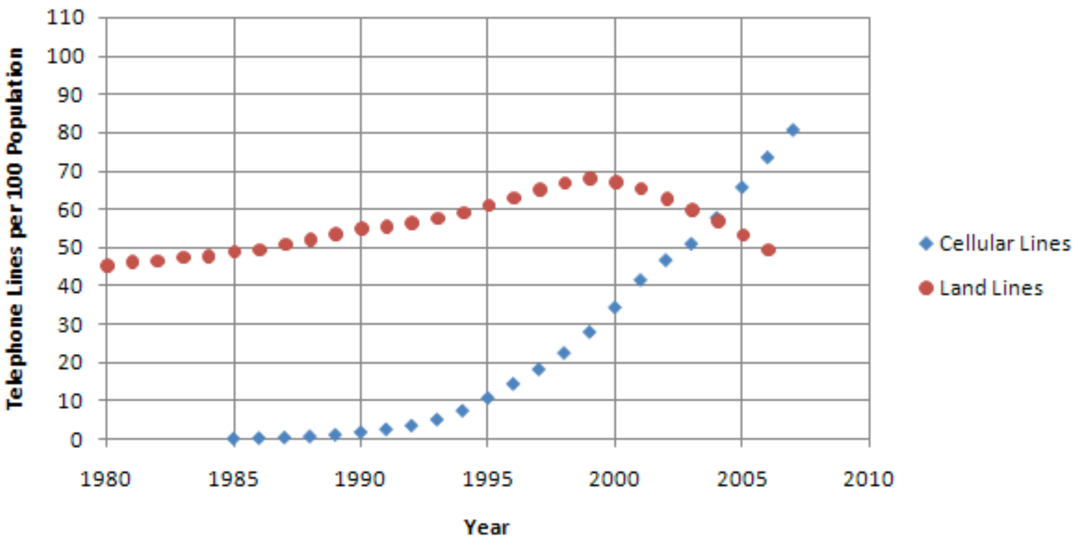

Figure 1. Historical data for phone ownership in the U.S.

As cellphones became popular, the number of landlines decreased. This suggests the data can be described with a differential competing-species model [11, 12, 13]. The competing-species model describes two species that both require a single finite resource and impede each other from acquiring it. The system of equations for this model is

$$\frac{dx}{dt} = x(a_1 - b_1 x - c_1 y), \qquad \frac{dy}{dt} = y(a_2 - b_2 y - c_2 x),$$

which cannot be solved analytically. In these equations, x is the population of one species, y is the population of the other species, and a_1 and a_2 are the unconstrained growth rates of the populations. The ratios a_1/b_1 and a_2/b_2 are the maximum populations for each species. The coefficients c_1 and c_2 are competition factors accounting for the negative effect that each species has on the growth of the other.

For our purposes, the two species are cellphones and landlines. The resource is market saturation among the proportion of the population of the U.S. willing to purchase phones. When total phone ownership exceeds the equilibrium value, one of the two types of phone will have to die out, or become obsolete. The competition model can be applied by taking Z_{cell} as the number of cellphones per hundred people and Z_{landline} as the number of landlines per hundred people. We determined appropriate coefficients for this model graphically by solving the equations numerically in Matlab [14]. The results are

$$\frac{dZ_{\text{cell}}}{dt} = Z_{\text{cell}}\left[0.315 - \left(\frac{0.315}{110}\right) Z_{\text{cell}} - \left(4.77 \times 10^{-4}\right) Z_{\text{landline}}\right],$$
$$\frac{dZ_{\text{landline}}}{dt} = Z_{\text{landline}}\left[0.21 - \left(\frac{0.21}{110}\right) Z_{\text{landline}} - \left(2.50 \times 10^{-3}\right) Z_{\text{cell}}\right].$$

With these coefficients, the model fits the historical data accurately from 1995, when cellphones reached a penetration level of 10 per hundred people, to 2006, the last year when data for both phone types was available. The graphical solution and its projection through 2030 appears in **Figure 2**. This model predicts that the market will support up to 1.1 cellphones per capita, or up to 1.0 landlines per capita. Based on an average of 2.37 telephones connected to each landline, there is a maximum of 2.37 landline phones per person. Included in these numbers are residential, commercial, nonprofit, and government owned phones. The "Cellphone Revolution" is taken as the time period from 1995, when cellphones first reached a saturation of 0.1 per capita, through 2025, when landlines drop below a saturation of 0.1 per capita.

Using this model, the total number of cellphones and the total number of landline telephones can be predicted for any future year. These numbers are used to determine the energy requirements in terms of gigawatt-hours per day (GWh/day). The power needed for each type of phone is in **Table 1**. For our purposes, a cordless phone is a landline telephone with either batteries or electronics, which draws constant power from the electrical grid. A corded phone gets all of its power from the telephone line. The energy to manufacture, ship, and dispose of a cellphone equals 180 MJ, or 50 kWh.

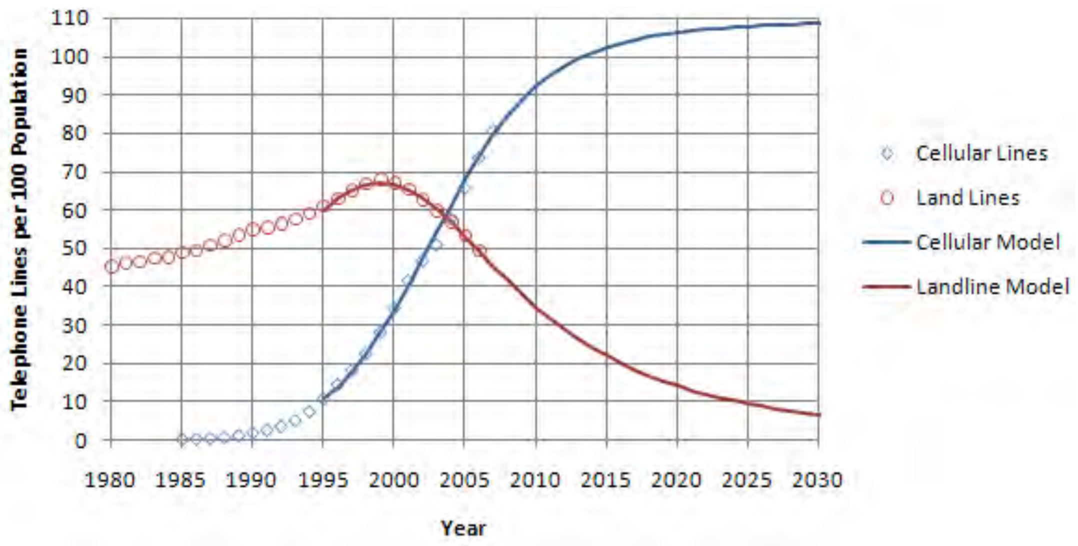

Figure 2. Historical data with projections from the competition model.

All power quantities are listed in terms of rate of primary energy use, which accounts for the fact that for every watt-hour drawn from the grid, 3.0128 watt-hours worth of fuel were used to produce it [9]. In this way, we account for inefficiencies in the power generation and distribution systems [15]. The three cellphone types listed correspond to the power required for their chargers when idle.

Table 1.

Primary power levels.

	Idle (W)	Active (W)	Active hours (h)	Fixed (Wh)	Life (d)	Daily energy (Wh)
Corded phone	0	0.452	1	25000	7200	3.92
Cordless phone	5.12	10.24	2	25000	3600	140.11
Cellphone (avg)	0.904	15.06	1	50000	540	128.44
Cellphone (new)	0.301	15.06	1	50000	540	114.59
Cellphone (5-star)	0.090	15.06	1	50000	540	109.74

The number of active hours corresponds to call time for corded phones, charging time for cellphones, and charging time for cordless phones. The manufacturing energy for a landline phone is assumed to be half that of a cellular phone, due to less-complex circuitry. A cordless phone has half the life of a corded phone, because it is more likely to get lost or broken. The final column of **Table 1**, showing lifetime average power per device in Wh/d, is calculated using

$$P = \frac{(\text{IdleWatts})(24 - \text{HoursActive}) + (\text{ActiveWatts})(\text{HoursActive}) + \text{Fixed}}{\text{Life}}.$$

The daily energy use for all phones is calculated using

$$\text{DailyEnergy} = \left(P_{\text{CellAvg}}\right)\left(N_{\text{cell}}\right) + \left[0.75\left(E_{\text{cordless}}\right) + 0.25\left(E_{\text{corded}}\right)\right]\left(N_{\text{landline}}\right).$$

There are 271,856,247 cellphones and 276,867,152 landline phones [20], meaning that the U.S. uses 64.3 GWh/d for telephones. **Figure 3** shows the total power produced for telephones during the transition period from 1995 through 2030. A baseline conservatively projects what power levels would have been needed if cellphones had not become popular.

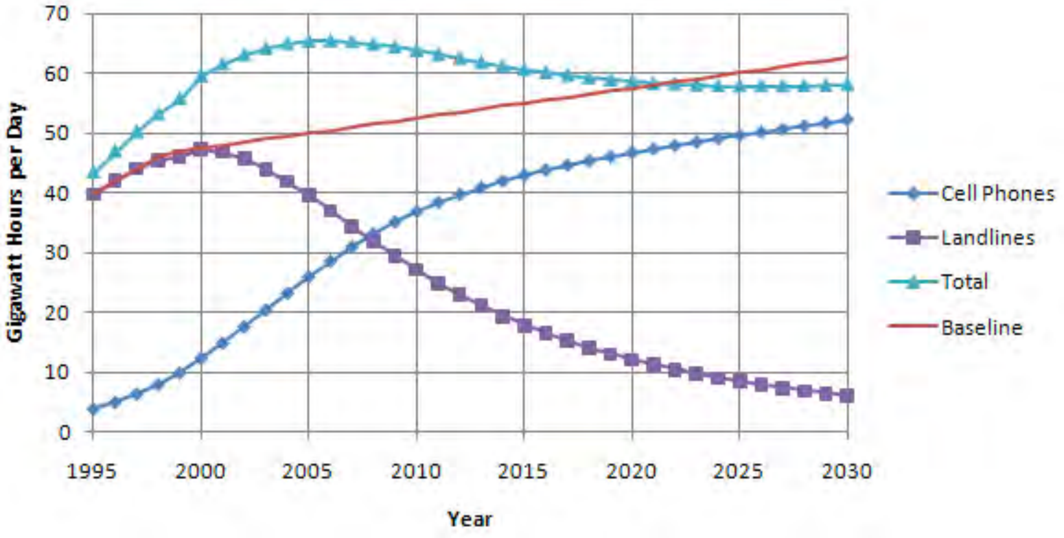

Figure 3. Energy for phones during transition period.

The power used by landlines begins to decline as cellphone power usage grows. The net change in power production during this transition is initially positive. After the year 2021, the transition state becomes more energy-efficient than the projected baseline, as seen in **Figure 4**. This occurs because cellphones require less primary energy per day than landlines.

Over the course of the transition period from 1995 to 2025, an additional 84 TWh of energy must be produced for telephones. However, starting in 2022, annual energy savings result.

Steady State

The steady state occurs when the entire market for telephones is satisfied. Based on the model of the transition period, this will include only cellphones. When that occurs, and the two types of phones are in equilibrium, the limiting value is 1.1 cellphones per person. We will have H = 126,316,181 households [8] with m = 2.37 members/household.

The total energy requirements for the steady state, based on the data in **Table 1**, are shown in **Table 2**. Energy-efficient chargers decrease the load.

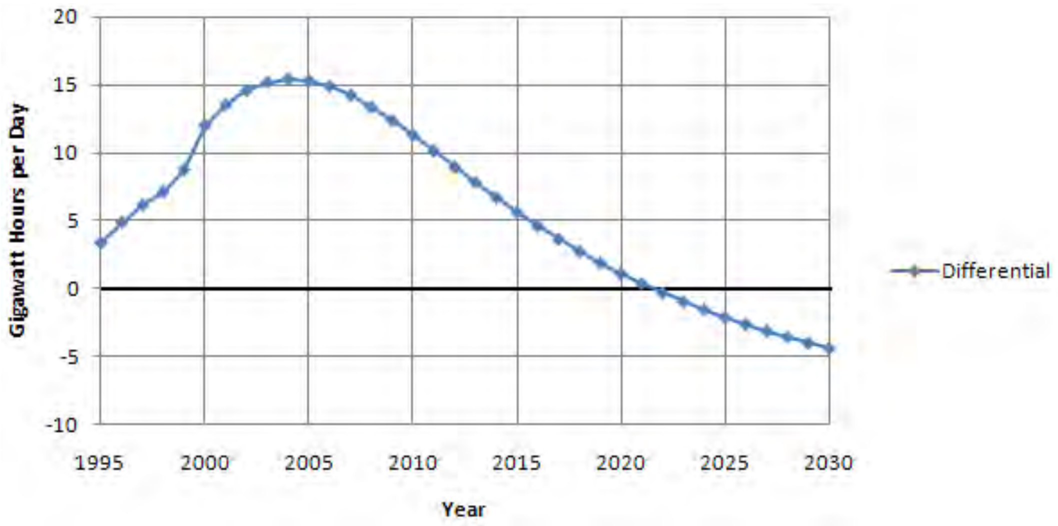

Figure 4. Difference in energy generated for phones during transition.

Table 2.
Energy requirements for steady state, by charger efficiency.

Device	Energy cost (GWh/d)
Cellphone (average)	42
Cellphone (new)	38
Cellphone (5-star)	36

Part 2: No Existing Infrastructure

Optimal State

To determine the optimal system for providing telephone service in a country roughly the same size as the U.S. but lacking existing communications infrastructure, we compare the power requirements of each type of phone. The fixed energy required to construct telephone infrastructure, averaged over the duration of the phone system, becomes negligible. The limiting values for landline and cellular phone penetration are 2.37 and 1.1 phones per person respectively, the same as in the U.S. The energy needed per day is the population multiplied by the phone penetration factor, and the energy per day per phone. **Figure 5** shows the projected power requirements for the country over time.

From these data, corded landline phones are the most energy-efficient, using about 3.2 GWh/d. However, universal use of corded phones is not a realistic scenario. When landline infrastructure is present, 75% of landline phone are assumed to be cordless, and 25% corded. Also, there are three levels of cellphone chargers to consider: the current average charger in the U.S., the more-efficient chargers currently being manufactured, and

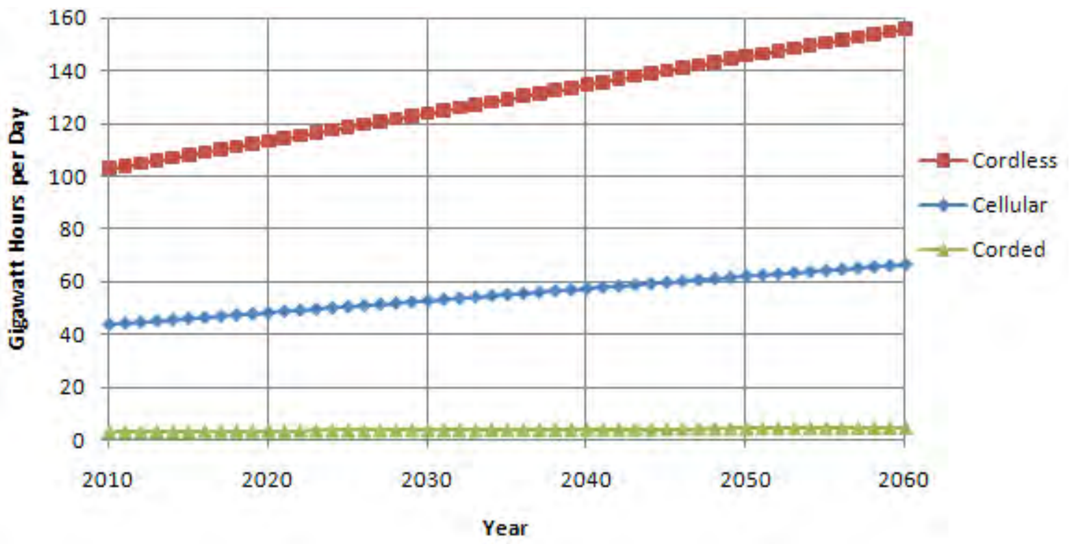

Figure 5. Phone energy need forecast for saturated market.

the energy-conserving 5-star chargers that are not yet common [21]. Calculating the energy use of these in the saturated market results in the power requirements shown in **Figure 6**.

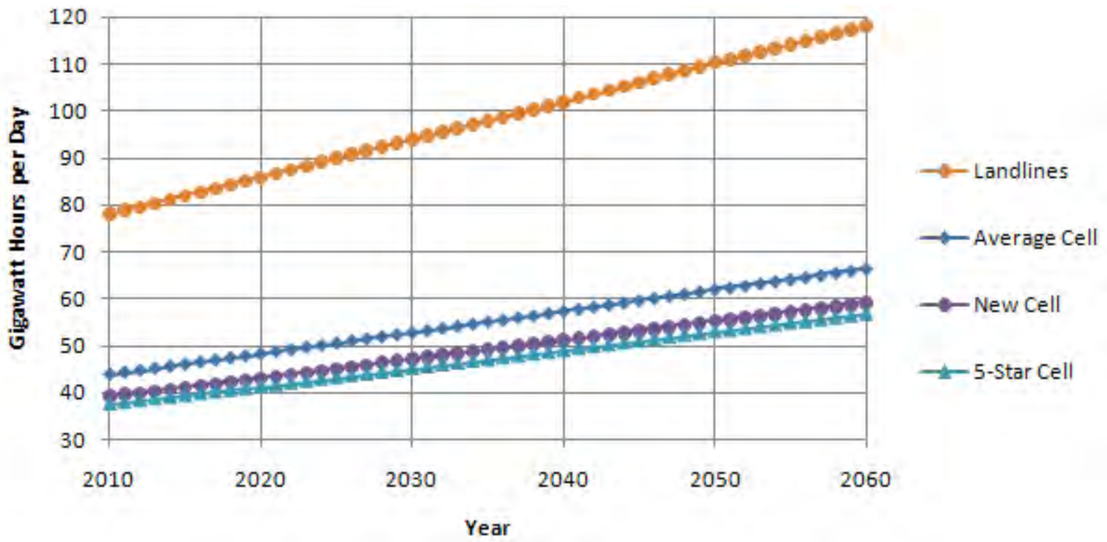

Figure 6. Realistic phone energy need forecast for saturated market.

From an energy perspective, it is most beneficial to create the infrastructure for a cellphone communication system. Passing legislation to decrease the amount of waste that chargers create could be used to make this state even more energy efficient.

Additional Factors

Outside of impacts on energy consumption, there are still numerous other factors that determine which type of phone will be favored by the general population:

- Cellphones provide greater mobility while increasing safety, especially while travelling alone or in a small group.
- Cellphones allow older children to have increased independence without putting themselves in danger [22].
- Impromptu scheduling changes and emergencies can be more easily handled with immediate communication available.
- Cellphones can make employees easier to reach; this could increase productivity by allowing employees to perform their jobs while not physically in the office.
- Cellphones are also used to replace watches, cameras, and alarm clocks, facts that may impact overall energy usage [23].

Cellphones also have negative consequences:

- It is suspected that cellphones contribute to brain cancer and tumors due to radiation from both cellphones and cell towers [24, 25].
- Cellphones can interrupt family life, straining relationships. Adults who use a cellphone for work sometimes let work interfere with family life, while children become attached to cellphones as a means of contacting peers, leading to more peer-based and fewer family-based activities [26].
- The nature of a cellphone can limit the ability to contact a group, such as a family. Instead of making a single call, it may be necessary to call each member separately, wasting time and effort, since there is no universal means of communication.
- Cellphone rings often interrupt family dinners, movies, classes, sporting events, and concerts, decreasing people's enjoyment of the experience; they are also a distraction to people while at work [27].
- Cellphones can increase response time for emergency vehicles, because a cellphone's position is much more difficult to locate than a landline's [28].
- Cellphones generally have higher prices, more expensive plans, and a shorter lifespan than landline phones [29].
- Cellphones are also more likely to be lost or stolen due to their transportable nature.
- Battery life is limited.

- With more cellphones in existence, there will likely be fewer pay phones or landlines in public places for use in emergencies.

Part 3: Effects of Charger Negligence

Often, cellphones are left overnight to charge; and in the morning when they are detached, the charger is left plugged into the wall, still drawing current. This practice wastes energy, since cellphones need to be charged for only a portion of the night. To determine the maximum amount of wasted energy by cellphone users in the U.S., we take into account both of these negligent practices in the equation below, where W_t is the total amount of wasted energy generated by cellphones through overcharging (W_o) and failing to unplug the charger when not in use (W_u):

$$W_t = W_p + W_u.$$

To quantify this, it is necessary to create models for both types of waste. The general format of the equation for any type of waste from a charger is

$$W = HCBW_pPhL.$$

The waste W is based on the number H of households, the average number C of chargers per household, the average amount of power P drawn during this time, and the hours h per day of wasteful practice. There are also conversion factors for the waste due to power plants (W_p), and the conversion of watts to barrels of oil. The value of L is 1.1 phones/person at the steady state.

To use this equation, the time that cellphone users waste must be calculated using the difference between the time charged and the charging time needed; the power also needs to be customized for this type of waste (P_v):

$$W_v = HCBW_pP_v\left(h_{\text{charging}} - h_{\text{needed}}\right)L.$$

The second form of waste can be modeled in a similar manner, using the number of hours that the charger is in the idle state (h_{idle}). The power consumption will also need to be specified (P_u).

$$W_u = HCBW_pP_uh_{\text{idle}}L.$$

This results in an overall model for the waste from cellphones in terms of barrels of oil:

$$W_c = HCBW_p\left[P_uh_{\text{idle}} + P_v\left(h_{\text{charging}} - h_{\text{needed}}\right)\right]L.$$

To calculate the total waste of cellphones, we use the values in **Table 3**, many of these values are reported data or calculated from reported data. The only values approximated are those for time per day spent charging.

Assuming that people leave their cellphones charging while they sleep, all cellphones would charge for approximately 8 hr/night, as noted above. As assumed earlier, each cellphone requires charging for only 1 hr/d. If the charger is left plugged in all of the remaining time, 16 hr/d is idle time. Using these values and assumptions, all of the cellphones within a country the size of the U.S. would waste the equivalent of 6,254 bbl/d of oil due to careless cellphone use.

Table 3.

Cellular charger waste components.

Factor	Value
H	126,316,181 households in U.S. [8]
C	2.37 cellphone chargers/household (one per person) [8]
B	1 barrel of oil /(1.6998×10^6 Wh) [30, 31]
W_p	3.0128 [9]
P_u	0.3 W [9, 17 ,18]
P_v	0.845 W [32]
h_{idle}	16 hr
h_{charging}	8 h
h_{needed}	1 hr
L	1.1 (cell), 2.37 (landline)

Part 4: Charger Negligence

Waste due to battery chargers applies to types of electronics beyond cellphones. We consider three types of chargers: cellphone chargers, cordless phone chargers, and other chargers, such as for laptops and MP3 players. Overall waste W_T is modeled as the sum for cellphones (W_c), cordless phones (W_l), and other types of chargers (W_o):

$$W_T = W_c + W_l + W_o.$$

The waste due to cellphones can be calculated as in Part 3. The waste due to cordless phone and other chargers is calculated similarly, although only waste due to the charger left idle should be accounted for:

$$W_l = HC_lBW_pP_lh_lL, \qquad W_o = HC_oBW_pP_oh_oL.$$

The values of power usage and hours left charging differ from those in the previous equations.

Overall waste can be calculated, in terms of barrels of oil, as

$$\begin{aligned} W_T &= HC_lBW_pP_lh_lL + HC_oBW_oP_oh_oL, \\ W_T &= HCBW_p\left[P_uh_{\text{idle}} + P_v\left(h_{\text{charging}} - h_{\text{needed}}\right)\right]L + HC_oBW_oP_oh_oL. \end{aligned}$$

The first equation corresponds to waste in a cordless-phone-dominant state, while the second models waste in a cellphone-dominant-state.

Relevant values are in **Table 4**, where a few values are reasoned. For instance, the number of hours that a cordless phone would be idle is based on the assumption that a phone charges 2 hr/d, is used 1 hr/d, so is idle 21 hr/d [2]. In addition, the amount of energy drawn by all other chargers is assumed to be constant and approximately the same as for an average cellphone charger (0.3 W) [9, 17, 18]. The number of chargers per household is the product of the average number of people in the household and an approximation for the number of chargers present and used within the household.

Table 4.

Charger waste components.

Factor	Value
H	126,316,181 households in U.S. [8]
B	1 bbl oil (1.6998×10^6 Wh) [30,31]
W_p	3.0128 [9]
C_l	2.37 cordless phone chargers/household [8]
P_l	1.7 W [16]
h_l	21 hr/d
C_o	3.318 chargers/household
P_o	0.3 W
h_o	16 hr
C	2.37 chargers/household (one per person) [8]
P_u	0.3 W [9, 17 ,18]
h_{idle}	16 hr
P_v	0.845 W [32]
h_{charging}	8 hr
h_{needed}	1 hr
L	1.1 (cell), 2.37 (landlines)

These values give an average waste of 49,000 bbl/d of oil for all chargers in a cordless-phone-dominated U.S., compared to 10,000 bbl/d of oil in a cellphone-dominated U.S., as shown in **Table 5**.

Table 5.

Charger waste.

