

EXTRA CREDIT ASSIGNMENT
MATH 340 FALL 2016

Instructions: Solve the following problems. Each problem will give you 2 extra points, which will be added to your Midterm 2 grade. Write out the solutions to the problems in as much detail as you can. Only detailed solutions will be graded. Turn in the assignment to your TA during your discussion or during their office hours before Dec 10th.

For problems 1, 2 and 3 read Section 8.1 from your textbook.

Problem 1. Consider a living organism that can live to a maximum age of 2 years.

Let $A = \begin{bmatrix} 0 & 0 & 10 \\