Charger type	bbl/d of oil $\times 10^3$
Cellphone	6.3
Cordless phone	45.9
Other	3.7
Total (cellphone state)	10.0
Total (cordless state)	48.6

Part 5: Economic and Population Growth

Based on the assumption that all m members of H households within the Pseudo U.S. have phones, the changes in economic status will not affect the total energy used by phones. To determine the energy usage projected over the next 50 years, we model the population of the Pseudo U.S. as equal the population of the actual United States. Based on data from the U.S. Census Bureau [33, 34, 35, 36, 37, 38, 39, 40], we create a regression describing population, $T_{\text{population}}$, as a function of time, X_{year}, as seen in **Figure 7** expressed by

$$T_{\text{population}} = 3216980X_{\text{year}} - 6156752732.$$

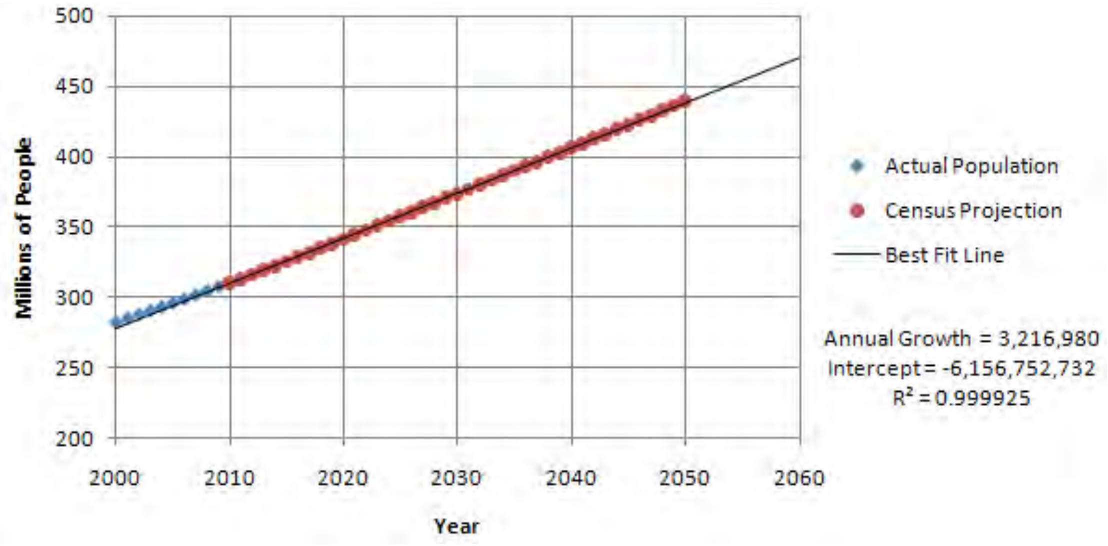

Figure 7. U.S. population.

This model matches the Census Bureau's predictions and can be used in conjunction with the energy equations developed in Part 1 to determine the total energy used by the Pseudo U.S. at any given time. To find the total energy used over each 10-year period, we integrate the population function from $X_{\text{year } n}$ to $X_{\text{year } (n+10)}$ and multiply by E_{phone}:

$$E_{\text{used}} = 365 \left(E_{\text{phone}} \int_{X_{\text{year n}}}^{X_{\text{year (n+10)}}} T_{\text{population}} \, dX_{\text{year}} \right).$$

Under the optimal scenario where cellphones with 5-star chargers saturate the market, the energy used for phone service each decade is listed in **Table 6**.

The total number of barrels of oil that must be provided to power plants over the next 50 years for this scenario is 503 million.

Table 6.

Total phone energy per decade.

Decade	Energy (bbl of oil $\times 10^6$)
2010s	84
20s	92
30s	101
40s	109
50s	117
Total	503

Analysis of the Model

Verification

To verify this model, one would have to obtain data for the next 10 years and compare the actual results to the predicted. Historical data cannot be used to verify this model, because data regarding the consumption of energy due to phone usage is not readily available. It was possible to verify the competition model for the data graphically. Most of the statistics used could be verified through additional research.

Strengths

- **Simplicity:** This model is simple enough that it entails a small amount of mathematical skill to operate. In addition, it is easily converted into an electronic form, such as Microsoft Excel or Matlab, and can therefore be visually displayed so that nearly no mathematical knowledge is necessary to understand the model.
- **Developed from historical data:** Population trends and the competition model were based off of real data from the Census Bureau and the CTIA Wireless Association.
- **Extendable:** To include additional factors, the model could be extended by additional terms with little impact on the functionality in the energy equations.
- **Flexible:** The equations used in this problem could be applied to other competing products that use energy.
- **Closed-form solution:** With the appropriate data, this model will generate numerical and graphical solutions.
- **Calculation time:** Due to the simplicity of the calculations, this model can be solved in a relatively short amount of time.

- **Includes variations:** This model accounts for cell, corded, and cordless phone usage, as well as a combination of the three. This allows for a more complete analysis.
- **Considers outside factors:** This study considers the implications of mobility and convenience for a realistic approach to energy efficiency.
- **Energy production costs:** The costs in the model take into account the inefficiencies of power generation by looking at total energy produced to render the energy consumed.

Weaknesses

- **Forecasting:** The model does not account for any changes in technology over the time period.
- **Infrastructure costs:** The initial infrastructure cost was assumed to defray to zero over time in order to decrease the number of inputs needed for the model. In reality these costs could potentially have an effect, especially in the short term.
- **Infrastructure maintenance costs:** The infrastructure maintenance and operations costs were not accounted for due to lack of data. For a more robust model, another term could be added to the energy equation to account for this energy consumption. Examples include the power used by each cellphone tower, approximately 1–10 kW, and the average power used to repair telephone lines damaged by storms.
- **Assumptions:** Simplifying assumptions had to be made in order to create a solvable model. In addition, some values used in the calculations had to be estimated.
- **Inputs:** This model requires a large amount of data, some of which is difficult to obtain.

Conclusion

Landlines are the most energy-feasible option only when all phones are corded phones. Otherwise, the most efficient means of providing telecommunication is through cellphones. This is based on:

- The steady state of the country with existing infrastructure would be 36–42 GWh/d.
- With no established infrastructure, it would be more energy-beneficial to have corded phones running on landlines (3.2 GWh/d); but other factors, such as preference for cordless technology, suggests that a cellphone infrastructure may be a safer investment.

- Cellphone and other charger negligence would cause a maximum of 10,000 bbl/d to be wasted. Cellphone charger negligence would cause a maximum of 6,000 bbl/d per day to be wasted, while cordless phone negligence would result in a waste of 45,000 bbl/d per day.
- The analysis of the telecommunications industry for the future shows that cellphones will be the most viable option, since they will require only 500 million barrels of oil over the next 50 years. Due to the social benefits of cellphones, as well as their energy efficiency relative to cordless phones, a cellphone dominant state should be accepted in the current infrastructure. Despite the fact that cellphones are less efficient than corded landline phones, they are more accepted by the general public.

References

[1] Discount Cell. 2008. Cell phone batteries. `http://www.discountcell.com/cellular/cell-phone-batteries.htm`.

[2] ACE3. n.d. Energy consumption of household products. `http://www.aceee.org/buildings/resappl_type/dtatbl.pdf`.

[3] U.S. Environmental Protection Agency. 2002. OSWER Innovations Pilot. The effectiveness of cell phone reuse, refurbishment and recycling programs. `http://www.epa.gov/oswer/docs/iwg/CellPhones.pdf`.

[4] ReCellular. n.d. Environmental data. Environmental statistic—Cell phone for soldiers. `http://www.att.com/Common/merger/files/pdf/CPFS_EarthDay/electronic_waste_fs.pdf`.

[5] Broussely, M., S. Herreyre, P. Biensan, P. Kasztejna, K. Nechev, and R.J. Staniewicz. 2001. Aging mechanism in Li ion cells and calendar life predictions. *Journal of Power Sources* 97–98 (July 2001): 13–21.

[6] Responsible Energy Corporation. 2008. Li-ion battery FAQs. Frequently asked questions about lithium ion (Li-ion) batteries. `http://www.greenbatteries.com/libafa.html`.

[7] Walmart Stores, Inc. 2009. Search results for "cordless phone". `http://www.walmart.com/search/search-ng.do?search_query=cordless+phone`.

[8] U.S. Census Bureau. 2007. State & County QuickFacts. `http://quickfacts.census.gov/qfd/states/00000.html`.

[9] Singhal, Pranshu. 2005. Integrated Product Policy Pilot Report. Stage I Final Report: Life cycle environmental issues of mobile phones. `http://ec.europa.eu/environment/ipp/pdf/nokia_mobile_05_04.pdf`.

[10] CTIA The Wireless Association. 2008. CTIA Advocacy. Wireless quick facts. `http://www.ctia.org/advocacy/research/index.cfm/AID/10323`.

[11] Giordano, Frank R., Maurice D. Weir, and William P. Fox. 2003. *Mathematical Modeling*. 3rd ed. Pacific Grove, CA: Brooks/ Cole.

[12] Meisterton-Gibbons, Michael. 1995. *A Concrete Approach to Mathematical Modelling*. New York: John Wiley & Sons.

[13] Farlow, Jerry, James E. Hall, Jean Marie McDill, and Beverly H. West. 2007. *Differential Equations and Linear Algebra*. 2nd ed. Upper Saddle River, NJ: Pearson Education.

[14] MathWorks. 2007. Matlab R2007b. `www.mathworks.com`.

[15] De Decker, Kris. 2008. The right to 35 mobiles. *Low-tech Magazine* (13 Feb 2008) `http://www.lowtechmagazine.com/2008/02/the-right-to-35.html`.

[16] MacKay, David. 2009. *Sustainable Energy—Without the Hot Air*. Cambridge, UK: UIT Cambridge. `http://www.inference.phy.cam.ac.uk/sustainable/book/tex/cft.pdf`.

[17] Motorola, Inc. 2008. Corporate responsibility report 2007. `http://www.motorola.com/mot/doc/7/7130_MotDoc.pdf`.

[18] Motorola, Inc. n.d. Motorola eco facts. `http://direct.motorola.com/Hellomoto/ecofacts`.

[19] Maxim Integrated Products. 1998. Draw 150 mW of isolated power from off-hook phone line. 6 July 1998. `http://www.maxim-ic.com/appnotes.cfm/an_pk/1923`.

[20] CTIA The Wireless Association. 2008. CTIA Advocacy. Wireless quick facts. `http://www.ctia.org/advocacy/research/index.cfm/AID/10323`.

[21] Begun, Daniel A. 2008. Phone makers monitor charger energy consumption. `http://hothardware.com/News/Phone-Makers-Monitor-Charger-Energy-Consumption`.

[22] Tobin, Declan. 2005. Teens and cell phones. `http://ezinearticles.com/?Teens-And-Cell-Phones&id=24519`.

[23] Kwan, Michael. 2008. Feature: Common uses for cell phones beyond voice calls. *Mobile Magazine* (22 October 2008) `http://www.mobilemag.com/content/100/340/C16461`.

[24] Levitt, Blake B. 1998. Cell-phone towers and communities: The struggle for local control. *Orion Afield* (Autumn 1998) `http://arts.envirolink.org/arts_and_activism/BlakeLevitt.html`.

[25] Quiring, Lynn. n.d. Beacons of harm: Cell phone towers and mobile phone masts. `http://omega.twoday.net/stories/4908113`.

[26] MSNBC.Com. 2006. Study links family problems with excessive cell phone use. `http://www.cellphones.ca/news/post001611/`.

[27] Gulli, Cathy. 2009. Help! My office is ring tone hell. *Maclean's* (26 January 2009): 52. `http://www2.macleans.ca/2009/01/21/help-my-office-is-ring-tone-hell/`.

[28] Channel 3000. 2008. Rock County 911 dispatchers say cell calls more difficult to pinpoint. `http://www.channel3000.com/news/16124111/detail.html`.

[29] Coombes, Andrea. 2004. Cutting the phone cord? Not so fast. `http://www.marketwatch.com/story/abandoning-landline-telephones-not-quite-there-yet`.

[30] Cooper, Alan H. 1999. Part III—Administrative, procedural, and miscellaneous. Internal Revenue Service, Dept. of the Treasury. `http://www.irs.gov/pub/irs-drop/n-99-18.pdf`.

[31] Energy Information Administration. 2001. Appendix H: Conversion factors. In *Annual Energy Outlook 2001*, 251–252. `http://www.eia.doe.gov/oiaf/archive/aeo01/pdf/apph.pdf`.

[32] MacKay, David. 2008. Phone chargers—the truth. `http://www.inference.phy.cam.ac.uk/sustainable/charger`.

[33] U.S. Census Bureau. 2008. U.S. population projections. National population predictions released 2008 (based on Census 2000). `http://www.census.gov/population/www/projections/summarytables.html`.

[34] U.S. Census Bureau. 2008. Table 1. Projections of the Population and Components of Change for the United States: 2010 to 2050. `http://www.census.gov/population/www/projections/files/nation/summary/np2008-t1.xls`.

[35] U.S. Census Bureau. 2008. Population estimates. `http://www.census.gov/popest/estimates.php`.

[36] U.S. Census Bureau. 2008. National and state population estimates. Annual population estimates 2000 to 2008. `http://www.census.gov/popest/states/NST-ann-est.html`.

[37] U.S. Census Bureau. 2008. Table 1: Annual Estimates of the Resident Population for the United States, Regions, States and Puerto Rico : April 1, 2000 to July 1, 2008. `http://www.census.gov/popest/states/tables/NST-EST2008-01.xls`.

[38] U.S. Census Bureau. 2004. Population estimates. 1980s. `http://www.census.gov/popest/archives/1980s/`.

[39] U.S. Census Bureau. 2008. Population estimates. 1990s. `http://www.census.gov/popest/archives/1990s/`.

[40] U.S. Census Bureau. 2008. Table 1. Projections of the Population and Components of Change for the United States: 2010 to 2050. `http://www.census.gov/population/www/projections/files/nation/summary/np2008-t1.xls`.

Jason Altieri, Katelynn Wilton, and Nevin Brackett-Rozinsky. Photo by Dominick DeSalvatore.

Energy Implications of Cellular Proliferation in the U.S.

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Summary

The U.S. has undergone a massive transformation in how it approaches telecommunications. In 30 years, it has gone from having an entirely landline-based phone system to one where 89% of the population uses cellphones, with 16% of households having replaced their landlines entirely. We set out to establish the key consequences and energy costs of this system.

By collecting data on wattages of cellphone chargers and modeling likely cellphone usage, we calculate that a cellphone might waste 86% of its energy intake through its charger, the equivalent of 754,000 bbl/yr of oil. Comparing that to the energy costs of landline phones, we model two transition scenarios as cellphones replace landlines. We conclude that the faster that landlines can be phased out, the more energy will be saved.

We find that a full cell network, combined with Voice over Internet Protocol (VoIP) technology, would be the best way to provide phone service to a Pseudo U.S. completely lacking in telecommunications. Doing this would save the cost of implementation of a landline infrastructure that would be rendered mostly redundant as cellphones became more popular. Because all the cellphone chargers in this Pseudo U.S. would be brand-new models with recent energy conservation features, cellphone waste would add up to only 234,000 bbl/yr of oil. We model the increase in cellphone energy consumption in this Pseudo U.S. for the next 50 years with two models: one accounts for the growth of the population, and another also factors in a rate of technological advance. In the first model, cellphone

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energy consumption would reach 1.53 million bbl/yr of oil by 2059, while in the second it would actually decrease to 525,000 bbl/yr by then, due to increases in battery efficiency and a reduction in standby power.

Cellphone chargers are a small part of standby-power waste in America. Using extensive wattage and usage data on consumer electronics, we calculate that these devices waste 99 million bbl/yr of oil.

These models show that although a single cellphone charger may waste only a small amount of energy (one author estimates leaving a charger plugged in for a day is about equal to driving a car for one second), the sheer magnitude of cellphone users means that this loss is significant.

Cellphone Chargers

We first consider the energy consumption of a single cellphone: the energy to recharge the cellphone, plus the energy used by the charger when it is left plugged into the wall. David MacKay convinced two engineers to measure a standard Nokia phone charger in a calorimeter—a much more accurate technique than anything we could devise. This method reported 0.472 W drawn while only the charger is plugged into the wall, 0.845 W wasted when a fully-charged phone is left attached, and, interestingly, 4.146 W lost as heat while the phone is charging. MacKay also suspects that older phone chargers may use 1–3 W [MacKay 2009]. IP.com [n.d.] reports 2.77 W consumed by a phone charger while charging and 0.45 W while not. Motorola [2008] lists its chargers' standby wattage at about 0.2 W. Since MacKay's experiment shows the charger drawing about twice as much power with the phone attached as without, we assume that brand-new chargers would do the same.

The average cellphone is replaced every 18 months [Stover 2008; ReCellular n.d.; Recycling for Charities 2008]. Fairly-new models and brand-new models are likely to be present in approximately equal numbers, while both will outnumber older chargers. If we assume that 20% of phone chargers are old, 40% use about 0.472 W, and 40% do not leak at all, then the average cellphone charger wastes about 0.589 W while plugged in without a phone and 0.938 W while left plugged in and attached to a fully-charged phone.

Next, we consider how long the charger is in each of these states. We construct a model with two expressions, one for each of two practices:

- **Users who unplug the charger when they detach the phone:** Let x be number of recharges per year and y the average number of hours that the phone remains plugged in after it has reached full charge. The annual amount of waste is $0.938xy$ W/yr.
- **Users who detach the phone from the charger but leave the charger plugged in:** These users still waste energy as above. We also assume that they rarely come back to unplug the charger later (perhaps a few

times a year when they need the outlet). The waste in this case is

$$[0.938xy + 0.589(8760 - 300 - xy)] \text{ W/yr},$$

where 8760 = 24 × 365 is the number of hours in a 365-day year and we subtract 300 hr as the charging time during a year and also subtract xy for the hours that the phone is still attached to the charger after being fully charged (so that it is leaking power at 0.938 W instead of 0.589 W).

We weight the two quantities, assuming that 25% of users unplug the charger and 75% don't.

Average talk time per charge is 5 hr and average standby time is 10 d [AT&T n.d.]. Average talk time of 1 hr/d and standby all the time would use up 60% + 30% = 90% of battery charge in 3 d; hence, about 100 recharges would be required per year. On the other hand, many users attach their phones every time they come home, in effect recharging the phone 365 times a year. We try both of these numbers in our model to see their effect on total energy consumption.

We choose 2 hr and 6 hr as two likely averages for how long users leave a phone attached after it is charged, since some leave a phone plugged in all night, producing 6–9 hr of waste after 1 hr of charge, while others may unplug within minutes. We select as suitable averages

175 recharges/yr, 4.5 hr attached after charging.

Table 1 shows the resulting total energy waste for the entire country for various combinations.

Table 1.
Total energy wasted in TWh/yr (1 terawatt-hour = 10^{12} Wh), by cellphone chargers, by recharges per year and hours before detaching the cellphone.

	Number of hours		
Recharges/yr	2	4.5	6
100	1.19		1.25
175		1.28	
365	1.27		1.51

From the five scenarios shown, 1.28 TWh/yr, or 754,000 bbl/yr of oil (where 1700 kWh = 1 bbl), is a fair estimate of the average waste; changing either variable has little effect.

How could we reduce waste? To find out, we assume that every user gets into the habit of unplugging the charger on detaching the phone. As **Table 2** shows, waste would be cut by 65–95%. The constant power drain of the charger plugged in to the wall simply outweighs the number of recharges or how long the phone is left attached.

Table 2.
Total energy wasted *if users unplug the charger* (in TWh/yr),
by number of recharges per year and hours before unplugging.

	Number of hours		
Recharges/yr	2	4.5	6
100	0.06		0.18
175		0.24	
365	0.22		0.65

How much energy per year is required in charging a cellphone? We estimate 300 hr/yr of charging time (100 recharges/yr × 3 hr/recharge). The average phone charges at 3 W, an average of new phones (many in range of 1 W) [AT&T n.d.] and older phones (with higher wattages); cf. MacKay's measurement of 4 W lost as heat during charging. Hence 3 W × 300 hr/yr = 0.9 kWh/yr is used to charge a cellphone. When combined with the 4.7 kWh/yr of waste determined above, we get 5.6 kWh/yr. This means that 84% of the of energy used on cellphones is wasted, which nicely splits the difference between three-year-old statistics (95% lost as waste [Richard 2005]) and statistics from November 2008 (67% lost as waste [Virki 2008]).

Transition from Landlines to Cellphones

U.S. cellphone use has grown logistically (**Figure 1**). As of February 7, 2009, there were 271,778,000 cellphone subscribers [CTIA…2009]. Meanwhile, the number of households using landlines is on a sharp decline. Between 2004 and 2008, the percentage of cellphone-only households rose from 4.5% to 16.4% [Dixon 2008]. We analyze the transition to a system of exclusively cellphones, evaluate the energy costs, and discuss the most efficient route to it.

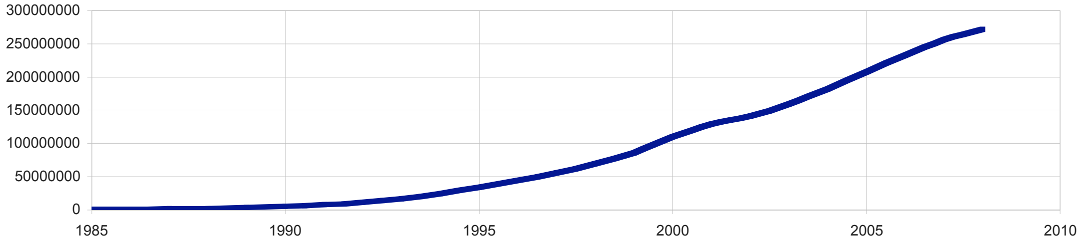

Figure 1. Cellphone subscribers in the U.S.

In considering the total energy costs of cellphones, we consider:

- **Charging** (as we have seen).
- **Production.** Energy consumed in production of cellphones is extremely

variable and poorly documented. A cellphone has an average life of 1.5 years, compared to 6 years for a cordless telephone. If the energy used in production is comparable, then over a given time period the production costs of cellphones will be four times that of cordless phones.

- **Towers.** We will calculate how many additional cell towers will be needed to support the growing number of cellphone subscribers, and also analyze some of the energy costs of maintaining the landline system.

Our primary focus in the ensuing models is the relative energy costs of keeping cellphones and cordless phones charged and usable. We would like to incorporate the energy costs, but lack of reliable data would create an error large enough to render the model unsuitable. Thus, we tackle the problem as energy usage by the consumer.

Basic Information and Assumptions

Approximately 2.5% of U.S. households use neither landlines nor cellphones [Bavdek 2008]. So we assume that a complete transition from landlines to cellphones will result in 97.5% of the population having cellphones, since variation in household size is negligible on such a large scale. So, of the 111.1 million households [U.S. Census Bureau n.d.], 108.3 million require some form of phone line. In 2008, 16.4% of total households opted to use only cellphones. Hence, the number of households using landlines in 2008 is $H = (.975 - .164) \times 111.1 = 90.1$ million. Furthermore, the average number of people per household is $m = 2.745$ [U.S. Census Bureau n.d.].

Our transition assumes that that every man, woman, and child receives a cellphone. However, approximately 6.9% of the population (21 million) is under the age of 5. They don't need cellphones; if we remove them from the number of subscribers, the U.S. would be close to complete transition already. With 272 million cellphone subscribers in a population of 305 million [U.S. Census Bureau] leaves 33 million people without cellphones; subtracting 21 million children leaves just 12 million to supply with cellphones.

The 272 million subscribers does not account for people with both a personal cellphone and a work cellphone. There are hardly any data on the number of such people; so we assume that the number of multiple-phone users is negligible.

Cell and Cordless Phone Energy Use

We consider three types of phones—cell, cordless home phone, and corded home phone. To establish the energy costs of each, we need five pieces of data for each type:

- $E_{\text{production}}$ = energy to produce it,
- E_{support} = energy to support it, per year;
- E_{charge} = energy to charge/power it, per year;
- LS = its average lifespan; and
- N = the number of such phones in use.

The first two pieces of data are nearly impossible to find to any degree of accuracy. So, we assume that it takes the same amount of energy to produce a cellphone as a cordless phone; the number of corded phones currently being sold is negligible.

Likewise, the energy that goes into phone support—cell tower construction, tower and landline upkeep, signals, etc.—is not well documented. We find two rough estimates:

- 0.12% of global primary energy use is by telecom companies [Ericsson 2007]. If this proportion holds true for the U.S., which uses 3,923 TWh/yr [Energy Information Association 2009], then U.S. telecom companies consume 470 TWh/yr. This figure does not tell us if the energy is going towards landlines or cellphones, but it gives an idea of the scale—much larger than the energy consumed by cellphones themselves.
- Japanese mobile telecommunications companies use 120 Wh per user per day [Etoh 2008]. If U.S. mobile companies do the same, usage would be 12 TWh/yr.

These numbers contradict each other—we find it hard to believe that a telecom company spends 40 times as much energy on noncellular aspects of its business as on mobile infrastructure. However, more accurate data is simply not available.

Since the first two quantities cannot be accurately determined, we use only the remaining three variables in our models. **Table 3** shows the values that we use. We have already done the estimates for cellphones. A cordless phone uses 28 kWh [Roth and McKenney 2007], a corded phone 2.2 kWh (= 0.25 W × 8760 hr/yr). (Every source we found said that corded phones used a "smidgen" or a "dab" of power, which we take to be 0.25 W.) We estimate the lifespans of those phones and assume that there are two cordless landline phones for every corded one.

To determine the number of each type of phone, we develop two models for relative change in the number of cellphones to the number of landline phones.

Model 1: Current Trends

We assume that current trends continue until 97.5% of people have cellphones and 0% use landlines. Using data since 2000, we project the trends

Table 3.

Parameters of phone usage.

	E_{charge} (kWh)	LS (yr)	N (units)
Cellphone	5.6	1.5	x
Cordless	28	6	$\frac{2}{3}y$
Corded	2.2	10	$\frac{1}{3}y$

shown in **Figure 2**, with landline data from Nielsen [2008] and cellphone data from Infoplease [2008].

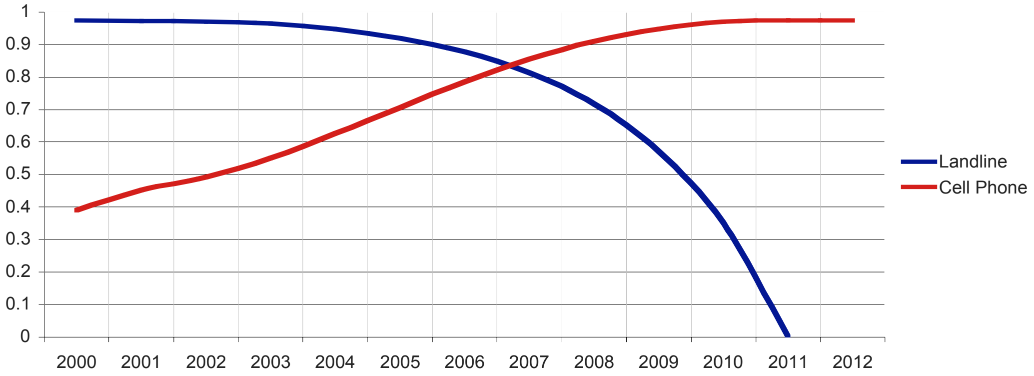

Figure 2. Cellphone and landline trends with projections: proportion of population vs. year.

We calculate the total energy consumption due to phone usage from 2009 to 2015. To do so, we need to find the areas under the two curves from 2008 to 2015; we use best-fit trend lines.

We get 19.4 kWh/yr for a landline as a weighted average for cordless and corded, with twice as many of the former as the latter.

Thus, the total amount of energy used from the beginning of 2008 through the end of 2015 is 19.3 TWh.

If each person (technically, 97.5% of the population) had a cellphone and no landline phones were used, the total energy used over this period would be only 12 TWh.

Model 2: Current Trends with Resistance to Extremes

Although cellphone usage will creep up to 97.5% of population, landline usage will in fact not drop to 0%: Many people feel more comfortable with the added security of a landline, in case their cellphone does not work (as during the aftermath of the September 11, 2001 attack on New York City), landlines are easier to talk on for long periods of time, some worry that cellphones may cause cancer, and senior citizens may resist technological changes. A substantial percentage of Americans may opt to stay with a landline as long as it is available. Using information from polls concerning feelings about having a landline, we offer an alternative projection in **Figure 3**.

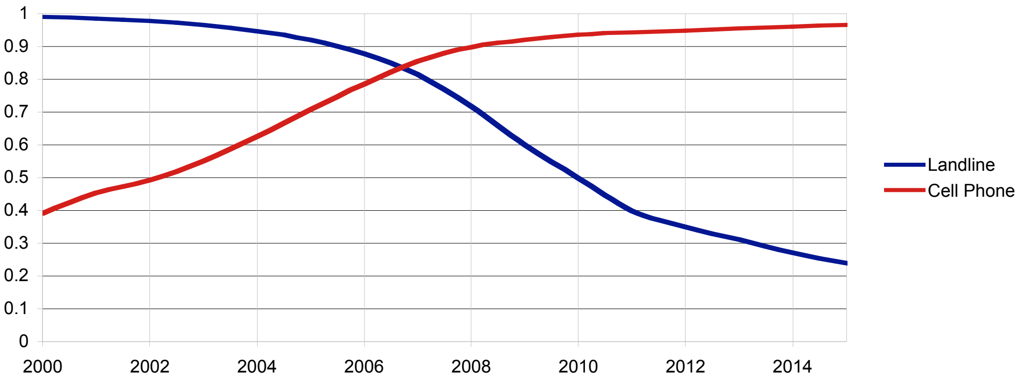

Figure 3. Current cellphone and landline trends adjusted: proportion of population vs. year.

We calculate as before, arriving at

$$E_{\text{cell}} = 11.3 \text{ TWh}, \qquad E_{\text{land}} = 17.6 \text{ TWh}, \qquad E_{\text{total}} = 28.9 \text{ TWh}.$$

This compares with 19.3 TWh under Model 1 and 12 TWh with only cellphones.

Bringing Phone Service to the Pseudo U.S.

We now consider a "Pseudo U.S.," a country with the same population, economic status, and infrastructure as the U.S. but with no phone system in place. Our goal is a strategy to implement a phone system that minimizes energy usage. In addition to providing a detailed analysis of the phone system, we will discuss the consequences of all current types of phone systems: landlines, cellphones, satellite phones, Voice over Internet Protocol technology (VoIP), and combinations.

Using only landlines connected to corded phones would be energy-efficient (since corded phones use less energy than cellphones), a landline system would also have lower maintenance costs, phones would be broken or lost less often, and there would be less social incentive to replace them with a more stylish or feature-filled new model every 18 months. There would also be fewer phones per person—many families could do with one or two instead of one for each family member.

However, Americans have already proven that they favor cordless phones over corded and that they are willing to replace landlines entirely with cellphones. Inevitably, the same landline-to-cell process would happen in Pseudo U.S. So it would be better to build a cellphone network in the first place.

Despite the advantages of cellphones, there are some drawbacks. Businesses would have to issue employees cellphones, and it would be difficult to prevent employees from using those for personal calls. Cellphones are vulnerable to hackers, interception (even by the government), and jamming.

A simple, cheap, and energy-efficient way to solve several of these problems is VoIP (Voice Over Internet Protocol); in the fourth quarter of 2008, VoIP provider Skype reported 405 million accounts worldwide [2008]. Assuming that Pseudo U.S. has the same technology level as the U.S., a network of Internet cables would be in place, connected to 37% of households [Energy Information Administration n.d., Table HC2.11]. Attaching the remaining 63% of homes to existing hubs would be easier than laying new phone lines, throwing up millions of phone poles, or constructing thousands of cell towers. Cordless VoIP phones might consume a fair amount of power, but the savings on construction and infrastructure costs would be enormous. VoIP would allow business to give employees phones for which they could be held accountable, and families could have a backup VoIP line.

Much of the energy cost of production of new cellphones could be alleviated by mandatory recycling of old ones. Recycling 100 million cellphones would save 0.215 TWh, enough energy to power 19,500 households for a year [Environmental Protection Agency n.d.].

Covering Pseudo U.S. with Cell Towers

The obvious energy-efficient choice seems to be the newly-released Tower Tube cell tower, with a range of 4 mi; it has a wind turbine attached, so it uses 40% less energy than conventional towers. It can be erected in days, has a small footprint, and is resistant to vandalism and the elements [Ericsson n.d.].

Knowing nothing about the geography of Pseudo U.S., we devise an optimal grid for cell-tower placement based on U.S. data, minimizing the number of towers while maximizing coverage. The most efficient design is a triangular lattice, as seen on the right of **Figure 4**, with each tower the vertex of an equilateral triangle, 6.93 mi from the next towers (determined from trigonometry). In reality, towers would be placed closer to ensure coverage despite geography.

Since each tower covers a unique hexagonal area of 41.6 mi^2 and the land area of the U.S. is approximately 3,540,000 mi^2, we would need approximately 85,000 towers.

Since a tower has a limit on how many users it can support, higher population density in cities demands higher density of towers. In the 601 U.S. cities with more than 50,000 people, the density of towers needs to be 16 times the average density of the rest of our grid, requiring 420 additional towers. Without data on the remaining cities with 10,000–50,000 people, we recommend a density of towers four times the density of the rest of the grid.

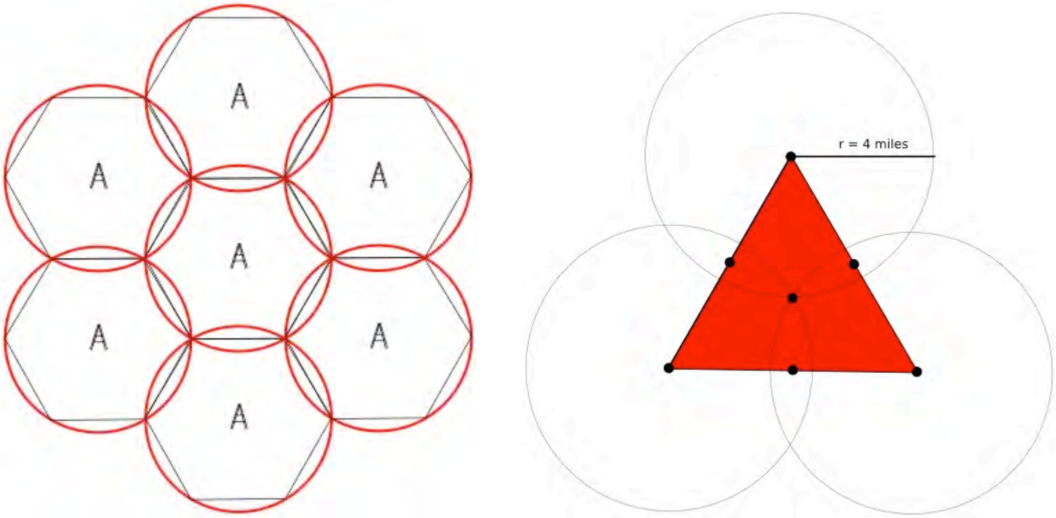

Figure 4. Cell tower lattice with lattice detail.

Information on cost or energy consumption of such a network is difficult to obtain. Using the Tower Tube with 10 kW of signal strength, and 86,000 towers, operating energy consumption would be at least 0.31 TWh.

Cellphone Chargers in Pseudo U.S.

Energy wasted by cellphone chargers in Pseudo U.S. can be determined using the same calculations as earlier for the real U.S. We simply disregard older phones and assume that every cellphone user has a charger that wastes only 0.2 W on standby left plugged in alone and 0.4 W attached to a fully-charged phone. **Table 4** shows the results.

Table 4.
Total energy wasted in Pseudo U.S. (TWh/yr) by cellphone chargers, by recharges per year and hours before detaching the cellphone.

	Number of hours		
Recharges/yr	2	4.5	6
100	0.36		0.39
175		0.40	
365	0.40		0.49

Pseudo U.S. would waste 30% less energy due to cellphone chargers than the real U.S. The 0.40 TWh/yr estimate corresponds to 234,000 bbl/yr of oil. This is encouraging, since the real United States will basically reach this state in a few years, as older cellphones are replaced.

If all users in Pseudo U.S. unplugged their chargers and phones at the same time, we get the results of **Table 5**.

Table 5.
Total energy wasted in Pseudo U.S. *if users unplug the charger* (in TWh/yr),
by recharges per year and hours before unplugging.

	Number of hours		
	2	4.5	6
100	0.02		0.07
175		0.09	
365	0.08		0.24

If users could refrain from recharging until absolutely necessary and unplug both phone and charger together within 2 hr of full charge, standby power waste would virtually disappear: 0.02 TWh/yr, or less than 2% of current U.S. waste.

Future Energy Needs for Pseudo U.S.

We model the electricity needs for the phone system over the next 50 years. This model depends heavily on both population growth and economic growth. We establish reasonable trends for each and describe why the population projections are more significant than the economic projections.

Population Growth

We project for Pseudo U.S. a population growth rate of 0.9%/yr, reflecting that of the U.S. [Central Intelligence Agency n.d.]; **Figure 5** shows such a projection for the U.S. We project maximum cellphone usage (97.5%) by 2015.

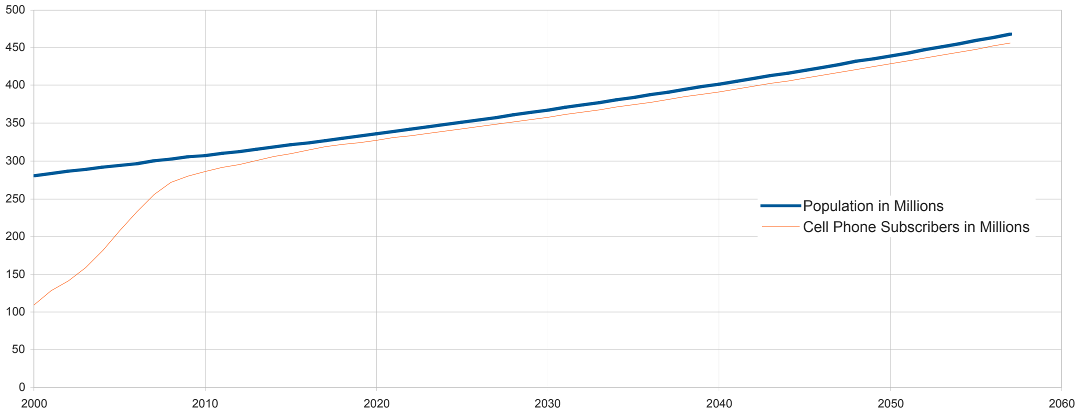

Figure 5. Population projection.

Economic Growth

Real GDP per capita for the U.S. shows a surprisingly robust trend (**Figure 6**). Despite the clearly-visible recession of the 1980s, there is a strong upward trend. Even with the current economic instability, these data suggest that over the next 50 years the U.S. economy will continue to grow at approximately the same rate as the past trend.

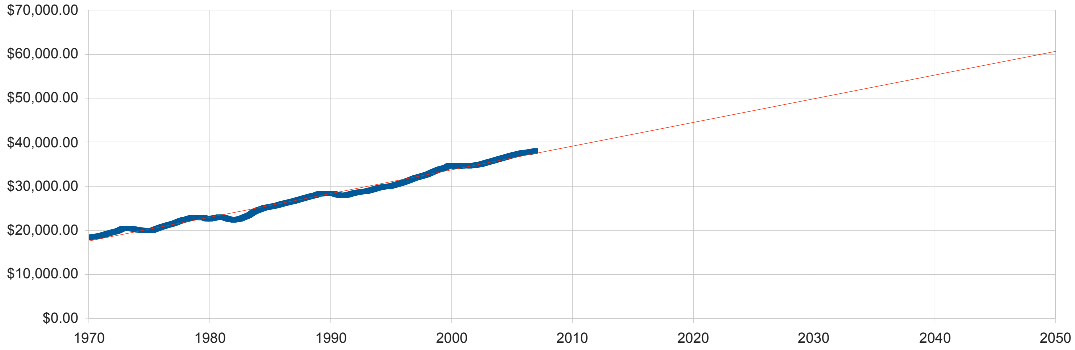

Figure 6. Real GDP per capita (in Year 2000 dollars) vs. year. [Measuringworth.com 2008].

Technological Advances

A cellphone network, combined with VoIP technology, would be more easily expanded and upgraded over the next 50 years than a landline network.

With improvements in satellite communications, satellites could be an attractive choice for covering the Rocky Mountain states and parts of the Midwest. We compiled a list of 15 contiguous states and Alaska that contains 14 of the 15 most-sparsely-populated states, excluding Maine but including Arizona. Together, these states contain 52% of the area of the United States but only 12% of its population [Wikipedia 2009]; since they are contiguous, they would be the easiest to cover efficiently with satellites. If such a satellite network could be made operational now, we could simply not build any of the 44,000 cell towers needed for that area.

Energy Needs Over the Next 50 Years

Taking the predictions of population and economic growth over the next 50 years, together with an assumed increase in energy efficiency of cellphone technology, we calculate the energy costs due to consumer use.

For the first model, we assume that cellphone efficiency remains constant.

For the second model, we anticipate that standby power waste would be nearly eliminated and batteries would last longer (so phones would need to be recharged less often), as cell technology became more advanced. We incorporate an exponential decay constant λ, projecting a λ proportion decrease in energy use per year. Our projection becomes

$$\text{Energy}[t] = 0.922(1.009)^t(1-\lambda)^t.$$

We set $\lambda = 2\%$.

We show the results for the two models in **Table 6**.

Table 6.

Energy costs (bbl $\times 10^6$ of oil) of consumer usage, for two projections.

	2009	2059
No change in energy efficiency	.92	1.53
Increasing energy efficiency	.92	0.53

Consumer Electronics

If cellphone chargers waste over 1 TWh/yr, how much energy is leaking out of other devices in U.S. households?

We could not find enough reliable data on the number and usage of light fixtures to study lighting.

We were able to study consumer electronics. The Energy Information Administration produced a lengthy study on the types of appliances and electronics in American homes [n.d., Table US-1]. Many other sources detail wattages and average kWh/yr consumed by various devices [Ames City Government 2002; Seattle City Light n.d.; ABS Alaskan n.d.; Fry 2006; Dot-Com Alliance n.d.; MacKay 2008; afterdawn.com 2007; Fung et al. 2002]. However, Roth and McKenney [2007] was by far the most thorough. The only major consumer electronics that they did not report on were digital TVs, and they could also provide only yearly consumption estimates for component stereos, printers, and modems. We checked all of their data against the other sources and filled in the gaps with corroborated data and estimates. We also updated the study (completed in January of 2007) as best we could, especially considering VCRs and game consoles, whose use has changed drastically in the last two years. **Table 7** shows total consumption by all electronic devices by type, with relative numbers of each device taken into account.

We offer a few notes on some kinds of devices.

- **Analog TVs:** They waste 4 W when turned off.
- **Digital TVs:** There is huge variation in wattage, from 100 W to 500 W. CNET.com [n.d.] shows an average for new HDTVs of 250 W, with standby power 1 W. We assume that in most cases the digital TV, being

Table 7.

Annual electricity consumption of consumer electronics (TWh).

	Active	Idle	Off	Total
Analog TVs	43.6	n/a	6.4	50.0
Digital TVs	37.8	n/a	0.4	38.2
Desktop computers	20.2	0.1	1.0	21.3
Set-top boxes	6.4	n/a	13.3	19.7
Compact audio	1.4	0.9	3.8	6.2
Component stereo	1.5	0.9	2.9	5.3
Game consoles	1.7	2.4	0.7	4.8
DVD players	0.5	1.2	2.6	4.3
VCRs	0.2	0.6	2.5	3.3
Laptop computers	2.3	0.1	0.4	2.8
Modems	0.7	n/a	1.8	2.5
Home theaters	1.5	0.6	0.1	2.2
Printers	0.3	0.5	0.2	1.0

the newest TV, would be used the most; so we apply usage data from Roth and McKenney [2007] for the most-used TV. About 19.25 million flat-screen TVs have been sold since the study was done, meaning that there are now 59.25 million digital TVs in the country [Burritt 2009].

- **Desktop computers:** We combine CRT and LCD monitors into a weighted total, taking into account time spent in screensaver and standby modes.
- **Set-top boxes** (cable, satellite, and other TV boxes): These waste more energy than any other type of electronics device—a surprising fact, since there are almost a million fewer set-top boxes than analog TVs—because they still use 15 W when off, presumably to stay in contact with the service provider and in some cases to perform services (e.g., to turn on at a certain time to record a show).
- **Compact audio:** We use data from Roth and McKenney [2007].
- **Component stereo:** Roth and McKenney [2007] estimated that a component stereo uses 115 kWh/yr, with an installed base of 50 million units. We decided that a stereo would have a usage pattern similar to a compact audio system, with wattage more like a home theater; we calculate that a stereo uses about 105 kWh/yr.
- **Game consoles:** Roth and McKenney [2007] reported only 2.6 TWh used by game consoles, with 1.0 TWh in active state, 1.3 in idle, and 0.4 while off. But game consoles have not only become more popular since January 2007, but the proportion of older-generation consoles to new ones has also gone down. This is important because newer ones are considerably more power-hungry. Roth and McKenney reported an average of 36 W for consoles, but multiple sources cite the new Xbox 360 at 173 W, the Playstation 3 at 190 W, and the Nintendo Wii at 18–19 W. Roth and

McKenney reported 64 million consoles, but since then Wiis have jumped from 1.5 million to 13.5 million, Xbox 360s from 4.8 million to 11.9 million, and PS3s from 0.8 million to 5.9 million [Brightman 2008]. With these 24 million new game consoles, we estimate that 12 million older ones have been removed from use. So, there are now about 52 million older consoles averaging 36 W, plus 12 million new Wiis at 19 W, 7 million new Xboxes at 173 W, and 5 million new PS3s at 190 W. Weighting appropriately gives current average wattage at 56 W.

- **DVD players:** They waste a staggering 87% of the energy that they use.
- **VCRs:** This was another area where we felt we had to correct for the two years since Roth and McKenney [2007]. They cited 5.0 TWh used by VCRs, but the number of VCRs in use has dropped since. Data from previous studies cited by them indicate that the number of VCRs decreased by 11.25% per year from 2001 to 2005. We extend this trend to the end of 2008, for an estimate of 71 million VCRs operational today. We also adjust their usage numbers downward by 15% to account for more families preferring to use a DVD player.

  VCRs turn out to be the energy-wasting champion by percentage, wasting over 95% of the energy that they consume.
- **Laptop computers:** Surprisingly efficient, laptops as a whole used one-seventh as much electricity as desktop computers, even though there are only twice as many desktops. A laptop uses only use 25 W while active instead of the 75 W that a desktop uses, despite the fact that the laptop wastes 18% of energy compared to only 5% for the desktop.
- **Modems:** Left on all the time, they have a low wattage (7 W), for 55 kWh/yr, close to the 53 kW of Roth and McKenney [2007]. Assuming that modems are used 6 hr/d (about 25% less than computers), only 0.7 TWh/yr is used by modems while people are actually connected to the Internet; the other 1.8 TWh lost as waste could be saved if people would unplug modems not in use.
- **Home theater:** This was one of the most efficient devices when off, probably due to Energy Star standby-power guidelines, since they are relatively new devices.
- **Printers:** Printers on average are in use for only a few minutes per day, but their idling wattage is quite high. A reasonable average is 300 W active and 12 W idle [Dot-Com Alliance n.d.]. Assuming that a printer is used 5 min/d and left on 4 hr/d, a printer would use 34 kWh/yr, close to the estimate of 30 kWh/yr of Roth and McKenney [2007].

This analysis reveals that it is the TV complex, not the computer complex, that is responsible for the bulk of waste: 4.7 TWh idling and 26.1 TWh off for TV-associated devices vs. 0.7 TWh idle and 3.5 TWh off for computer-associated devices.

If the TV and related devices were plugged into a power strip that was turned off when the electronics are not in use, households would use 18% less energy on electronics. Since the average household uses 11% of its energy on household electronics, this would represent a 2% reduction in overall residential electricity usage. A power strip could even be fitted with a remote-control switch—the strip would consume slight standby power waiting for the remote signal, but the devices plugged into it would not. This would be a convenient way to turn off electronics that would also save electricity.

In all, this selection of household electronics consumes 169 TWh/yr of electricity, or the equivalent of 99 million bbl/yr of oil—considerably more than the 1.3 TWh/yr wasted by cellphone chargers. Of the 169 TWh, 125 TWh is for devices in use, 7 TWh idle, and 37 TWh waste. By percentage, 26% of a house's energy spent on electronics is wasted: 22 million bbl/yr of oil wasted by standby power and 4 million by electronics on but idle. David MacKay attempted to minimize his standby power waste by unplugging everything he could, finding that he could save 1.1 kWh per day [2009]. Our data suggest that the average American could save just about as much (376 kWh/yr, or 1.0 kWh/d) by doing the same.

References

ABS Alaskan. n.d. Power consumption table. `http://www.absak.com/library/powerconsumption-table`.

afterdawn.com. 2007. Power consumption of common home electronics devices. `http://my.afterdawn.com/ketola/blog_entry.cfm/2423/home_electronics_power_consumption`.

Ames City Government. 2002. Common household appliance energy use. `http://www.cityofames.org/electricweb/Energyguy/appliances.htm`.

AT&T. n.d. Cellphones and devices. `http://www.wireless.att.com/cell-phone-service/cell-phones/index.jsp`.

Bavdek, Maureen. 2008. Nearly 1 in 5 U.S. households have no phone. Reuters News Service (17 December 2008). `http://www.pcmag.com/article2/0,2817,2337125,00.asp`.

Brightman, James. 2008. Wii U.S. installed base now leads Xbox 360 by almost 2 million. `http://www.gamedaily.com/articles/news/wii-us-installed-base-now-leads-xbox-360-by-almost-2-million`.

Burritt, Chris. 2009. Super Bowl may spur fewer TV sales as retailers fight slump. `http://www.bloomberg.com/apps/news?pid=20601103&sid=a4OnWOI6j7.c&refer=news`.

Central Intelligence Agency. n.d. United States. In *CIA World Factbook*. `https://www.cia.gov/library/publications/the-world-factbook/print/us.html`.

CNET. n.d. The chart: 139 HDTV's power consumption compared. `http://reviews.cnet.com/4520-6475_7-6400401-3.html?tag=rb_content;rb_mtx`.

CTIA The Wireless Association. 2009. Estimated current US wireless subscribers. `http://ctia.org/`.

Dixon, Kim. 2008. U.S. cell-only households keep climbing. `http://www.reuters.com/article/technologyNews/idUSTRE4BG5GH20081217`.

Dot-Com Alliance. n.d. ICT power consumption reference tables. `http://www.dot-com-alliance.org/POWERING_ICT/Text/hotwords/Data_on_Power_Consumption.html#Table6`.

Energy Information Association. 2009. Electricity basic statistics. `http://www.eia.doe.gov/basics/quickelectric.html`.

________. n.d. Table HC2.11 Home electronics characteristics by type of housing unit, 2005. `http://www.eia.doe.gov/emeu/recs/recs2005/hc2005_tables/hc11homeelectronics/pdf/alltables.pdf`.

________. n.d. Table US-1. Electricity consumption by end use in U.S. households, 2001. `http://www.eia.doe.gov/emeu/recs/recs2001/enduse2001/enduse2001.html`.

Environmental Protection Agency. n.d. Recycle your cellphone. It's an easy call. `http://www.epa.gov/epawaste/partnerships/plugin/cellphone/index.htm`.

Ericsson. 2007. Sustainable energy use in mobile communications. `http://www.ericsson.com/technology/whitepapers/sustainable_energy.pdf`.

Ericsson. n.d. Ericsson Tower Tube. `http://www.ericsson.com/campaign/towertube/`.

Etoh, Minoru. 2008. A power consumption ratio, 1:150. `http://micketoh.blogspot.com/2008/03/power-consumption-ratio-1150.html`.

Fry, Jason. 2006. A hunt for energy hogs. *Wall Street Journal* (18 December 2006). `http://online.wsj.com/public/article/SB116603460189049162-04zk0DLUxdchdcCNfF4I_Q8jDZU_20071218.html?mod=blogs`.

Fung, Alan S., Adam Aulenback, Alex Ferguson, and V. Ismet Ugursal. 2002. Standby power requirements of household appliances in Canada. *Energy and Buildings* 35 (2) 217–228. `http://www.sciencedirect.com/science?_ob=ArticleURL&_udi=B6V2V-45X2RF8-D&_user=`

6046335&_rdoc=1&_fmt=&_orig=search&_sort=d&view=c&_acct=C000069202&_version=1&_urlVersion=0&_userid=6046335&md5=1c63a4b640ba73c78ed66a358c89df32.

Infoplease. 2009. Cellphone subscribers in the U.S., 1985–2005. http://www.infoplease.com/ipa/A0933563.html.

IP.com. n.d. Zero power consumption from battery chargers when not in use. http://www.priorartdatabase.com/IPCOM/000146735/.

MacKay, David J.C. 2009. *Sustainable Energy—without the Hot Air*. Cambridge, UK: UIT Cambridge. http://www.withouthotair.com/.

MeasuringWorth. 2008. US real GDP per capita. http://www.measuringworth.org/datasets/usgdp/graph.php.

Motorola. 2008. Charger energy efficiency. http://www.motorola.com/content.jsp?globalObjectId=9392.

Nielsen Company. 2008. Call my cell: Wireless substitution in the United States. http://www.nielsenmobile.com/documents/WirelessSubstitution.pdf.

ReCellular. n.d. Wireless recycling. Frequently asked questions. http://www.recellular.com/recycling/faqs.asp.

Recycling for Charities. 2008. Cellphone recycling facts. http://www.recyclingforcharities.com/blog/?p=9.

Richard, Michael Graham. 2005. Treehugger homework: Unplug your cellphone charger. http://www.treehugger.com/files/2005/11/treehugger_home_2.php.

Roth, Kurt W., and Kurtis McKenney. 2007. Energy consumption by consumer electronics in U.S. residences. http://www.ce.org/pdf/Energy%20Consumption%20by%20CE%20in%20U.S.%20Residences%20(January%202007).pdf.

Seattle City Light. n.d. Stretch your energy dollar. http://www.ci.seattle.wa.us/light/accounts/stretchyourdollar/ac5_appl.htm#Entertainment.

Skype. 2008. Skype fast facts Q4 2008. http://ebayinkblog.com/wpcontent/uploads/2009/01/skype-fast-facts-q4-08.pdf.

Stover, Dawn. 2008. Cellphone recycling for cash a win-win, or is it? MSNBC (23 January 2008). http://www.msnbc.msn.com/id/22671798/.

U.S. Census Bureau. n.d. Population clocks. http://www.census.gov/.

Virki, Tarmo. 2008. Cellphone industry eyes charger power savings. Reuters (19 Nov 2008). http://www.reuters.com/article/technologyNews/idUSTRE4AI2T520081119.

Wikipedia. 2009. U.S. states. `http://en.wikipedia.org/wiki/U.S._states`.

Advisor Michael Hitchman, with team members Benjamin Coate, Nathaniel Landis, and Zachary Kopplin.

Modeling Telephony Energy Consumption

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Summary

The energy consequences of rapidly changing telecommunications technology are a significant concern. While interpersonal communication is ever more important in the modern world, the need to conserve energy has also entered the social consciousness as prices and threats of global climate change continue to rise. Only 20 years after being introduced, cellphones have become a ubiquitous part of the modern world. Simultaneously, the infrastructure for traditional telephones is well in place and the energy costs of such phones may very well be less. As a superior technology, cellphones have gradually begun to replace the landline but consumer habits and perceptions have slowed this decline from being an outright abandonment.

To evaluate the energy consequences of continued growth in cellphone use and a decline in landline use, we present a model that describes three processes—landline consumption, cellphone consumption, and landline abandonment—as economic diffusion processes. In addition, our model describes the changing energy demands of the two technologies and considers the use of companion electronics and consumer habits. Finally, we use these models to determine the energy consequences of the future uses of the two technologies, an optimal mode of delivering phone service, and the costs of wasteful consumer habits.

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Introduction

The telephone has become a fundamental part of our social fabric. In the past couple of decades, we have seen a shift from fixed landline telephones, generally one per household, to individual ownership of cellphones. We attempt to determine the impact of this change on American energy consumption.

The factors that go into accurately modeling telephony energy consumption are complex. We need to take into account also the energy consumption of peripheral devices, such as answering machines for landline phones and chargers for cellphones. Moreover, landline phones are not a uniform product. Cordless phones consume considerably more energy than their corded counterparts. Likewise, the total energy cost of cellphone usage is complicated by such factors as recharging, replacement, and battery recycling. Our model takes all of these factors into account, and additionally attempts to use the limited real-world data available to chart the changes in each of these factors over time.

Perhaps the most complex factor to model is adoption of technological innovations in a population. This is relevant not only to landline adoption and cellphone adoption, but additionally de-adoption of landline phones in the face of cellphone usage can be considered an independent innovation and modeled accordingly. Research into the phenomenon indicates that it can be modeled globally by the differential equation

$$\frac{dP}{dT} = rP\left(1 - \frac{P}{K}\right),$$

where P is the proportion of the population that has adopted the innovation at time t, r is the adoption rate, and K is the saturation point for the innovation.

Using the descriptions of such a model, we arrive at an accurate fit to available data and can predict future demand for cellphones and landlines. Determining the cost for these respective technologies we arrive at the total energy burden. Briefly, we explore how this question relates to the energy consumption of other household electronics, and how much waste is generated therein. Additionally, we explore the caveat that technological development has been and continues to be wildly unpredictable, and the consequences of this reality.

A separate question is how best to distribute landline and cellphones throughout a population committed to neither, so as to minimize energy consumption while not violating social preference. This problem is explored through an optimization with respect to energy usage, in which we discover that a country, here a "Pseudo-U.S.," which supports a cellphone-only communicative infrastructure minimizes its total energy consumption, and also does not violate social demand for novel technologies. Finally, we estimate the total energy consumption by such a nation over the next 50 years.

Model Overview

We examine two approaches to modeling technology diffusion through a population. The first attempts to gauge technology adoption at the household level and aggregate these results to model global trends. However, this approach is unsuccessful, and we explain why. The second approach models technology adoption at the global level; it

- accurately models past and present telephony energy consumption,
- makes future predictions of cellphone saturation and landline de-adoption consistent with previous technological replacement paradigms, and
- encompasses a broad range of pertinent factors in telephony energy consumption.

Model Derivation

Adoption of Innovations

Our model describes U.S. usage rates for landlines and cellphones as three diffusive innovation curves. Consider the adoption of an innovation Y. At small times after the development of this innovation, adoption of Y throughout a population is minimal. As the innovation spreads, demand increases until a saturation point is reached. Thus, the spread of Y throughout a population is proportional to its synchronous prevalence, but is checked from exponential growth by an upper bound to its saturation in a population. At its simplest, we can model this as

$$\frac{dY}{dt} = Y(1 - Y).$$

Of course, adoption is not uniform between different technologies, and saturation rates likewise vary. By introducing constants r for adoption rate and S for saturation rates, we can refine our model to

$$\frac{dY}{dt} = rY\left(1 - \frac{Y}{K}\right),$$

which has a solution in form of the logistic equation. Therefore, for each of the processes we assume a model of the form

$$Y(t) = \frac{A}{1 + Be^{-Ct}}.$$

The sigmoidal form of adoption processes is well-known and has been observed in the specific case of cellphone adoption and wireless-only lifestyle adoption.

Proceeding globally, we initially model the consumption of telephones from their inception by the equation:

$$p_l(t) = A\left(\frac{1}{1+Be^{-C(t-D)}} + \frac{1}{1+Ee^{-F(t-G)}} - 1\right), \tag{1}$$

where the D and G parameters are chosen so that time is shifted relative to the onset of cellphone adoption. This expression is essentially the addition of two sigmoid curves. The first models the adoption of the landline phone as a new innovation; and the second models the de-adoption of landlines as an independent innovation of a "wireless-only" lifestyle, which has a subtractive effect total landline usage.

Likewise, the consumption by cellphones is given by

$$p_c(t) = \frac{J}{1+e^{-K(t-L)}}, \tag{2}$$

where again L is a time shift chosen to make the model coincide with cellphone adoption.

We tried to model this at the microscopic level, but that proved to be an intractable approach. From census data, the number of households with m members over the course of history is readily available [U.S. Census Bureau 2007]. Equally accessible are the rates of penetration and average costs of cellular and landline communications penetration [U.S. Census Bureau 2001; Eisner 2008]. With this abundance of data, one may be tempted to propose an econometric forecast of telephony usage that is driven by the marginal cost-benefit analysis that a household performs. However, determining the functional form that defines the behaviors that are muddled by habits and irrationality are troubling. When reduced to a first-order approximation, such a model still requires the calibration of numerous parameters [Koyck 1954]. After attempting such an approach several times, we abandoned it. We believe the above model captures the data equally well without making undue assumptions.

Energy Cost of Landlines

Together these two functions model three processes: landline adoption, wireless adoption, and wireless only adoption. Additionally, they describe the long-term behavior of these processes as they reach a steady state. To approach the question of annual energy consumption by telephony products, we combine these functions with models for energy expenditure by landline phones and their peripherals, as well as cellphones and their peripherals. The formula for energy consumption by landline phones and peripherals is

$$E_l(t) = Pp_l h(\pi_a e_a + \pi_b e_b + \pi_c e_c + \pi_d e_d).$$

Table 1 delineates the variables and their explanations. The time variable t is normalized so that $t = 0$ denotes 1960.

Table 2.

Variables and their meanings.

Variable	Description
$P(t)$	Population of U.S. in year t
$p_l(t)$	Landlines per person in U.S.
$h(t)$	Handsets per landline
$\pi_a(t)$	Percentage of landline owners with corded phones
$e_a(t)$	Yearly Energy Consumption (YEC) (kWh) by corded phones
$\pi_b(t)$	Percentage of landline owners with cordless phones
$e_b(t)$	YEC by cordless phones
$\pi_c(t)$	Percentage of landline owners with combination cordless phone/answering machines
$e_c(t)$	YEC by combination cordless phone/answering machines
$\pi_d(t)$	Percentage of landline owners with separate answering machines
$e_d(t)$	YEC by separate answering machines

Due to a lack of relevant data, we make several assumptions:

- All yearly energy consumption functions are constant over time. Because corded phones draw their energy solely from phone lines, there is little room for variation in their power draws, so this at least seems reasonable. However, answering machines, cordless phones, and combinations of the two do not have this restriction, and it seems likely that they are becoming more energy efficient with time. However, no data were available to support this hypothesis, so we fixed YEC based on available sources.
- The adoption of cordless vs. corded phones and answering machines no doubt follows its own sigmoidal curve, but again no data are available. So the variables $h, \pi_a, \pi_b, \pi_c, \pi_d$ are all modeled as first-order linear approximations.

Regardless, results produced by the model agree well with available data for energy consumption.

Energy Cost of Cellphones

The energy cost for cellphones can be modeled as

$$E_C(t) = Pp_c(E_{c1} + E_{c2}),$$

where

$$E_{c1}(t) = f_C(C_{\text{charge}} t_{\text{charge}} + C_{\text{standby}} t_{\text{standby}})$$

and

$$E_{c2}(t) = R_{\text{cell}}R(t).$$

Table 2 describes each relevant variable.

Table 2.

Variables and their meanings.

Variable	Description
$P(t)$	Population of U.S. in year t
$p_c(t)$	Number of cellphones per person
E_{c1}	YEC by cellphones and chargers
E_{c2}	YEC by cellphone recyclers
f_C	Frequency of cellphone charging
C_{charge}	Charger wattage during charging
t_{charge}	Daily charger time spent charging
C_{standby}	Charger wattage during standby
t_{standby}	Daily charger time spent in standby
R_{cell}	Energy needed to recycle one cellphone battery
$R(t)$	Percentage of cellphones recycled in year t

The immediate contributions to cellphone energy consumption are charging the phone and leaving the charger plugged in with no phone attached. It is difficult to find data on cellphone charging frequency. Rosen et al. [2001] argue that people charge their phone 50 times each year at their residence (noting that many people charge the phone in their car); but this figure seems very low. Newer phones with a multitude of features require more-frequent charging. Since charging the cellphone has developed into a habit for most people, we assume that people charge the phone every night and keep the charger attached to an outlet all the time.

Rosen et al. [2001] observe that the average time to charge a cellphone is 2 hrs, which seems low in comparison to other data, which suggest 3–4 hrs to charge to 80% and an additional 8 hrs to charge to 100%. However, a phone charged every night is unlikely to have a nearly-empty battery. We assume that the overnight charging does not affect the 2-hr charging time. That 50% of cellphone batteries are lithium-ion batteries, which do not allow for overcharging, justifies this assumption [Fishbein 2002; Rosen et al. 2001]. Once a lithium-ion battery is charged, the power drawn differs negligibly from that when no phone is connected to the charger [Rosen et al. 2001]. Therefore, we feel justified in adopting Rosen et al.'s statistic.

To model the energy cost of recycling used cellphone batteries, we consider the batteries to be recycled by the Rechargeable Battery Recycling Corporation, justified by its significant market share and the fact that it recycles batteries in the U.S. [Office Depot 2004].

Energy Optimization

Given the above functions for energy costs for cellular and landline telephone usage, we can optimize energy consumption. A Pseudo U.S. with the approximate size of the U.S. would likely have a similar distribution of household size.

Let H_m be the number of households with m members and l_m the fraction of households with m members that have landline service. If we assume that the communication needs of every family are satisfied by either having a landline or by each member possessing a cellphone, the numbers of required cellphones T_c and landline phones T_l can be calculated as

$$T_l = \sum_{m=1}^{7} l_m H_m, \qquad T_c = \sum_{m=1}^{7} m(1 - l_m) H_m.$$

We believe that in the absence of a landline, members of a household will not share cellphones.

The total telephony energy demand of the proposed plan for Pseudo U.S. is

$$E(t) = E_l(t) + E_c(t).$$

Using only landlines would minimize the number of telephone units required; however, landline phones and their companion technologies are much less energy-efficient than cellphones. Using only cellphones would maximize the number of telephone units required; and though the energy cost per unit is reduced, the overall increase in units may have deleterious consequences. Therefore, we optimize the variables l_m to yield the best communications strategy from an energy perspective.

We could modify the above summations to consider roles played by cellphones that are not achievable by a landline. For example, suppose that a single landline cannot serve a large family. If n is the number of people a single landline can serve in a household, we may assume that a family of m with one landline will need to purchase $(m - n)$ cellphones. Then we have

$$T_c = \sum_{m=1}^{7} m(1 - l_m) H_m + \sum_{m=n+1}^{7} l_m H_m (m - n),$$

where the second term gives the fraction of families too large to be served by a single landline. Implicit in this formula is an assumption that no family obtains a second landline. This is reasonable, since the average number of landlines per household in the U.S. is only 1.118 [Eisner 2008].

Likewise, we could further complicate the cost function by asserting that not every family member requires a cellphone if a landline is absent.

However, we find that such a modification does not enrich the conclusions of our optimization.

Results

Energy Consumption

Using the above information, we create an energy consumption function:

$$E(t) = E_c(t) + E_l(t).$$

To make this specific, we must estimate parameter values for $A, \ldots, G$ in **(1)** and **(2)**. Using an optimization algorithm described in the methods section below, we arrive at the conclusions in **Table 3**.

Table 3.

Values of parameters, as fitted from data in Eisner [2008].

Parameter	Value
A	1.1263
B	1.0924
C	0.0423
D	27
E	0.0109
F	0.1587
G	30

Moreover, functions can be described for parameters for E_l and E_c. **Tables 4** and **5** give values for the variables and parameters in **Tables 1** and **2**.

Table 4.

Values for variables in **Table 1**. Source: Rosen et al. [2001].

Variable	Value
$P(t)$	Population growth as predicted by the Census Bureau
$p_l(t)$	As defined in **(1)**
$h(t)$	$1.89\text{E}^{-3}t + 1.076$, $t \leq 40$; $-1.20\text{E}^{-3} + 1.152$, $t > 40$
$\pi_a(t)$	$1 - \pi_b(t) - \pi_c(t)$
$e_a(t)$	20 kWh
$\pi_b(t)$	$\max(0{,}1.45\text{E}^{-2}t - 1.45\text{E}^{-1})$, $t \leq 40$; $.44{,}t > 40$
$e_b(t)$	28 kWh
$\pi_c(t)$	$\max(0{,}1.07\text{E}^{-2}t - 1.07\text{E}^{-1})$, $t \leq 40$; $.32{,}t > 40$
$e_c(t)$	36 kWh
$\pi_d(t)$	$\max(0{,}2.31^{-2}t - 2.31\text{E}^{-1})$, $t \leq 40$; $.69{,}t > 40$
$e_d(t)$	36 kWh

Table 5.

Values for variables and parameters in **Table 2**. Source: Rosen et al. [2001].

Variable	Value
$p_c(t)$	as defined in **(2)**
E_{c1}	$0.365(4 \cdot 2 + 0.6 \cdot 24)$ kWh
E_{c2}	$-0.0283e^{\frac{-(t-1993)}{17.1573}} + 0.00037$ kWh
f_C	365
C_{charge}	4 W
t_{charge}	2 hr
C_{standby}	0.6 W
t_{standby}	24 hr
R_{cell}	0.0037 kWh
$R(t)$	$-7.639e^{-(t-1993)/17.1573} + 0.0999$

From our model, we expected that by 2050 cellphones will have completely replaced landlines in the U.S. Thus, we estimate steady-state energy consumption as $E(90) = 2.99$ TWh/yr, equivalent to 1.7 million bbl/yr of oil.

Energy Optimization Results

From our optimization results for the distribution of telephone types in the Pseudo U.S., we find that it is almost always preferable to have a cellphone-only state, in terms of energy efficiency. Even assuming a landline can service an unlimited number of people in a household, our optimization finds that only for families of size 7 or larger it is energy-efficient to own a single landline and peripherals in place of a cellphone for each family member.

The cost of leaving cellphone chargers on standby when not active would amount to approximately 62$\%$ of the total YEC, or 862,000 bbl/yr of oil.

Energy Waste by Other Household Electronics

We also discuss the impact of leaving devices plugged in when the device is not in use. From Rosen et al. [2001], we adopt the following approach. First, we investigate the average wattage used in standby mode by the devices under consideration and the time spent in standby mode, respectively. Then we find saturation and penetration values to find the total energy expenditure in the U.S. We consider computers, TVs, set-top boxes (digital and analog), wireless set-top boxes, and video-game consoles.

We take the data for the three types of set-top boxes and the video-game console from Rosen et al. [2001]. Furthermore, the average American spends an average of 4.66 hours watching television and 4.4 hours using a computer every day [Bureau of Labor Statistics 2009]. Average power drawn by computers and television sets turns out to be 4 and 5.1 W [Rosen et al. 2001]. The first two columns of **Table 6** give our data set. We use that information, along with saturation rates and household penetration rates [Eisner 2008], to arrive at the figures in the third column.

Table 6.
Data used for power consumption of household electronics. Source: Thorne and Suozzo [1998].

Device	Standby time (proportion)	Power drawn in use (W)	Standby power consumption (TWh/yr)
Set-top box, analog	.78	10.5	3.2
Set-top box, digital	.78	22.3	0.6
Wireless receiver	.78	10.2	1.4
Video-game console	.98	1.0	0.5
TV	.80	5.1	10.3
Computer	.81	4	3.3

We conclude that wasteful energy expenditure due to appliance standby in the U.S. consumes approximately 11.4 million bbl/yr of oil.

Future Predictions

Assuming moderate economic and population growth, **Table 7** shows results for the Pseudo U.S., using population projections from the U.S. Census Bureau [1996a; 1996b].

Table 7.
Projected energy use in Pseudo U.S.

Year	Energy ($\times 10^6$ bbl oil)
2010	1.14
2020	1.24
2030	1.66
2040	1.77
2050	1.89

However, we believe that such an analysis is of limited use. Predicting the future of so many variables for a 50-year period is extremely difficult, especially in the realm of technology, where it is commonplace for innovations to change social paradigms. For example, consider an attempt in the 1950s to model the growth of computer usage. Any such attempt would have been unlikely to foresee personal computers, the Internet, or cellphones (which today are rapidly replacing many of the functions of personal computers). Likewise, the energy cost of cellphones may vary greatly due to changes in technology: Social awareness about energy efficiency may drive them to ever-lesser energy consumption, but also they may gain additional features or be replaced by miniaturized computers that result in more energy consumption.

Conclusions

Recommendations

From and energy perspective, we find that it is more efficient to abandon landlines in favor of cellphones. This suggestion is reinforced by the model prediction, which suggests an elimination of landlines in the near future by consumer adoption of a wireless-only lifestyle.

Finally we find that the waste generated by chargers on standby (i.e., not charging a device) are a significant source of energy waste. We therefore advocate that efforts be made to forgo convenience and unplug devices when in standby.

Model Strengths and Weaknesses

Strengths

- The model reproduces sigmoidal innovation-adoption behavior without making undue assumptions about the underlying processes.
- The model incorporates a broad span of indirect sources of energy consumption: battery recycling, commuters with cellphones, landline companion technologies.

Weaknesses

- Our model captures only global adoption behavior. This exclusion of underlying behavior is a detriment in capturing deviations from the standard behavior, as was exemplified by the underestimation in the 1990s, when economic expansion may have driven telephone adoption.
- Due to lack of data, the model relies on interpolation of data related to cellphone and landline energy costs.
- For simplicity, the model excluded other possible communications technologies. As noted earlier, paradigm shifts in technology are commonplace yet hard to predict.
- The perspective excludes other communications technologies.
- The model fails to capture any benefit of landlines not provided by cellphones. It may be that landlines are associated with a certain degree of security, which mediates the current prediction that landlines will be completely abandoned.

Future Work

- We believe that a model at the microscopic level that takes into consideration consumer perceptions and habits, in addition to economic data, would perform the best.
- We also believe with Bagchi [2008] that modeling cellphones and landlines as more directly competing products with reference to economic data would provide better data fits and predictions.
- The analysis is limited to the household level. Landline phones will persist in many businesses, and we believe that this persistence will be a significant factor in energy consumption.

References

Bagchi, Kallol. 2008. The impact of price decreases on telephone and cell phone diffusion. *Information and Management* 45 (2008): 183–193.

Eisner, James. 2008. Table 16.2: Household Telephone Subscribership in the United States. In Trends in Telephone Service (March 2008), Federal Communications CommissionIndustry Analysis and Technology Division, Wireline Competiton Bureau. `http://hraunfoss.fcc.gov/edocs_public/attachmatch/DOC-284932A1.pdf`.

Fishbein, Bette. 2002. *Waste in the Wireless World: The Challenge of Cell Phones*. New York: INFORM, Inc. `http://www.p2pays.org/ref/19/18713.htm`.

Koyck, L.M. 1954. *Distributed Lags and Investment Analysis.* Amsterdam: North-Holland.

Office Depot. 2004. Office Depot launches free cell phone and rechargeable battery recycling program. `http://mediarelations.officedepot.com/phoenix.zhtml?c=140162&p=irol-newsArticle&ID=674008`.

Rosen, Karen, Alan K. Meier, and Stephan Zandelin. 2001. Energy use of set-top boxes and telephony products in the U.S. `http://repositories.cdlib.org/cgi/viewcontent.cgi?article=1950&context=lbnl`.

Thorne, Jennnifer, and Margaret Suozzo. 1998. Leaking electricity: Standby and off-mode power consumption in consumer electronics and household appliances. `http://www.aceee.org/pubs/a981.htm`.

U.S. Bureau of Labor Statistics. 2009. American time use survey summary—2008 results. `http://www.bls.gov/news.release/atus.nr0.htm`.

U.S. Census Bureau.. 1996a. Projections of the number of households and families in the United States: 1995 to 2010. Current Population Reports P25–1129. `http://www.census.gov/prod/1/pop/p25-1129.pdf`.

________. 1996b. Population projections of the United States by age, sex, race, and Hispanic origin: 1995 to 2050. Current Population Reports P25–1130. `http://www.census.gov/prod/1/pop/p25-1130.pdf` .

________. 2001. Construction and housing. Chapter 25 in Statistical Abstract of the United States: 2000. `http://www.census.gov/prod/2001pubs/statab/sec25.pdf` .

________. 2007. U.S. households by size, 1790–2006. Reproduced at `http://www.infoplease.com/ipa/A0884238.html` .

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America's New Calling

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Summary

The ongoing cellphone revolution warrants an examination of its energy impacts—past, present, and future. Thus, our model adheres to two requirements: It can evaluate energy use since 1990, and it is flexible enough to predict future energy needs.

Mathematically speaking, our model treats households as state machines and uses actual demographic data to guide state transitions. We produce national projections by simulating multiple households. Our bottom-up approach remains flexible, allowing us to

- model energy consumption for the current U.S.,
- determine efficient phone adoption schemes in emerging nations,
- assess the impact of wasteful practices, and
- predict future energy needs.

We show that the exclusive adoption of landlines by an emerging nation would be more than twice as efficient as the exclusive adoption of cellphones. However, we also show that the elimination of certain wasteful practices can make cellphone adoption 175% more efficient at the national level. Furthermore, we give two forecasts for the current U.S., revealing that a collaboration between cellphone users and manufacturers can result in savings of more than 3.9 billion barrels-of-oil-equivalent (BOE) over the next 50 years.

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Problem Background

In 1990, less than 3% of Americans owned cellphones [International Telecommunication Union n.d.]. Since then, a growing number of households have ditched their landline in favor of cellphones for each household member. We develop a model for analyzing how the cellphone revolution impacts electricity consumption at the national level. In particular, we

- assess the energy cost of the cellphone revolution in the U.S.,
- determine an efficient way of introducing phone service to an nation like the U.S.,
- examine the effects of wasteful cellphone habits, and
- predict future energy needs of a nation (based on multiple growth scenarios.)

Assumptions

- The population of the U.S. is increasing at roughly 3.3 million people per year [U.S. Census Bureau 2009].
- The relatively stable energy needs of business and government landlines, payphones, etc. have a negligible impact on energy consumption dynamics during the household transition from landlines to cellphones.
- No household member old enough to need phone service is ever without it.
- Citizens with more than one cellphone are rare enough to have a negligible energy impact.
- The energy consumption of the average cellphone remains constant. Future changes in cellphone energy requirements depend largely on changes in user habits and in manufacturing efficiency, so are difficult to predict. However, we drop this assumption in our final section.

Energy Consumption Model

Our approach involves three steps:

- We model households as state machines with various phones and appliances.
- We use demographic data to determine the probability of households changing state.

- By simulating multiple households, we extrapolate national energy impacts.

Households

The basic component of our model is the household. Each household has the following attributes:

m : number of members old enough to need a telephone.

t : number of landline telephones.

c : number of members with cellphones.

The state of each household can be described in terms of the above values. We generate m from available demographic data and hold it constant.

A household can exist in one of four disjoint states at a time; each state has two associated conditions:

- Initial State: when a household uses only landline telephones:
  $t > 0,\ c = 0$
- Acquisition State: after a household acquires its first cellphone:
  $t > 0,\ 0 < c < m$
- Transition State: after all household members have their own cellphone but the landline is retained:
  $t > 0,\ \ c = m$
- Final State: after the household abandons landline telephones:
  $t = 0,\ c = m$

These states are disjoint. We do not assume that all states are reached during the timeline of a household. We assume that cellphones, once acquired, are never lost, and that landlines, once dropped, are never readopted. Thus, a household will never re-enter a state that it has left. Thus, a household will reach one or more of the above states in the order listed.

Consider a household with three members ($m = 3$), one landline telephone ($t = 1$), and no cellphones yet (c = 0). **Figure 1** shows the timeline of such a household as it moves through the four phases.

Our model generates household state-transition probabilities from demographic data. However, this process is simulation-dependent, as we discuss later.

Nations

Households are only part of the story. We model the national timeline during the country-wide transition from landlines to cellphones as a composition of multiple overlapping household timelines. Furthermore, the

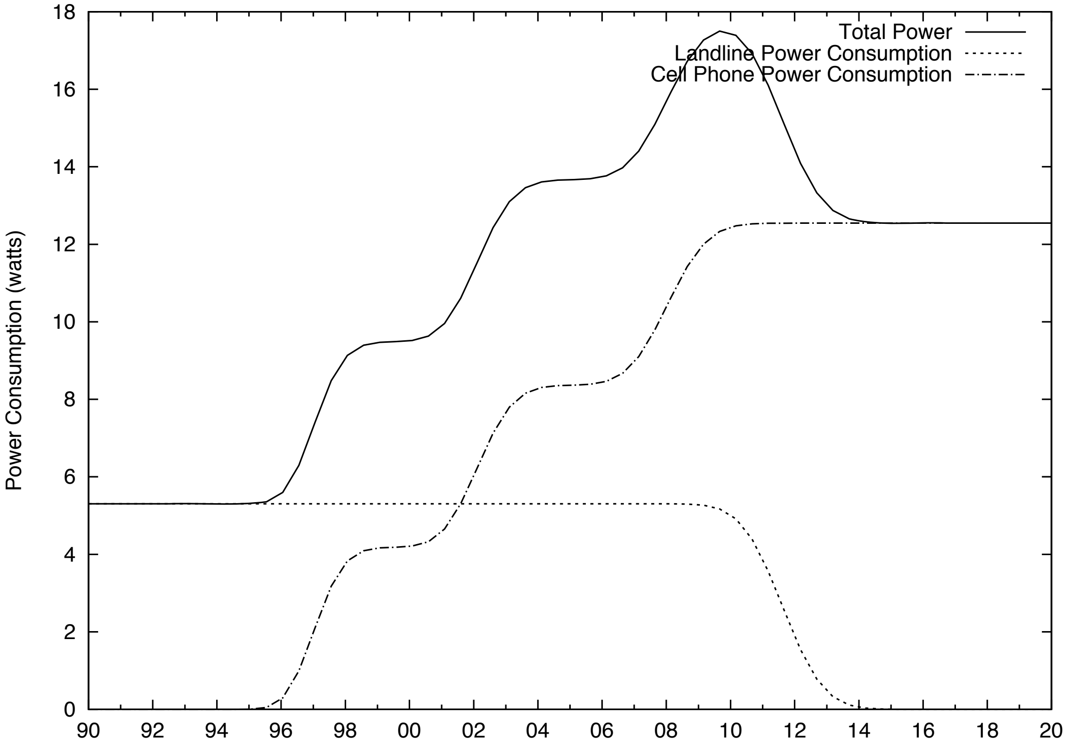


Figure 1. Power consumption timeline for a hypothetical household.

decisions that households make regarding when to acquire cellphones and when to abandon landlines depend on the larger national context. For example, a household would be much more likely to acquire its second or third cellphone in 2008 than it would have been in 1990.

A hypothetical nation with only three households might have the timeline composition of **Figure 2**. That the three household power usages converge is a result of there being three members in each household.

We proceed to construct such a timeline for the U.S. We average the power consumption over all households in the U.S. to generate a national timeline like that in **Figure 3**.

The Current U.S.

Using Technological Data

To use our model in conjunction with relevant data, we have to calculate:

C_{wattage} : the average power consumption of a cellphone over its lifetime.

L_{wattage} : the average power consumption of a landline phone over its lifetime.

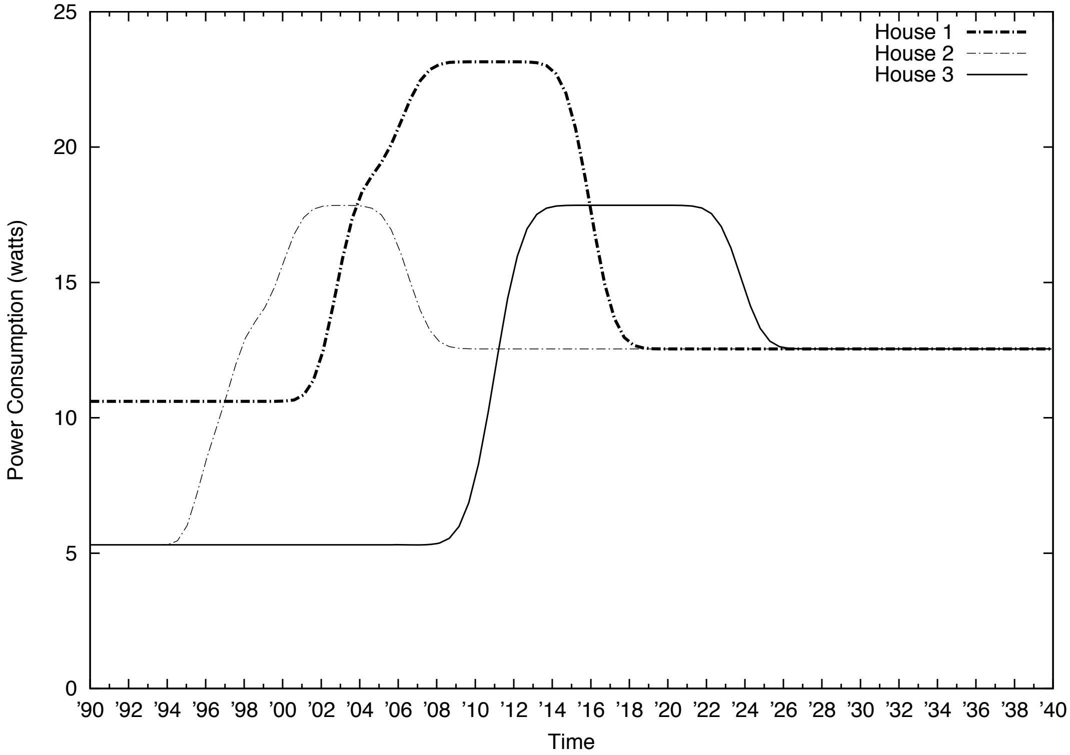


Figure 2. Power consumption timeline for three hypothetical households.

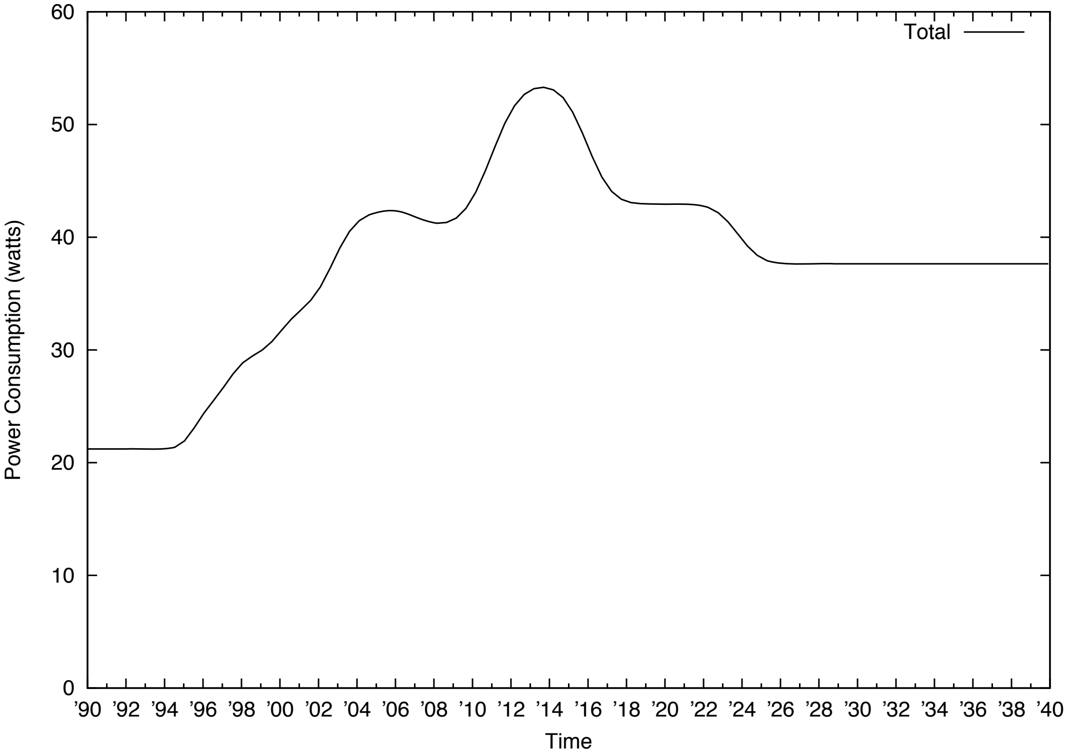


Figure 3. Power consumption timeline for a hypothetical nation consisting of the three households shown previously.

We deal with only cordless landline phones because corded phones use minimal levels of energy and are ignored in the literature that we consulted [Frey et al. 2006; Rosen et al. 2001].

We derive C_{wattage} as follows:

$$C_{\text{wattage}} = \text{Charger}_{\text{wattage}} + \frac{C_{\text{upfront}}}{C_{\text{lifetime}}}.$$

We add to the usage wattage of the charger the upfront energy cost (in joules) manufacturing a cellphone, amortized over the lifetime of a cellphone (in seconds). The bulk of energy consumption occurs in manufacturing and use [Frey et al. 2006], so we ignore the rest of a phone's life cycle (e.g., shipping).

For cellphones, Frey et al. [2006] give:

$$C_{\text{upfront}} = 148 \text{ MJ}, \qquad C_{\text{lifetime}} = 2 \text{ yr}, \qquad \text{Charger}_{\text{wattage}} = 1.835 \text{ W}.$$

Analogously, for corded phones we have:

$$L_{\text{wattage}} = \text{Cordless}_{\text{wattage}} + \frac{L_{\text{upfront}}}{L_{\text{lifetime}}}.$$

Though there are many different kinds of cordless phones, we use the values for cordless phones with integrated answering machines, as determined by Rosen et al. [2001]:

$$L_{\text{upfront}} = 167 \text{ MJ}, \qquad L_{\text{lifetime}} = 3 \text{ yr}, \qquad \text{Cordless}_{\text{wattage}} = 3.539 \text{ W}.$$

Thus, our simulation uses the following values:

$$C_{\text{wattage}} = 4.182 \text{ W}, \qquad L_{\text{wattage}} = 5.304 \text{ W}.$$

Demographic Data

We need demographic data to guide the transition of household states over the course of a simulation. We could allow houses to decide randomly when and whether to adopt new cellphones, as well as when and whether to drop their landline. However, we prefer to use actual penetration data to probabilistically weight household decisions.

Consider the household decision of whether to purchase a cellphone in month M. We use a three-step process to produce the cellphone acquisition probability function $a(M)$ employed in our simulation:

1. Find historic data about the number of cellphone owners over time.
2. Interpolate between data points.

3. Define $a(M)$, the probability of a simulated household acquiring a cellphone in month M.

For step 1, we use data from the International Telecommunication Union [n.d.]. In step 2, we use linear interpolation between available data points to make a continuous function from 1990 (the start of our simulation) to 2009, as shown in **Figure 4**.

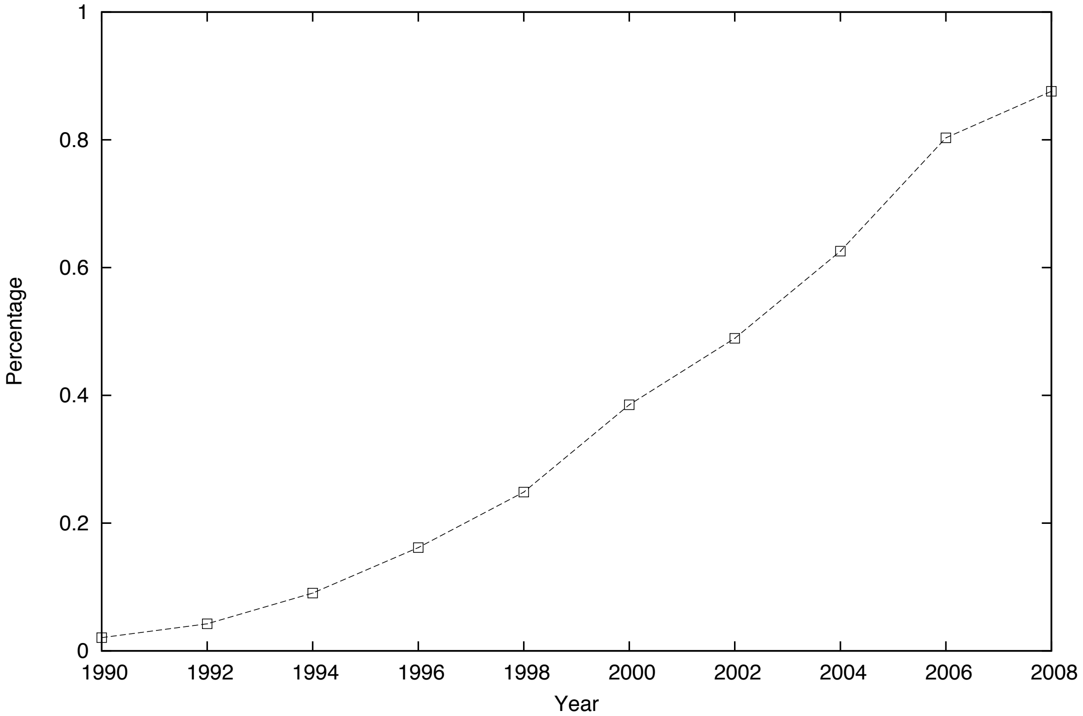


Figure 4. Cellphone penetration demographics.

Then we use a linear regression to extrapolate between 2009 and 2040. Call this function f. Then, for step 3, we have

$$a(M) = f(M) - \frac{\sum_{H\in\text{Houses}} c(H,M)}{\sum_{H\in\text{Houses}} m(H,M)}, \qquad \textbf{(1)}$$

where

$c(H,M)$ is the number of cellphones owned by members of simulated household H in month M,

$m(H,M)$ is the number of members in simulated household H in month M, and

the summations are over all households in the simulation.

In essence, **(1)** subtracts the current simulated cellphone penetration during month M from the approximated market penetration, $f(M)$, which is derived from available data.

Using $a(M)$, the households in our simulation make decisions that approximate historical data. As the second term in **(1)** approaches the historical value returned by $f(M)$, the chance of a simulated household buying a cellphone decreases to zero.

We perform an almost identical process with historic landline ownership data to determine the probability of a household dropping its landline in month M; we omit the details. Mnemonically, however: $a(M)$ is the probability of acquiring a cellphone, and $d(M)$ is the probability of dropping a landline.

Simulating the Current U.S.

The historical demographic data help guide our simulation, and technological data help us calculate power consumption at any point during the simulation.

We algorithmically generate household timelines as follows:

```
While month M is before end date
  For every house H do
    if H is in 'initial' or 'acquisition' state
      get a new cellphone with probability a(M)
    if H is in 'transition' state
      get rid of landline with specified probability d(M)
  End For
  Calculate power consumption.
  Let M = M + 1 month
end while
```

The power consumption is calculated from C_{wattage}, L_{wattage}, and current phone ownership.

Figure 5 shows the timeline of power consumption for the U.S. over the past 19 years, with future projections. Interesting features of this graph are:

- The steep energy consumption as Americans acquire cellphones yet retain their landlines.
- The drop after cellphone penetration slows and landlines are abandoned.
- The rising slope after households have dropped their landlines and the population grows.

At first, most households tend to be in an Acquisition State, having both landlines and an increasing number of cellphones. Next, households begin to progress to a Transition State, slowly dropping landlines while retaining cellphones—hence, overall consumption drops. The final upward slope represents the steady state, in which population growth (and associated cellphone acquisition) is the only factor affecting energy consumption.

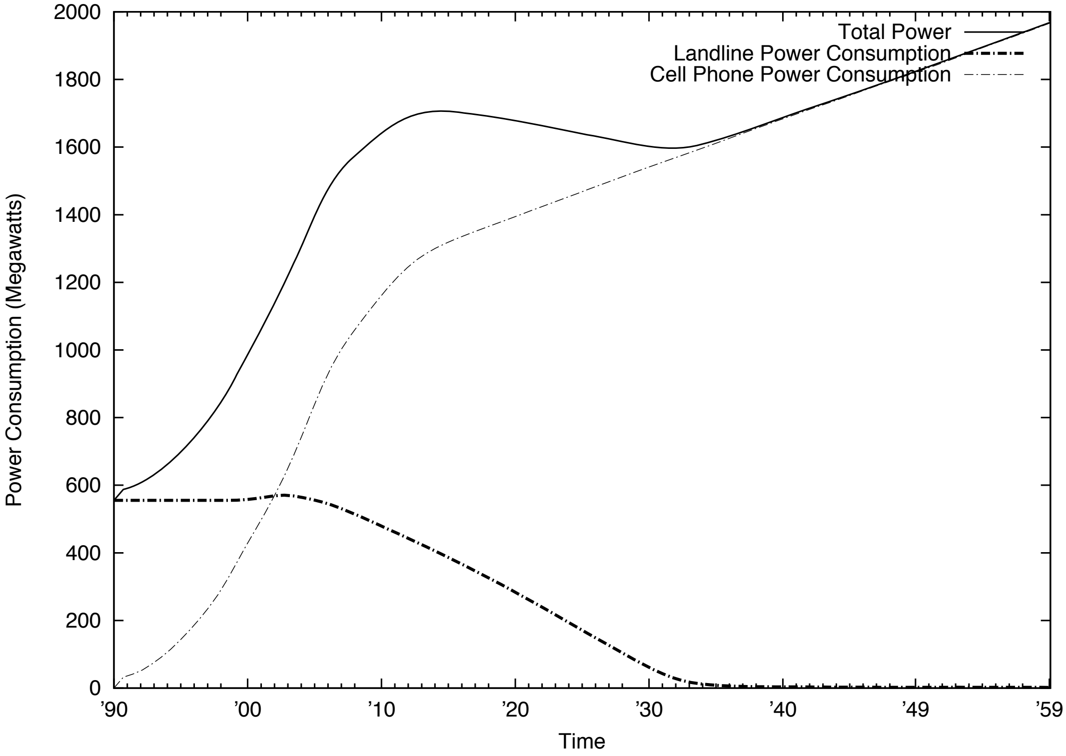


Figure 5. Energy consumption timeline for the U.S. over the past 19 years, with future projections.

Optimal Telephone Adoption

For an emerging nation without phone service but with an economic status roughly similar to the current U.S., we examine two hypothetical scenarios for introducing phone service:

- Cellphones Only, or
- Landlines Only

Because it took Russia roughly 6 years for cellphone penetration to go from 2% to 105% [International Telecommunication Union. n.d.], we assume a similar timescale for introducing cellphones to our hypothetical nation. Furthermore, a country with the same economic status as the U.S. should be capable of making a similarly quick adoption of either cellphones or landline phones, even though landline phone infrastructure involves the extra complexity of laying cables.

Cellphones Only

For our cellphone introduction plan, we assume that 0% of the population in 2009 have cellphones and that 100% of the population in 2015 have cellphones. If we interpolate linearly between these two dates, we can derive the number of people with a cellphone in any month during the 6-year period. If we assume that the rate at which cellphones consume energy remains roughly the same between 2009 and 2015, then we have all the information we need to run our simulation.

The only major change that we make to our model is that the Initial State of a household now involves having no phones at all, and the Final State involves each household member owning a cell phone.

The steep slope levels off when cellphone market penetration reaches 100%, and the only relevant factor after that is population growth (**Figure 6**, top curve).

Landlines Only

We alter our model so that the Initial State of a household still involves having no phones and the Final State involves having one landline (**Figure 6**, bottom curve).

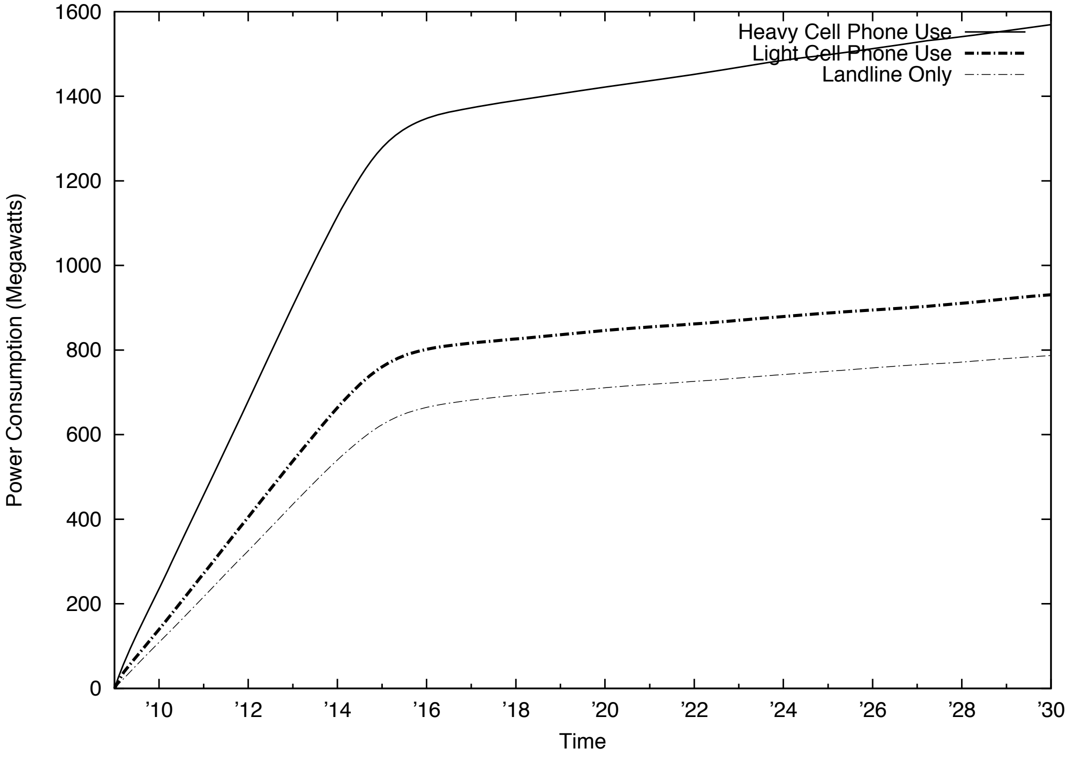


Figure 6. Power consumption comparison of adoption plans: cellphone only (top curve), "cellphone light" use (middle curve), and landline only (bottom curve).

The Landlines Only plan requires less than half the power of the Cellphones Only plan. However, we prefer to delay our recommendation. First, we examine a way to make cellphone adoption more energy efficient.

Waste and "Vampire" Chargers

Although the above comparison shows Landlines Only to be a clear winner, we should take into account that the rate at which cellphones consume energy varies depending on the practices of users. Until now, we have assumed that the energy consumption of a cellphone is equal to the consumption of its charger—even though many people do not use their charger as conservatively as they could. We now relax this assumption and assess the total cost of certain wasteful practices by supposing that our hypothetical nation's citizens

- never charge a cellphone after it is finished charging and
- never leave their charger plugged in when not charging the phone.

The value for C_{wattage} that we calculated earlier was based on the assumption of Frey et al. [2006] that cellphone chargers spend their lifetimes plugged in—mostly in standby ("vampire") mode. **Figure 7** shows, in barrels-of-oil-equivalent (BOE), the amount of energy wasted each month by vampire charging.

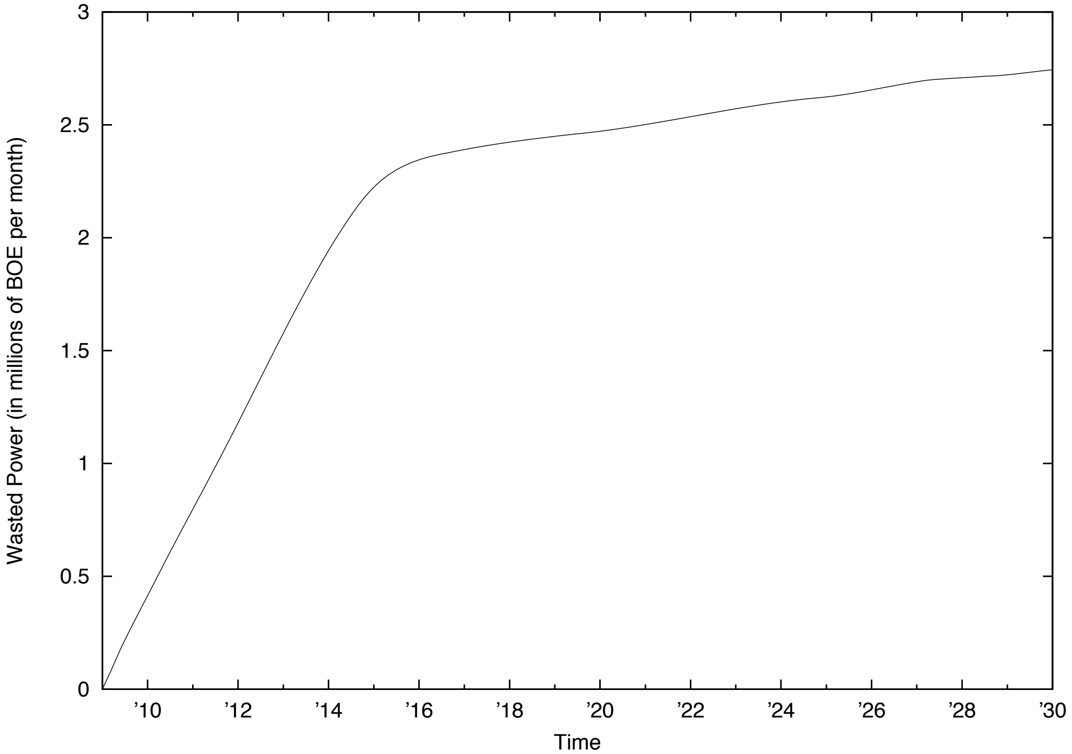


Figure 7. Barrels-of-oil-equivalent (BOE) wasted due to vampire charging.

We now derive a new value for C_{wattage} based on Roth and McKenney [2007], which shows that the average cellphone needs to spend only 256 hr/yr charging. In short, we make C_{wattage} depend strictly on its minimum battery requirements and assume that users charge their phones only enough to keep them charged for the entire day. Roth and McKenney also suggest that chargers require 3.7 W when charging.

$$C'_{\text{wattage}} = \text{Battery}_{\text{wattage}} + \frac{C_{\text{upfront}}}{C_{\text{lifetime}}}, \qquad \textbf{(2)}$$

$$\text{Battery}_{\text{wattage}} = \frac{\text{Time spent charging}}{\text{Lifetime}} \times \text{Wattage when charging}$$

Thus, $\text{Battery}_{\text{wattage}} = \frac{256(\text{hr})}{8760(\text{hr})} \times 3.7\,\text{W} = 0.108\,\text{W}.$ The second term in **(2)** is the same as in **(1)**. So,

$$C'_{\text{wattage}} = 2.455\,\text{W}.$$

Recall that our previous value was

$$C_{\text{wattage}} = 4.182\ \text{W}.$$

The middle curve of **Figure 6** shows the lower energy expenditure of this "cellphone light" use.

Other Household Appliances

Generalizing our previous analysis, we now assume that households do not simply use cellphones and/or landlines. They also each have the following common appliances:

- 0 or 1 computer (50% have 1) [Newburger 2001],
- 0 or 1 DVD player (84% have 1) [Nielsen. . . 2007], and
- 2 or 3 TVs [Nielsen. . . 2007].

We select these appliances because they are responsible for a significant amount of household energy consumption [Floyd n.d.]. The "vampire" energy leakage from these appliances is:

Computer	2.63 W	[Roth and McKenney 2007]
DVD player	3.64 W	[Roth and McKenney 2007]
TV	6.53 W	[Floyd n.d.]

The graph of a single household might look like **Figure 8**. **Figure 9** shows our hypothetical nation's wasted power, interpreted in barrels-of-oil-equivalent (BOE).

Clearly, telephone-related energy loss is a significant contributor to the overall energy consumed by the U.S. However, other electrical appliances have a larger impact.

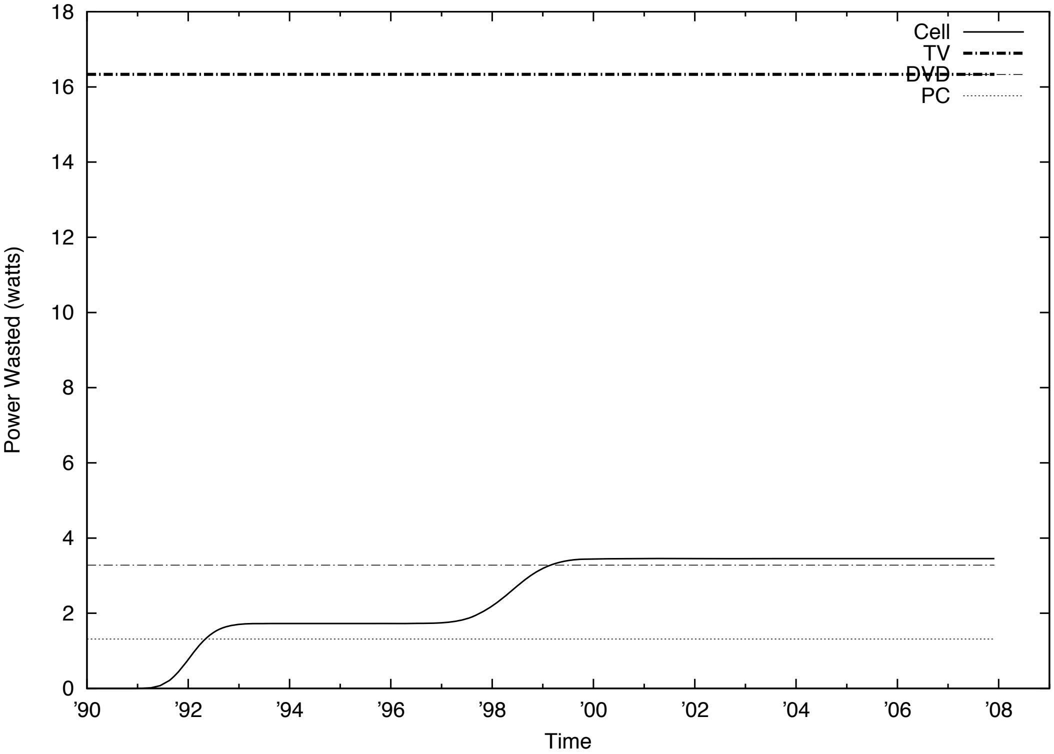


Figure 8. Energy consumption timeline for a household with various appliances, transitioning from landlines to cellphones.

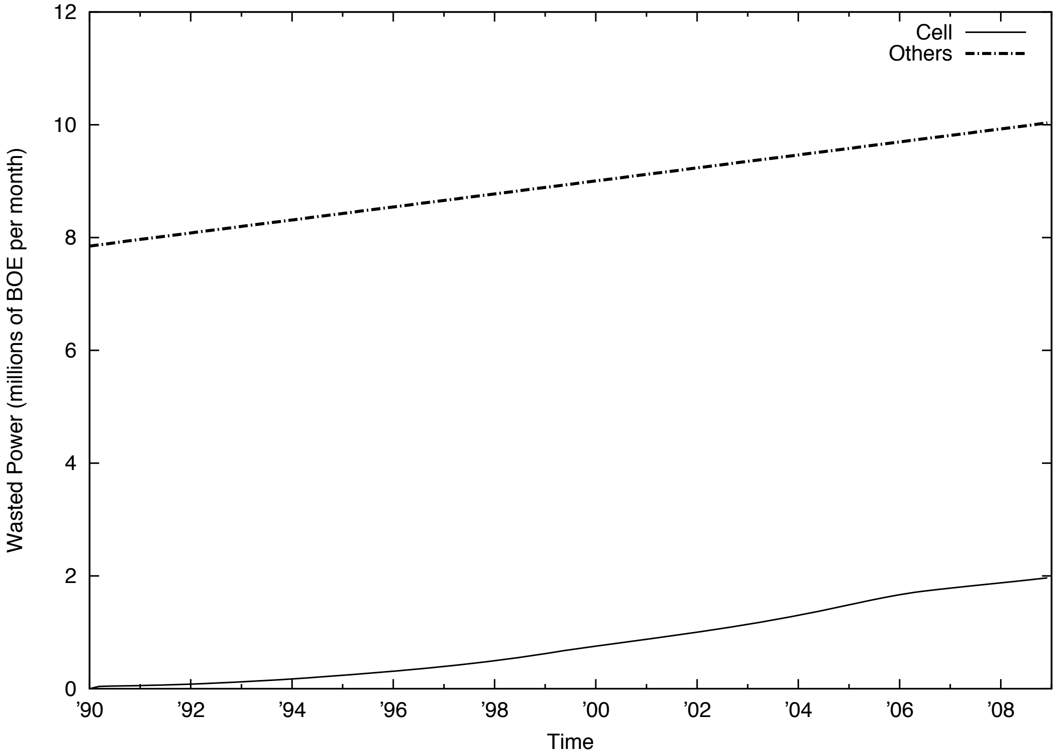


Figure 9. Energy consumption timeline for a household with various appliances, transitioning from landlines to cellphones.

Predictions

Here we tie our previous work together into a predictive simulation that investigates the energy impact of the following eventualities:

- Cellphone efficiency stays the same.
- Cellphone efficiency decreases (e.g., with the introduction of smartphones).
- People save 50% of energy currently lost to "vampire" charging.
- People do not stop "vampire" charging.

In all cases, the population of the nation is assumed to grow at 3 million people per year—a rate comparable to that of the current U.S.

Optimistic Prediction

For our optimistic prediction, we assume that cellphone energy requirements remain constant with each successive generation of cellphones and the population eliminates 50% of energy consumption due to "vampire" charging.

Recall that our best-case value for the use-phase power consumption of a cell phone (no vampire charging) is

$$\mathrm{Battery}_{\mathrm{wattage}} = 0.108 \text{ W},$$

and our worst-case scenario (charger always plugged in) is

$$\mathrm{Charger}_{\mathrm{wattage}} = 1.835 \text{ W}.$$

We choose a use-phase value half-way between the two:

$$\mathrm{Realistic}_{\mathrm{wattage}} = 0.9715 \text{ W}.$$

As in **(1)** and **(2)**, we add this to the manufacturing-phase energy cost to obtain an optimistic (but not too optimistic) average cellphone wattage. With this value, we graph in **Figure 10** the power consumption over the next 50 years.

Landline telephone usage still contributes significantly to the total power consumption of the nation until the year 2030. The cellphone power consumption trend may not be meaningful until looked at alongside the pessimistic prediction.

Pessimistic Prediction

We assume that cellphone energy requirements increase with each successive generation of cellphones at a rate comparable to the increase from regular cellphones to smartphones. In short, we are modeling the transition from landlines to cellphones to smartphones. We also assume that the population does not manage to avoid "vampire" energy loss.

Because smartphone technology exists in a state of relative infancy, technical information about it is scarce. Thus, we make an estimate of the average wattage of a smartphone based on the fact that for all tasks (emailing, text messaging, idling, etc.) a smartphone requires more than twice as much power as a regular cellphone [Mayo and Ranganathan 2005]. Endeavoring to be conservative, we assume that smartphone manufacturing costs are the same as for cellphones, even though they are likely much higher. Thus, we borrow most values from **(1)** to calculate average smartphone wattage:

$$S_{\text{wattage}} = 2 \times \text{Charger}_{\text{wattage}} + \frac{C_{\text{upfront}}}{C_{\text{lifetime}}}.$$

With $S_{\text{wattage}} = 6.017$ W, and smartphones becoming widespread at around 2025, we are ready to make our comparison.

Comparison

The two predictive scenarios above are represented together in **Figure 10**, which graphs the nation's total power consumption.

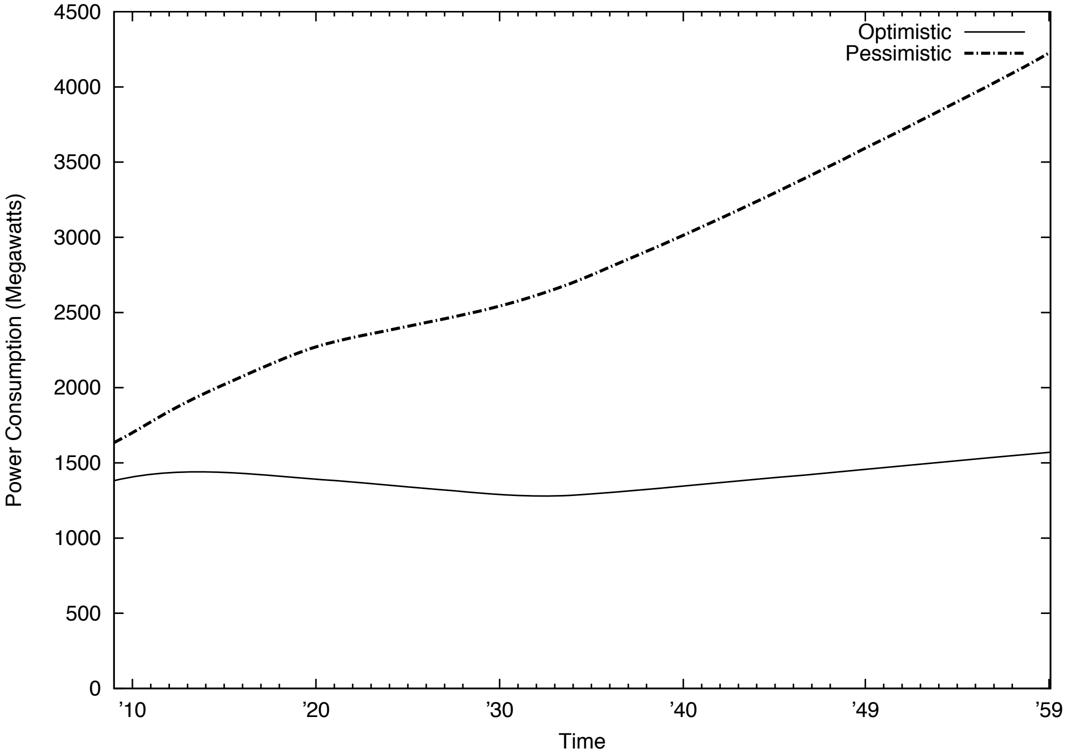


Figure 10. Comparison between optimistic prediction and pessimistic prediction.

Our model leads us to recommend the adoption of conservative practices (on the part of cellphone users) and research into greater phone efficiency (on the part of cellphone manufacturers). A 50% reduction in vampire phone charging and a dedication to energy-efficient phones, according to our simulation, would result in the conservation of 3.9 billion BOE over the next 50 years. Even our pessimistic scenario is not as pessimistic as it could be, since we chose a deliberately low value for the energy cost of smartphones; our optimistic scenario is not as optimistic as it could be, since we assumed only a 50% reduction in vampire energy losses.

Conclusion

Modeling the cellphone revolution can benefit from a bottom-up approach. The basic components of this approach are households undergoing a series of transitions such that each member acquires a cellphone and, eventually, the household abandons its landline.

For an emerging nation adopting a new telephone system, landline adoption would be twice as efficient as cellphone adoption. However, if the nation enforces conservative cellphone energy use, the cellphone plan can be almost comparable to the landline plan.

Also, our model is capable of showing a vast divergence between an optimistic future scenario and a pessimistic one. This being the case, we must recommend a concerted energy conservation effort on the part of cellphone makers and cellphone consumers. Doing so would result in savings of over 3.9 billion barrels-of-oil-equivalent (BOE) over the next 50 years.

Strengths and Weaknesses

Strengths

- **Uses demographics**. Our model simulates the decisions of households based on historic data, making it a good model for assessing the energy consumed to-date.
- **Incorporates manufacturing**. We combine the energy cost of a phone's manufacturing-phase with the phone's use-phase wattage, thereby increasing the simplicity of our model without ignoring the significant energy consumption during manufacturing.
- **Retains flexibility**. Because our model is a bottom-up approach, various details at the household level can easily be incorporated into national simulations. We did this, for example, to assess the cost of "vampire" chargers and to assess the cost of non-telephonic appliances.

Weaknesses

- **Ignores infrastructure**. We do not examine the energy cost of cellular infrastructure (towers, base stations, servers, etc.) as compared to the energy cost of landline infrastructure (i.e., telephone lines and switches).
- **Extrapolates naively**. Though we use demographic data to guide household decisions before 2009, we use simple regression techniques to forecast future demographic information. Using better forecasts would make predictions more accurate. Data that we extrapolated are: cellphone energy-use changes, cellphone penetration dynamics, and landline abandonment rates.
- **Simplifies households**. Our model doesn't examine all household member dynamics—e.g., members getting born, growing old enough to need cellphones, moving out, starting households of their own, etc.

References

Floyd, David B. n.d. Leaking electricity: Individual field measurement of consumer electronics. `enduse.lbl.gov/info/ACEEE-Leaking.pdf` .

Frey, S.D., David J. Harrison, and H. Eric Billet. 2006. Ecological footprint analysis applied to mobile phones. *Journal of Industrial Ecology* 10 (1–2): 199–216. `http://www3.interscience.wiley.com/journal/120128219/abstract` .

International Telecommunication Union. n.d. ITU World Telecommunication/ICT Indicators Database. `http://www.itu.int/ITU-D/ICTEYE/Reports.aspx` .

Mayo, Robert N., and Parthasarathy Ranganathan. 2005. Energy consumption in mobile devices: Why future systems need requirements-aware energy scale-down. In *Power-Aware Computer Systems*, edited by Babak Falsafi and T.N. Vijaykumar, 26–40. New York: Springer. `www.hpl.hp.com/personal/Partha_Ranganathan/papers/2003/2003_lncs_escale.pdf` .

Newburger, Eric C. 2001. Home computers and Internet use in the United States: August 2000. `www.census.gov/prod/2001pubs/p23-207.pdf`.

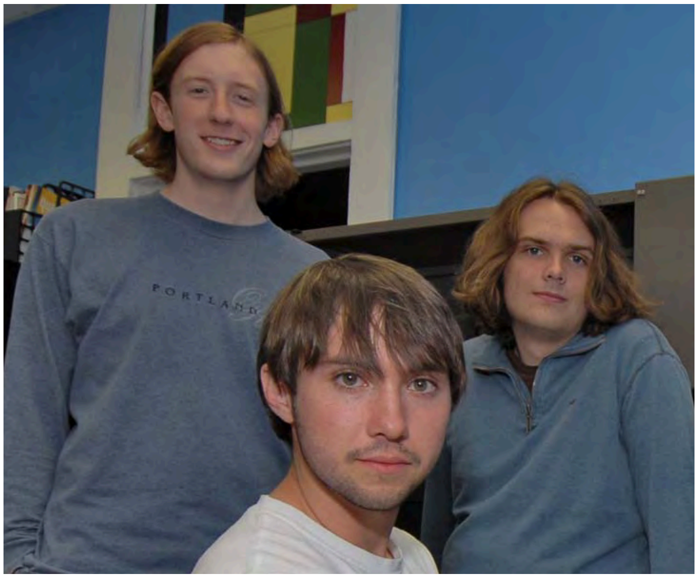
Nielsen Media Research. 2007. Average U.S. home now receives a record 104.2 TV channels, according to Nielsen. `http://www.nielsenmedia.com/nc/portal/site/Public/menuitem.55dc65b4a7d5adff3f65936147a062a0/?vgnextoid=48839bc66a961110VgnVCM100000ac0a260aRCRD`.

Rosen, Karen B., Alan Meier, and Stephan Zandelin. 2001. Energy use of set-top boxes and telephony products in the U.S. `http://eetd.lbl.gov/ea/reports/45305/45305.pdf` .

Roth, Kurt W., and Kurtis McKenney. 2007. Energy consumption by consumer electronics in U.S. residences. `http://www.ce.org/pdf/Energy%20Consumption%20by%20CE%20in%20U.S.%20Residences%20(January%202007).pdf` .

Singhal, Pranshu. 2005. Integrated Product Policy Pilot Project Stage I Report. Finland: Nokia Corporation.`http://www.esm.ucsb.edu/academics/courses/282/Readings/Singhal-Nokia-2005a.pdf` .

U.S. Census Bureau. 2009. State & county QuickFacts. `http://quickfacts.census.gov/qfd/states/06000.html` .



Tommy Rogers, Stephen Foster, and Bob Potter.

Wireless Networks: An Easy *Cell*

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Summary

The number of cellphones worldwide raises concerns about their energy usage, even though individual usage is low (< 10 kWh/yr). We first model the change in population and population density until 2050, with an emphasis on trends in the urbanization of America. We analyze the current cellular infrastructure and distribution of cell site locations in the U.S. By relating infrastructure back to population density, we identify the number and distribution of cell sites through 2050. We then calculate the energy usage of individual cellphones calculated based on average usage patterns.

Phone-charging behavior greatly affects power consumption. The power usage of phones consumes a large part of the overall idle energy consumption of electronic devices in the U.S.

Finally, we calculate the power usage of the U.S. cellular network to the year 2050. If poor phone usage continues, the system will require 400 MW/yr, or 5.6 million bbl/yr of oil; if ideal charging behavior is adopted, this number will fall to 200 MW/yr, or 2.8 million bbl/yr of oil.

Introduction

As energy becomes a growing issue, we are evaluating current infrastructure to locate inefficiencies in power consumption. The increase in cellphone usage in the past decade raises concern about greater energy consumption compared to landline phone networks.

By modeling subscriber growth and trends, we can get a clearer picture of the energy consequences of our mobile network. By correlating the

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The UMAP Journal 30 (3) (2009) 385–402.

growth of mobile subscribers with changes in our mobile infrastructure, we can strategically develop our current communications network to meet energy-efficient guidelines.

Current Cellular Network Model

Assumptions

- The FCC database contains all relevant and major cell sites in the U.S.
- Cell sites serve areas of homogeneous population density, characterized by the population density at the exact location of the site.
- All cell sites can communicate to 50 km (approximately the limit of modern technologies).
- The strength of a cell tower depends primarily on the number of antennas (we lack information about transmission power).

Communication Standards

CDMA and GSM, the two primary standards for mobile phones in the U.S., require different antennas, so different cell sites exist for each standard. However, to simplify our models, we assume that all mobile phones use one generic standard.

Network Model and Component Power Usage

A simplified cellular network model and corresponding energy usage requirements are shown in **Figure 1**. Cellphones connect directly to cell sites, which may or may not be mounted on antenna towers. We consider each antenna mounted on a tower as a separate cell site. A tower can handle a range of calls at once (about 200–500 users, using 600–1000 W [Ericsson 2007]) and pass them along to Mobile Switching Stations (MSCs). Communication between MSCs and cell sites can be accomplished through fiber-optic networks or microwave connections. Each MSC can handle approximately 1.5 million subscribers and consumes about 200 kW. MSCs connect directly into the communications backbone of the country. Since a fiber-optic backbone is necessary in any scenario (or in any Pseudo U.S.), we do not consider it in energy estimates.

Cell Site Registration Databases

All cellular radio transmitters greater than 200 m in height are required to be registered in the FCC Universal Licensing System Database [Fed-

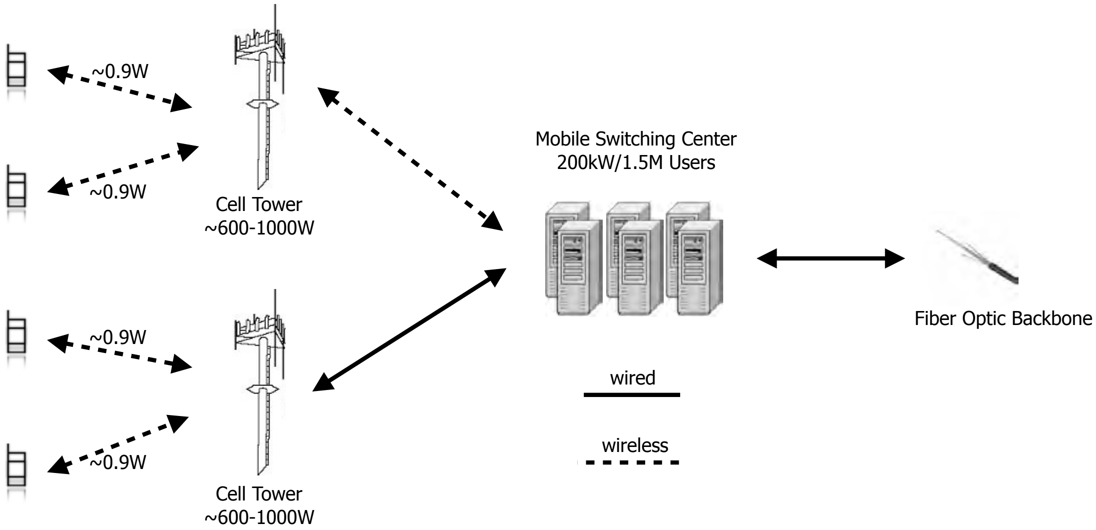


Figure 1. Simplified network model for infrastructure calculations. Each component of each type (cell sites and MSCs) is assumed to be identical for all carriers and geographies.

eral Communications Commision 2009], ensuring that a majority (but not all) cell sites are included. The database contains approximately 20,000 cell site locations comprising about 130,000 individual cell sites.

Tower Location

We show cell-site location and population density in **Figure 2**. Interestingly, several cell towers seem to be in the Gulf of Mexico and in the Atlantic Ocean (either due to errors in registrations or to the use of ships and/or oil rigs). Also interesting is the single tower at the center of Dallas (northern Texas), which contains 25 antennas and suggests a series of smaller sites spread throughout the city.

Antennas per Cell Site

Many cell sites in urban areas use more antennas and higher transmission power. Although some Effective Radial Power (ERP) data is included in the FCC database, many sites have no published information and several have a negative ERP (impossible). Many sites have similar transmission power, likely due to FCC regulations. To quantify the power of a cell site, we use the number of antennas. While most sites have only a single antenna, many have several, and a few have as many as nine (**Figure 3**).

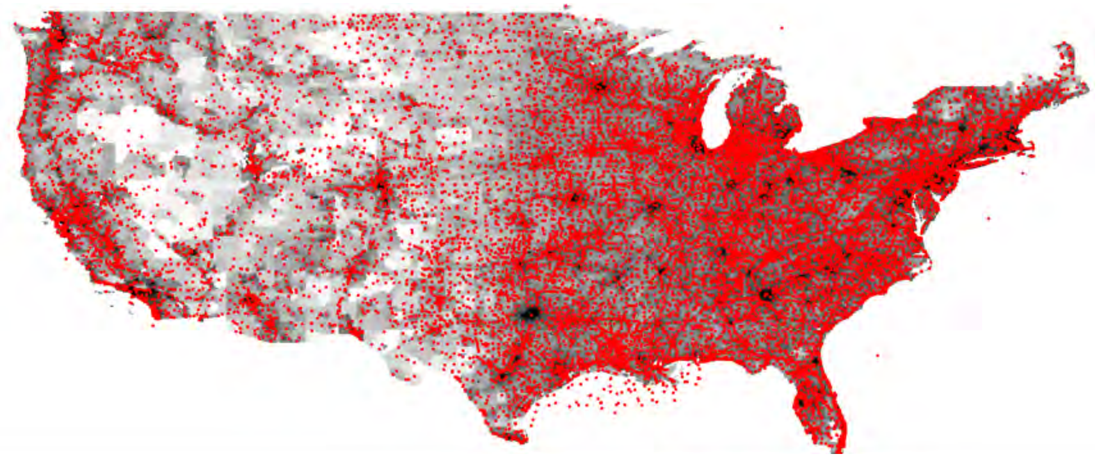


Figure 2. Cellphone towers (red) and population density (grays).

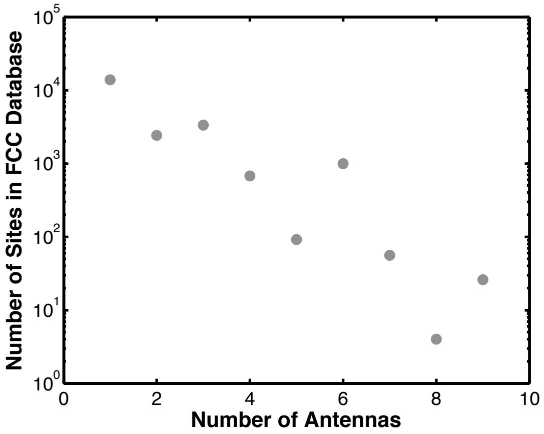


Figure 3. Distribution of the number of antennas per tower.

Tower-Antenna-Population Density Relations

To calculate how many cell sites are used on average in regions of varying population density, we use the site locations to interpolate densities from the maps. Binning the data for population density, we get in **Figure 4** the relationship between antenna density and population density. The initial portion of the graph approximately shows a steady increase in the number of towers, with one antenna per tower. However, above 150 people/km^2, the number of towers levels off and the number of antennas per tower rises to compensate for the increased population.

Coverage Overlap

We investigated overlapping coverage by determining the number of nearby cell sites at a range of locations; the method is illustrated in **Figure 5**.

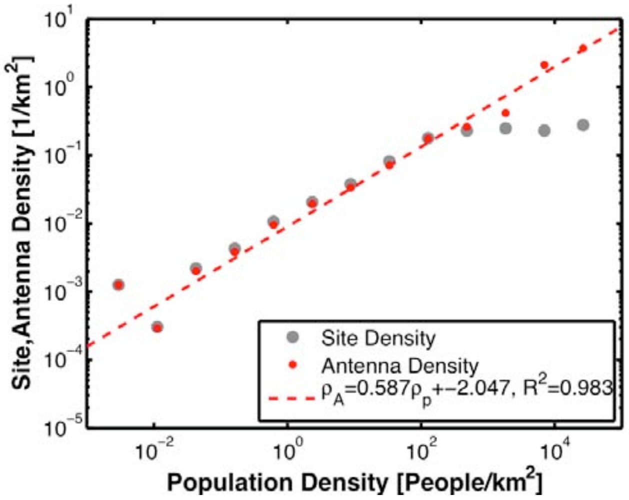


Figure 4. Antenna density vs. population density.

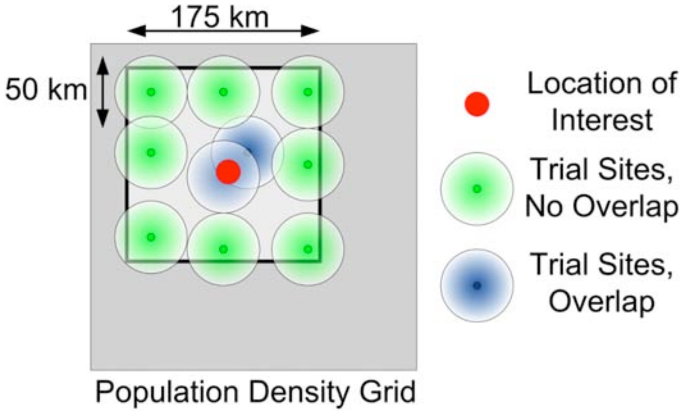


Figure 5. Illustration of algorithm to determine number of overlapping cell sites. The figure does not represent the eccentricities of the grid due to changing longitudinal lengths.

For each cell in the population density grid, we construct a trial list of all towers within a reasonable range (towers within 1° latitude, 3° longitude, or approximately 100–200 km in each direction). For each candidate tower, we calculate the great-circle distance (in km) between the location (latitude δ_1, longitude λ_1) and the tower (latitude δ_1, longitude λ_1) [Weisstein n.d.]:

$$d = 6378 \cos^{-1}[\cos\delta_1 \cos\delta_2 \cos(\lambda_1 - \lambda_2) + \sin\delta_1 \sin\delta_2].$$

If the great circle distance is less than the maximum range of a tower (approximately 50 km), the region is considered to be in the tower's plausible range. We thus calculate for each location the number of cell sites within range (**Figure 6**). While some cities have a large degree of overlap, others accomplish full connectivity by using many smaller rooftop sites or higher-power antennas. Also noticeable are several regions in the Western U.S. with no current connectivity.

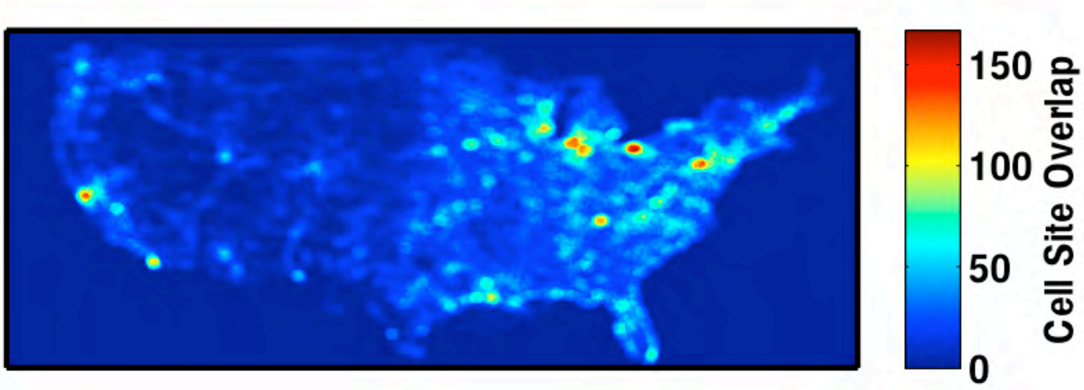


Figure 6. Results of overlap calculations for the known grid of cellsites as reported by the FCC. Most urban regions have higher overlap of cell towers to cope with an increased population load.

Model for Cellphone Usage

Basic Assumptions

Our investigations uncover three main components of electricity consumption by cellphones:

- powering the phone during talking and standby,
- powering the charger with a phone attached, and
- powering the charger without a phone attached.

Therefore, we model the cellphone usage of an average person as a function of three different characteristics:

- At what remaining battery level (0–100 %) does the user recharge the cellphone?
- How long does the cellphone remain connected to the charger after the battery is completely charged?
- Does the user unplug the charger from the outlet upon completion of battery charging?

The possible power consumption states of a phone adapter are displayed in **Table 1**.

Cellphone Information and Usage Behavior

Battery Capacity

Table 2 displays the average battery capacity, power consumption during talking, and standby power consumption for batteries of the nine largest

Table 1.

Cellphone charger states and energy consumption.

State	Consumption (W)
Unplugged	0
Plugged in, no phone	0.5
Phone attached, not charging	0.9
Phone attached, charging	4.0

mobile phone manufacturers in the U.S. We determined averages using manufacturer information about more than 150 popular cellphones, approximately 15 phones per provider [IDC 2008]. Power consumption is calculated using battery capacity and estimates of talk time and standby time for individual phones, assuming each phone has a 3.7 V lithium-ion battery. The last line shows an overall average weighted by 2008 U.S. market share.

Table 2.

Average capacity and energy consumption for popular U.S. cellphones.

Rank	Manufacturer	Market share (%)	Battery capacity (mAh)	Talk power, (W)	Standby power, (W)
1	Samsung	22.0	980 ± 228	0.0138 ± 0.0051	0.875 ± 0.293
2	Motorola	21.6	826 ± 122	0.0108 ± 0.0023	0.655 ± 0.292
3	LG	20.7	890 ± 106	0.0116 ± 0.0036	0.923 ± 0.242
4	RIM	9.0	1216 ± 276	0.0145 ± 0.0060	1.065 ± 0.348
5	Nokia	8.5	1066 ± 192	0.0122 ± 0.0032	0.735 ± 0.334
6	Sony Ericsson	7.0	1015 ± 214	0.0085 ± 0.0039	0.431 ± 0.110
7	Kyocera	5.0	900 ± 000	0.0200 ± 0.0030	0.970 ± 0.080
8	Sanyo	4.0	810 ± 89	0.0161 ± 0.0037	0.908 ± 0.152
9	Palm	2.2	1500 ± 346	0.0167 ± 0.0042	1.402 ± 0.353
	Weighted average		960 ± 166	0.0127 ± 0.0039	0.829 ± 0.263

Cellphones Per Person

The average number of cellphones owned per person is determined using historical population and mobile phone data and extrapolated to the year 2050 [Federal Communications Commission 2008; U.S. Census Bureau 2008]. **Figure 7a** displays the total number of cellphone subscribers normalized by the population of the U.S. The historical data fit a sigmoidal curve, assuming that the ratio will eventually reach a value of 1 cellphone per person (complete saturation). **Figure 7b** compares the yearly increase in U.S. population to that of cellphone users. By 2015, the predicted number of cellphone owners reaches the total number of people and continues to grow with the population.

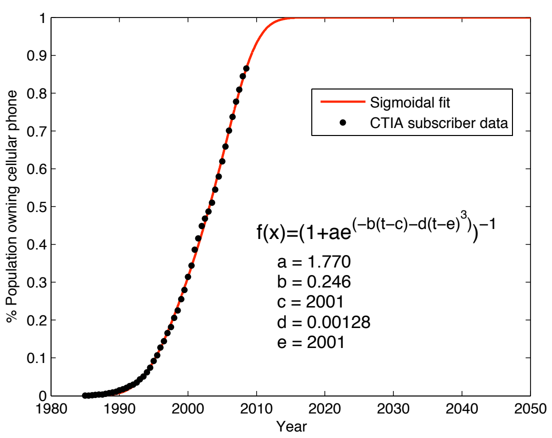


Figure 7a. Sigmoidal fit for the average number of cellphones per person in the U.S.

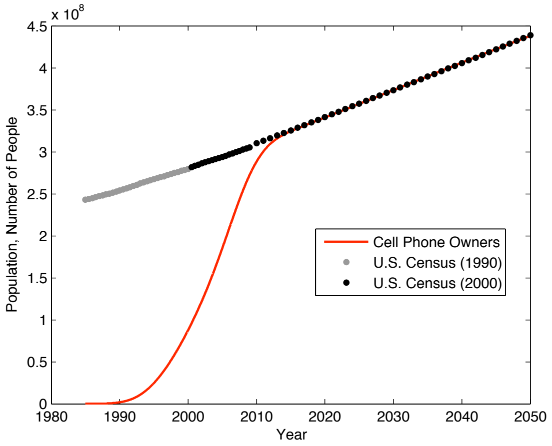


Figure 7b. Predicted growth and saturation of cellphone owners in the U.S.

Average Talk-Time per Person

The average talk time of an individual user between 1991 and 2050 is determined in a fashion similar to the average number of cellphones per owner. **Figure 8** displays the trends in landline and cellphone usage in terms of total minutes used per year between 1991 and 2007 [CTIA 2008; Federal Communications Commission 2008], together with our extrapolation. We assume that average usage will eventually saturate to some value, and a first-order exponential growth function is employed to model this behavior. **Figure 9** displays the predicted growth of cellphone usage assuming saturation at 15, 20, 25, and 30 minutes per person per day.

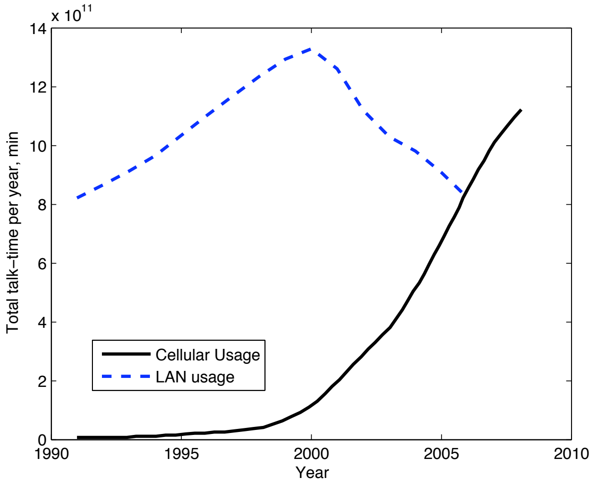


Figure 8. Historical behavior of landline and cellphone usage in the U.S.

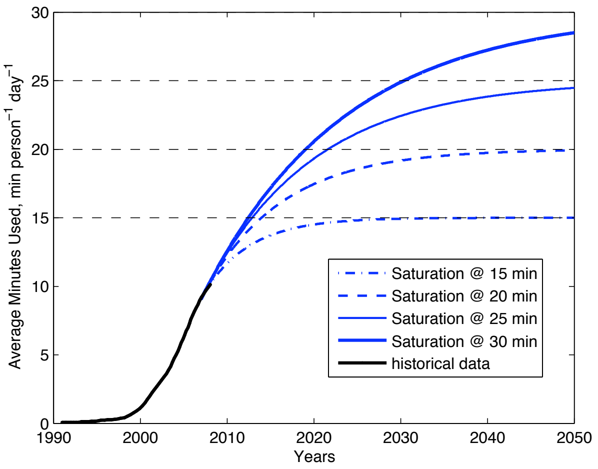


Figure 9. Predicted saturation behavior of average daily mobile cellphone usage.

Recharge Probability and Duration

We model the battery level at which a person is likely to charge their phone as a Gaussian distribution (**Figure 10**), based on cellphone behavior data [Banerjee et al. 2007]. Users tend to recharge their phone batteries at between 25% and 75% of full capacity.

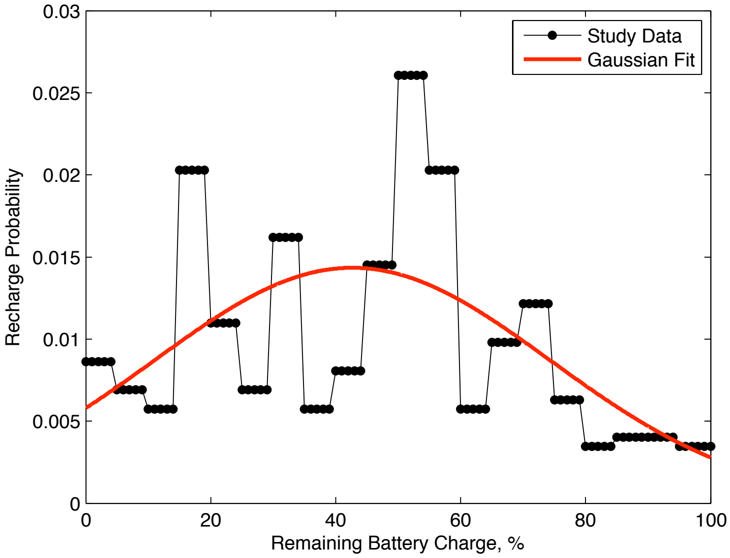


Figure 10. Fitted Gaussian distribution for recharging behavior of users.

The time to charge a lithium-ion battery is typically not proportional to the remaining charge to be added. Therefore, we assume that the battery charge increases exponentially as a function of charge time, as depicted in **Figure 11**.

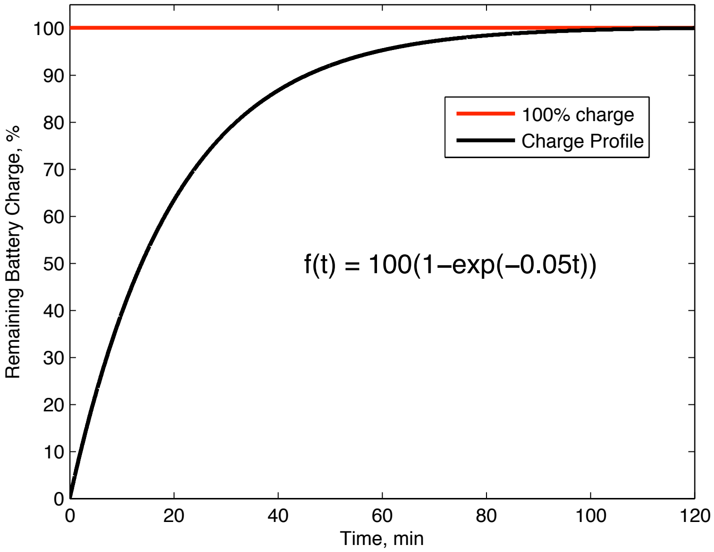


Figure 11. Typical charge profile for lithium-ion battery.

Calculation of Average Energy Consumption

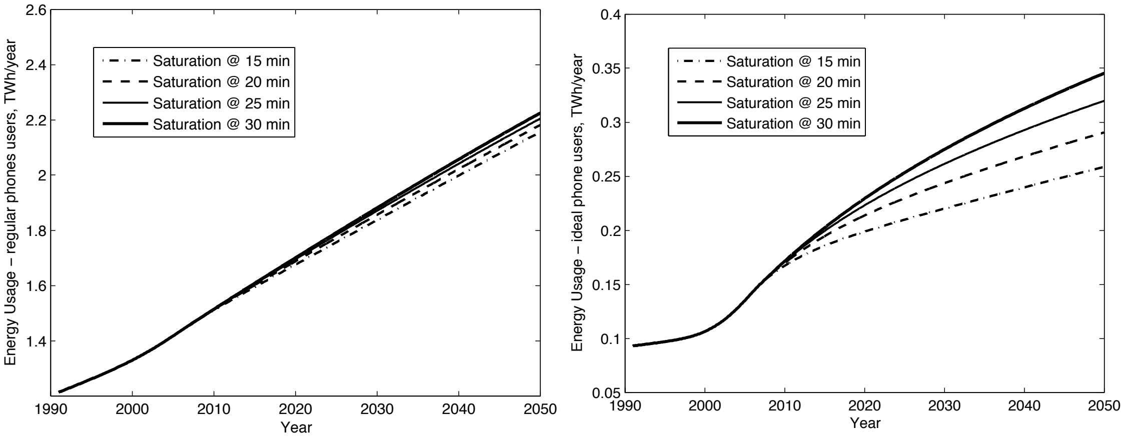
We calculate the energy consumed by the average cellphone user over the course of a year by employing the battery and usage behavior extrapolations discussed earlier. We assume that the full range of remaining battery charge (0–100%) can occur before charging is initiated, depending on the type of user ("regular" or "ideal"). The total energy consumption is calculated from battery capacity and different power states of a charge-adapter. The duration that the adapter stays in a particular power state is determined by the frequency of charging (number of charge cycles per year), which is approximated by the power consumption during periods of cellphone talking and standby. Furthermore, the power consumption during talking/standby is weighted by the average number of minutes a person talks on the phone per day (see **Figure 8**). Finally, the average energy consumption across the entire population of cellphone users is determined using a weighted sum of energy at each remaining battery level and the probability distribution that charging starts at that battery level.

We assume that there are only two types of users:

- the "regular" user, who charges for 8 hr at a time (at the probability given by the fitted Gaussian distribution) and always leaves the charge-adapter plugged in; and
- the "ideal" user, who charges for only the time to reach 100% charge (at the probability distribution centered at 15–20% battery levels) and never leaves the charger plugged in when not charging.

Energy Usage of Cellphones

The yearly energy consumed by cellphone charging between 1991 and 2050 for the "regular" user and for the "ideal" user is displayed in **Figure 12**. The yearly consumption of the "ideal" user is less than one-fifth that of the "regular" user. This drastic difference is primarily a consequence of unplugging the charger after charge completion. As a result of the increased energy savings of the "ideal" behavior, we see an increased sensitivity to the cellular usage saturation at different values of minutes per person per day. These trends are more difficult to see with the regular behavior since the majority of energy consumption is wasted by the charger.



a. "Regular" user. **b.** "Ideal" user.

Figure 12. Yearly energy consumption of "regular" user and "ideal" user, assuming different user saturation times (15, 20, 25, and 30 min/person/d).

Pseudo-U.S. Model

Assumptions

- A communication infrastructure is entirely nonexistent.
- A power grid already exists.
- Each household must have television and Internet service.
- Each household has either one landline phone per person or one cellphone per person.

Comparison of Fiber-optics to Wireless Networks

We compare the energy usage per person for an entirely wireless network to the cost of running a competitive fiber-optic network. Since the choice of wireless vs. fiber optic affects the energy usage of TVs, computers, and phones in a household, we consider all three of these communication methods. The estimated power usage for each system is summarized in **Table 3**. Based on current estimates for each electronic device [Rosen and Meier 1999], a completely wireless approach could be energy competitive against a fiber-optic solution, due to the energy inefficient link necessary in every household.

Table 3.

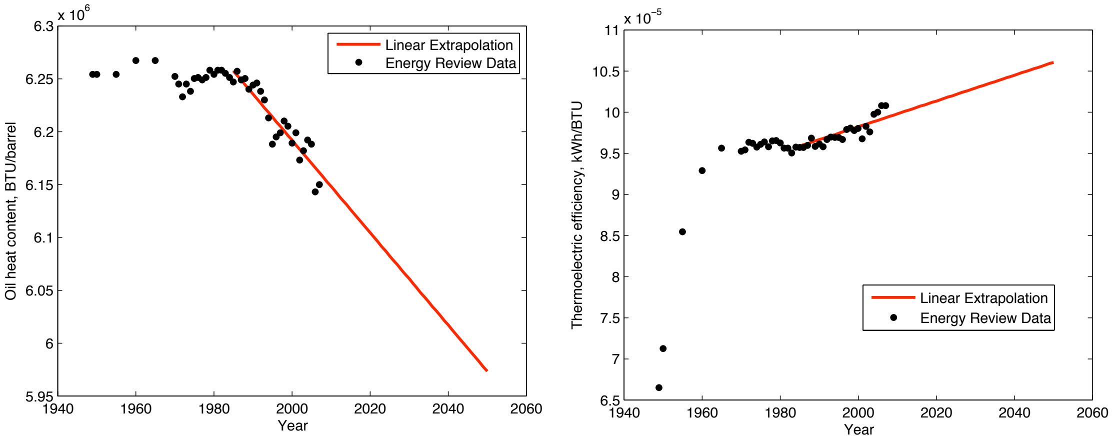
Electricity usage for fiber-optic and wireless approaches, per household of 2.5 members with one computer, one TV, and one phone per person.

Category	Fiber-optic usage	Wireless usage
General	Fiber-optic link (16 W)	
TV		DTV converter (5 W)
Internet		2.5 × WIMAX card (1 W) 2.5 × transmission (0.75 W)
Phone		2.5 × cellphone (0.75 W) 2.5 × transmission (0.75 W)
Total	16 W	13 W

Energy to Oil Conversion

We determine the amount of electrical energy available per barrel of oil using historical data [Energy Information Administration 2008; Taylor et al. 2008]. **Figure 13a** shows the heat content per barrel of oil from 1949 to 2007 with linear extrapolation to 2050. Heat content is decreasing, possibly due to a decreasing proportion of energy-rich oil in the global market. The thermoelectric efficiency (i.e., the efficiency of converting heat created by burning fuel into electricity) is displayed in **Figure 13b** with extrapolation. Using the heat content and thermoelectric efficiency data, the total electricity produced per barrel of oil is obtained and displayed in **Figure 14**. From the extrapolation, we find that one barrel of oil will produce approximately 628 kWh of electricity in 2050.

While a considerable amount oil is needed to create 1 TWh or more of electricity, it is very unlikely that oil would be used to create this electricity. In **Figure 15**, we see that oil at its peak use (1977) accounted for only 17% of the electricity produced in the U.S. Today, oil accounts for less than 4% of electricity and this value appears to be decreasing slowly.



a. Heat content. **b.** Thermoelectric efficiency.

Figure 13. Heat content and thermoelectric efficiency data for oil, with extrapolations.

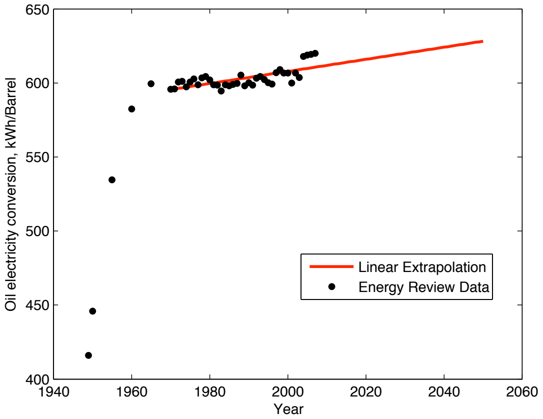


Figure 14. Electricity per barrel of oil, over time.

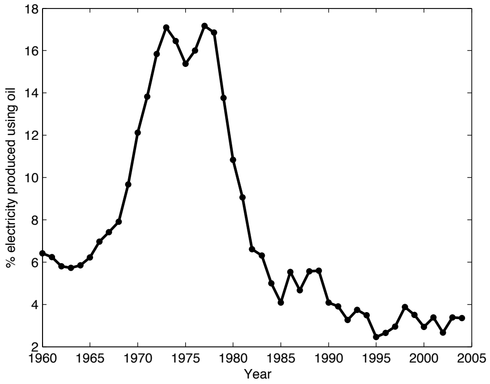


Figure 15. Trends in U.S. electricity production from oil.

Overall Charger Power Usage

To gauge the inefficiency of cellphones compared to other electronics, we compare results of our analysis with Rosen and Meier [1999]. With updating to reflect 2008 cellphone usage, the results are shown in **Figure 16**. Although the energy usage of cellphones chargers is significant (2 TWh / yr), it is only a small portion of the overall energy wasted by idle electronics (34 TWh / yr), or 54 million barrels per year using the conversions established above.

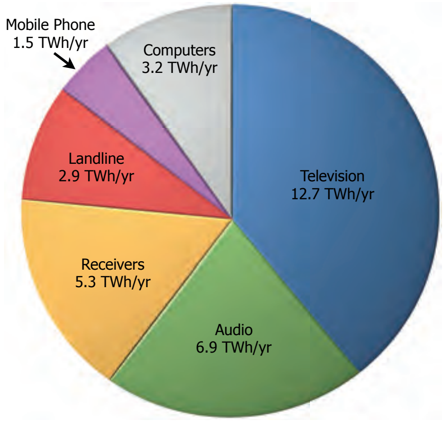


Figure 16. Usage of various electronics according to Rosen and Meier [1999], with cellphone energy usage updated to 2008 per our model.

Cellular Network Growth Through 2050

Assumptions

- No new (radically disruptive) technologies will be introduced past 3G (third generation of cellphones). Current technology will improve until a minimum necessary energy usage is achieved.
- Population density growth will follow similar trends to 2050.
- The number of towers necessary for a given population density will remain constant through 2050.

Technology Improvements

The power requirements of cellular networks has fallen drastically since the 1980s. Until 2050, similar reductions in power usage will be likely, either through improvements in the electronics of cell sites (computers and such) or more-efficient communication strategies (antenna transmissions). To characterize this reduction in energy, we use information on energy usage of past technologies [Ericsson 2007], as shown in **Figure 17**. Technologies following the primary upgrade path (1G to 2G and beyond) are leveling out in their minimum energy usage. Although the introduction of 3G initially caused a large increase in power consumption, it seems to have a greater potential for reducing energy consumption. Since future technologies cannot be accurately quantified, we assume that all future networks will be based on a variation of 3G architecture. Calculated from **Figure 17**, the relevant efficiencies for each decade are shown in **Table 4**.

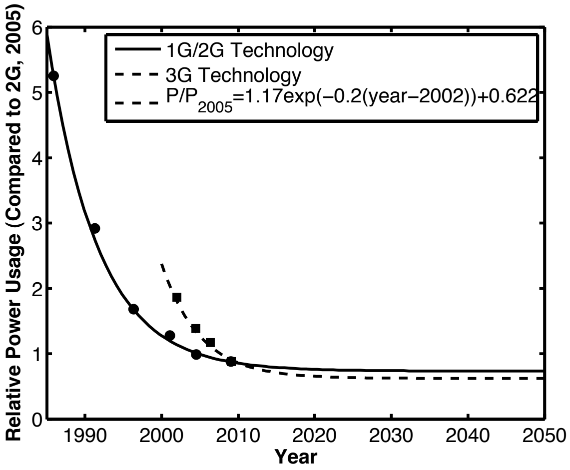


Figure 17. Characterization of technological improvements in cellular infrastructures on energy usage, for two different sets of technology, with corresponding exponential fits of the form $a\exp(-bx)+c$ projecting to 2050 [Ericsson 2007].

Table 4.

Network technology efficiency.

Year	Relative power usage
2005	1
2010	.85
2020	.66
2030	.63
2040	.62
2050	.62

Infrastructure Improvements

As the population grows and the use of cellphones increases, more cell sites and related infrastructure will be necessary. To model the increasing number of towers, we combine tower density / population density relations with population density predictions. The resulting increase in towers is seen in **Figure 18**. These predictions assume that tower capacity will not grow directly but instead improve through energy efficiency.

Overall Energy Usage

We calculate total energy usage of the U.S. cellular network using the predicted increase in cell sites, observed trends in technology, predicted usage patterns, and recent energy statistics. Final predictions are shown for two usage scenarios in **Figure 19**. If chargers are used inefficiently power consumption will grow to approximately 400 MW, or 5.6 million

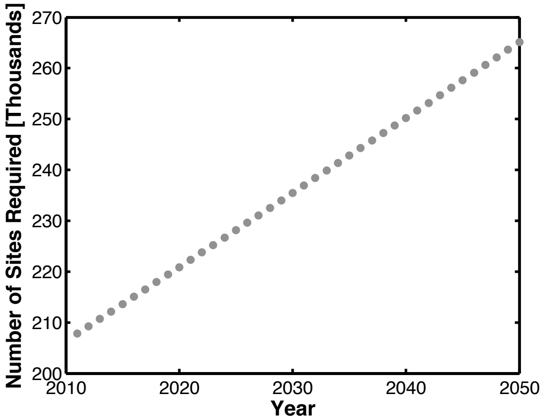


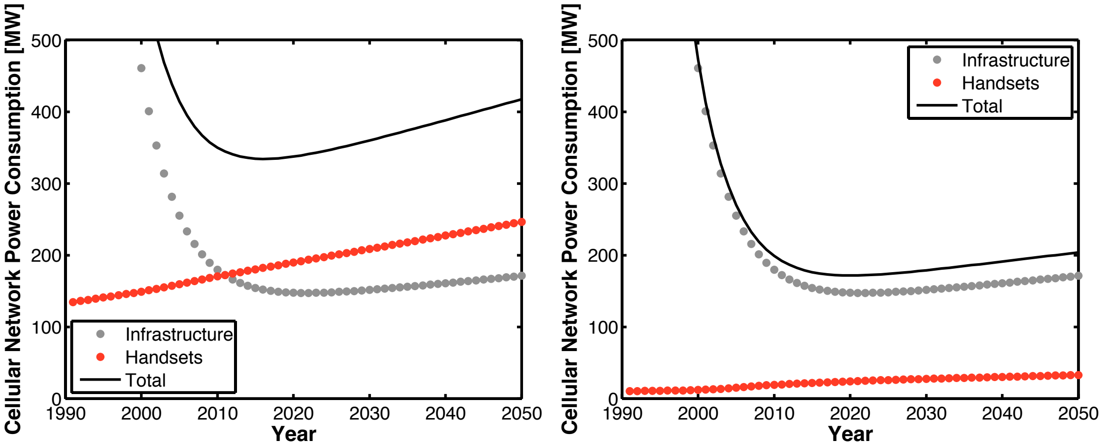
Figure 18. Predicted number of cellphone towers from 2007 to 2050.

bbl/yr. However, if consumers utilize chargers efficiently, consumption by 2050 will be approximately 200 MW/yr (2.8 million bbl/yr of oil).

Conclusion

We estimate power consumption of the U.S. cellular network, based on

- models of usage trends,
- current infrastructure,
- population projections, and
- technology improvements.



a. Inefficient charger usage.

b. Ideal charger usage.

Figure 19. Predictions for the energy usage of the U.S. cellphone network for two different charge scenarios.

Technological developments will cause energy usage to decrease until 2015, after which increasing population will demand more power usage.

We assess the optimal communications network for a country similar to the U.S. A wireless network (to houses) comprising voice, data, and TV service would draw less electricity than a fiber-optic approach and hence be optimal, as long as wireless communication can provide sufficient bandwidth (likely).

We compare energy consumption for "regular" users and "ideal" users in terms of charging practices. A "regular" user today wastes 4.8 kWh/yr through inefficient charging.

We model energy wasted by various idle household electronics, including cellular network usage. A person today wastes 125 kWh/yr through idle electronics.

We model energy needs for phone service through 2050 and calculate the number of new cell towers and other infrastructure necessary.

If inefficient charging strategies are used, cellular networks in 2050 will require 400 MW/yr of electricity (5.6 million bbl/yr of oil). If more-efficient chargers are introduced or people change their habits, only 200 MW of power (2.8 million bbl/yr of oil) will be required.

References

Banerjee, Nilanjan, Ahmad Rahmati, Mark D. Corner, Sami Rollins, and Lin Zhong. 2007. *Users and batteries: Interactions and adaptive power management in mobile systems*. In *UbiComp 2007: Ubiquitous Computing,* edited by J. Krumm et al., 217–234. Lecture Notes in Computer Science, vol. 4717/2007. Berlin / Heidelberg, Germany: Springer. `http://www.springerlink.com/content/t2m30643713220k6/`.

Center for International Earth Science Information Network (CIESIN), Socioeconomic Data and Applications Center, Columbia University. 2005. Gridded population of the world, version 3 (GPWv3): Population density grids. `http://sedac.ciesin.columbia.edu/gpw/global.jsp`.

CTIA: The Wireless Association. 2008. 2008 CTIA semi-annual wireless industry survey. `http://files.ctia.org/pdf/CTIA_Survey_Mid_Year_2008_Graphics.pdf`.

Energy Information Administration. 2008. Annual energy review (AER) 2007. Technical Report DOE/EIA-0384(2007). `http://www.eia.doe.gov/aer/`.

Ericsson. 2007. Sustainable energy use in mobile communications. White paper, August 2007. `http://www.ericsson.com/campaign/sustainable_mobile_communications/downloads/sustainable_energy.pdf`.

Federal Communications Commission, Industry Analysis and Technology Division, Wireline Competition Bureau. 2008. Trends in telephone service. URL `http://hraunfoss.fcc.gov/edocs_public/attachmatch/DOC-284932A1.pdf`.

________. Database downloads: Antenna structure registration: Cellular—47 CFR Part 22: Licenses. URL `http://wireless.fcc.gov/antenna/index.htm?job=uls_transaction&page=weekly`.

Interactive Data Corp. 2008. Worldwide quarterly mobile phone tracker. April 2008. URL `http://www.idc.com/getdoc.jsp?containerId=IDC_P8397`.

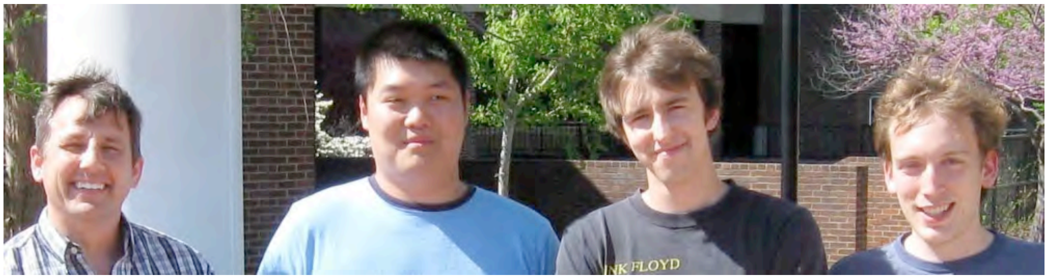
Rosen, Karen, and Alan Meier. 1999. Energy use of U.S. consumer electronics at the end of the 20th century. Technical report, Lawrence Berkeley National Laboratory. `http://eetd.lbl.gov/EA/Reports/46212/`.

Taylor, Peter, with Olivier Lavagne d'Ortigue, Nathalie Trudeau, and Michel Francoeur. 2008. Energy efficiency indicators for public electricity production from fossil fuels. International Energy Agency. `http://www.iea.org/textbase/papers/2008/En_Efficiency_Indicators.pdf`.

U.S. Census Bureau. 2004. Tble HH-6. Average population per household and family: 1940 to present. `www.census.gov/population/socdemo/hh-fam/tabHH-6.pdf`.

________. 2008. Population projections: U.S. interim projections by age, sex, race, and Hispanic origin: 2000 to 2050. `http://www.census.gov/population/www/projections/usinterimproj/`.

Weisstein, Eric W. n.d. Great circle. From MathWorld—A Wolfram Web Resource. `http://mathworld.wolfram.com/GreatCircle.html`.



Advisor Louis Rossi with team members Bob Liu, Jeff Bosco, and Zachary Ulissi.

Judges' Commentary: The Outstanding Cellphone Energy Papers

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General Remarks

As in past years, the diverse backgrounds of the undergraduate participants yielded many interesting modeling approaches to the stated problem. The judges assessed the papers on the breadth and depth of analysis for all major issues raised, on the validity of proposed models, and on the overall clarity and presentation of solutions.

The Executive Summary is often still below par; it should motivate the reader to read the paper. It must not merely restate the problem, but indicate how it was modeled and what conclusions were reached, without being unduly technical.

Assumptions must be clearly stated and justified where appropriate. Some teams overlook important factors and/or make unrealistic assumptions with no rationale. It should be made clear in the model precisely where those assumptions are used.

Graphs need labels and/or legends and should provide information about what is referred to in the paper. Tables and graphs that are taken from other sources need to have specific references. Failure to use reliable resources and to properly document those resources kept some papers from rising to the top. The best papers not only list trustworthy resources but also document their use throughout the paper.

The UMAP Journal 30 (3) (2009) 403–407.

Requirements and Selected Modeling Approaches

The Cellphone Problem involved the "energy" consequences of the cellphone revolution, and five Requirements were delineated. To receive an Outstanding or Meritorious designation, teams had to address issues raised in each of these Requirements. Additionally, Outstanding papers considered both wireless and wired landlines and the infrastructure to support cellphones and landlines.

Requirement 1

Teams were first asked to estimate the number of U.S. households in the past that were served by landlines and also to estimate the average size of those households. They were then to consider the energy consequences, in terms of electricity use, of a complete transition from landline phones to cellphones, with the understanding that each member of each household would get a cellphone.

To address this problem, the energy used by current landlines had to be considered. Whereas corded landline phones use relatively little electricity, the same cannot be assumed about cordless landline phones. The top papers researched this issue and arrived at documented estimates of the number of corded vs. cordless landline phones and the average number of each per household. This led to a more realistic appraisal of the energy used by the landline phone system.

With regard to cellphones, teams that rose to the top considered the infrastructure necessary—for example, the building of numerous additional communication towers if cellphones are to replace landline phones completely. This was of special importance in the analysis of the transitional phase. Also, the varying amount of electricity required by different types of cellphones was a consideration in the transitional and steady-state phases.

Interesting models were constructed for the transitional phase of the cellphone "takeover." Some teams considered the spread of cellphones as the spread of a disease and used the Verhulst model for logistic growth, using the population of the U.S. as the carrying capacity and estimating the rate of growth of cellphones from published reports on the growth of cellphone use. Other teams generalized this to an SIR model or used the Lotka-Volterra predator-prey model, with cellphones as the predators and landline phones as the prey. A few used the competing-species model. The judges looked very favorably upon models for which sufficient rationale was given as to why that model might be appropriate in this circumstance. Interpretation of the parameters and solutions as they applied to the problem at hand was essential.

Many papers ignored the transition phase and only considered the energy comparison for the steady state in order to determine the energy consequence. Some teams merely talked their way through the issues and did not construct a mathematical model. After estimating energy costs associated with landline phones and cellphones, many teams used linear equations to model the total costs associated with the numbers of phones.

Requirement 2

Teams were asked to consider a "Pseudo U.S."—a country similar to the current U.S. in population and economic status, but with neither landlines or cellphones. They were to determine the optimal way, from an energy perspective, to provide phone service to this country. The teams were also to take into account the social advantages that cellphones offer and the broad consequences of having only landlines or only cellphones.

Once again, consideration of the infrastructure for phones was important. In addition to landline phones and cellphones, many teams considered the VoIP (Voice over Internet Protocol) communication option. Not every team that considered VoIP took into account the costs for laying the cables; some assumed that such cables were already in place (a questionable assumption). However, failure to consider the VoIP method of phone service may have kept a Meritorious paper from becoming an Outstanding paper. If one were to assume that households would already have one or more computers with Internet access, the energy costs associated with VoIP would be quite small.

In terms of finding the optimal way to provide phone service from an energy perspective, some teams used linear programming, using the costs determined in Requirement 1 and quantifying in various ways the social advantages of cellphones, as well as the preference for landlines in certain circumstances. Other teams used AHP (Analytic Hierarchy Process), which worked well to get parameters used in the optimization routine but did not work as an optimization tool in itself. Teams that considered the advantages and disadvantages of various phone types not just for individuals, but for businesses also, demonstrated a thoroughness that was commendable. Another factor that some teams considered was the number of children under 5 who would have no need for cellphones.

Requirement 3

This was an extension of Requirement 2, asking teams to consider the electricity wasted when cellphones are plugged in that do not need charging and when chargers are left plugged in after the phone is removed. Teams were to continue to assume that they were in the Pseudo U.S. and were to interpret energy wasted in terms of barrels of oil used.

To determine the amount of energy wasted, teams had to first estimate the number of hours that a "typical" cellphone charger is in the state of charging a phone, left plugged into a phone not in need of charging, and left plugged in when the phone is not connected to it. Some teams did their own informal surveys, but better estimates were arrived at from publications and surveys.

In some papers, probability distributions were used to describe this behavior, but use of such distributions was not always justified.

Teams that were more comprehensive took into account the fact that some cellphones and chargers use less power than to do others, based on brands, age, and capabilities of the phones and chargers. Assuming that all electrical energy is generated by oil, translating kilowatts of energy into barrels of oil used was a straightforward transformation.

Requirement 4

This requirement extended the concepts in Requirement 3 and asked teams to estimate the amount of energy wasted by all electronic chargers. Since this question was very open-ended, contest papers showed a wide variety of estimates for the energy wasted in terms of barrels of oil. The top teams estimated the average hours that appliances are left plugged in, charging and not charging, and also the number of hours that chargers are plugged in without the appliance.

More-comprehensive papers considered many other kinds of electronic devices and by comparison showed that the amount of energy wasted by cellphones is relatively small.

Requirement 5

For this part, students were to consider the population and economic growth of the Pseudo U.S. for the next 50 years and predict energy needs for providing phone service based on their analysis in the first three Requirements. Predictions were to be interpreted in terms of barrels of oil used.

Papers needed to consider both economic growth and population growth in order to estimate energy needs in the future. Various types of regression fits were applied to historical population data and economic data such as GDP. Using earlier estimates of energy requirements, coupled with the regression equations from historical data, predictions were made for the amount of energy used every decade for the next 50 years. Some teams predicted greater efficiency in future phones and the development of chargers that would use less electricity.

Papers showed estimates for the number of barrels of oil used on a per-day basis or perhaps on a per-year basis. There was no one right answer, and answers given depended on assumptions made at the start. Some papers contained graphs displaying future values but did not give tables. A table together with a graph is a better way to display information.

Concluding Remarks

Mathematical modeling is an art that requires considerable skill and practice in order to develop proficiency. The big problems that we face now and in the future will be solved in large part by those with the talent, the insight, and the will to model these real-world problems and continuously refine those models.

The judges are very proud of all participants in this Mathematical Contest in Modeling, and we commend you for your hard work and dedication.

About the Author

Marie Vanisko is a Mathematics Professor Emerita from Carroll College in Helena, Montana, where she has taught for more than 30 years. She was also a visiting professor at the U.S. Military Academy at West Point and taught for five years at California State University Stanislaus. While in California, she co-directed MAA Tensor Foundation grants on Preparing Women for Mathematical Modeling, a program encouraging more high school girls to select careers involving mathematics, and was also active in the MAA PMET (Preparing Mathematicians to Educate Teachers) project. Marie serves as a member of the Engineering Advisory Board at Carroll College, is on the advisory board for the Montana Learning Center for mathematics and science education, and is a judge for both the MCM and HiMCM COMAP contests.

Judges' Commentary: The Fusaro Award for the Cellphone Energy Problem

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Introduction

MCM Founding Director Fusaro attributes the competition's popularity in part to the challenge of working on practical problems. "Students generally like a challenge and probably are attracted by the opportunity, for perhaps the first time in their mathematical lives, to work as a team on a realistic applied problem," he says. The most important aspect of the MCM is the impact that it has on its participants, and, as Fusaro puts it, "the confidence that this experience engenders."

The Ben Fusaro Award for the 2009 Cellphone Energy problem went to a team from the Lawrence Technological University in Southfield, MI. This solution paper was among the top Meritorious papers and exemplified some outstanding characteristics:

- It presented a high-quality application of the complete modeling process.
- It demonstrated noteworthy originality and creativity in the modeling effort to solve the problem as given.
- It was well written, in a clear expository style, making it a pleasure to read.

The UMAP Journal 30 (3) (2009) 409–412.

The Problem: Energy Consequences of the Cellphone Revolution

The Cellphone Energy Problem involved many facets of the "energy" consequences of replacing landlines with cellphones and five Requirements were delineated. Teams had to address issues raised in each of the five Requirements. Additionally, the best papers considered both wireless and wired landlines and the infrastructure to support cellphones and landlines.

The Lawrence Technological University Paper

Assumptions

The team began with a page of assumptions, most of which were well-founded and enabled them to determine parameters in their models. However, certain assumptions made were unrealistic and these led to results that did not reflect the real-world situation. In particular, in the eyes of the judges, assuming that all landline phones are cordless was a serious shortcoming that greatly impacted the issue of energy use. Furthermore, while the team did address the issue of infrastructure, the assumption that infrastructure for cellphones is equal to that for landline phones seemed to ignore the need for the large number of additional communication towers if cellphones were to replace landlines.

Requirement 1: Mathematical Formulation for the Transition

In Requirement 1, teams were to consider the energy consequences in terms of electricity utilization of a complete transition from landline phones to cellphones, with the understanding that each member of each household would get a cellphone. The Lawrence Tech team shone in mathematically modeling this transition! For their first model representing the transition from landline to cellphones, the team used the basic logistic differential equation to model the rate of change in the number of cellphones over time. They used the total population as the carrying capacity and determined the intrinsic rate of growth of cellphones from published results. This was very well done, though references for the tables and graphs should have been included. The second model introduced was a predator-prey system of differential equations, and the team is to be commended on their clear statement of rationale for using this model, with cellphones causing the demise of landlines. However, this model quickly became complicated, so they headed "down a different route." And, once again, their rationale for the equations used and parameters arrived at was commendable.

In modeling the energy used by cellphones, the team considered three distinct models of cellphones and did a good job of researching the habits of individuals of different ages regarding talking times. The assumption they made that the average number of calls is directly related to the talk time per call might be questionable, but this was not considered a serious deficiency and it enabled them to estimate needed parameters in their model.

Requirements 2, 3, 4, and 5

Documented sources were used to estimate energy used for charging batteries, and these were translated into barrels of oil used. Energy usage comparisons were demonstrated for landline cordless phones and cellphones. This was taken forward into Requirement 2 and they seemed to conclude that the optimal mix of landline and cellphones would be the state where the same amount of energy was used by cordless landline and cellphones.

For Requirement 3, after gathering data on energy consumption by phone chargers, the team demonstrated an interesting comparison of energy consumed by daily vs. weekly charging and charger left plugged in or not, and from this they estimated the long-term consequences of avoiding wasteful practices in the charging of cellphones. The team introduced a percentage comparison of energy wasted by various charging methods.

Requirement 4 extended the concepts in Requirement 3 and asked teams to estimate the amount of energy wasted by all idle electronic appliances. Since this question was very open-ended, contest papers showed a wide variety of estimates for the energy wasted. The Lawrence Tech team limited themselves to the average hours that computers, televisions, DVD players/VCRs, and game consoles are left plugged in and the resulting annual oil consumption from such wasteful practices. A linear pattern of growth was projected up to 2059. More-comprehensive papers considered many more electronics and, by comparison, showed that the amount of energy wasted by cellphones is relatively small compared to many other electronic devices. Thus, when the team referred to cellphones as the "most energy consuming devices" in the Executive Summary, judges questioned the credibility of the paper.

For Requirement 5, students were to consider the population and economic growth of a Pseudo U.S. for the next 50 years and predict energy needs for providing phone service based on their analysis in the first three Requirements. Predictions were to be interpreted in terms of barrels of oil used. To their credit, the Lawrence Tech team had numerous appendices with data tables (but again without reference).

Recognizing Limitations of the Model

Recognizing the limitations of a model is an important last step in the completion of the modeling process. The students recognized that their model failed to look at technological changes, including advances in battery and cellphone technology. They also acknowledged that assuming that every member of a population has a cellphone puts cellphones into the hands of infants and ignores the fact that some individuals have more than one cellphone.

Conclusion

Although there were some deficiencies in Requirements 2–5, the quality of the mathematical modeling done in Requirement 1, coupled with the excellent use of resources to answer the questions posed throughout, made the Lawrence Technological University paper one that the judges felt was worthy of the Meritorious designation. The team is to be congratulated on their analysis, their clarity, and their use of the mathematics that they knew to create and justify their own model for the cellphone revolution problem.

About the Authors

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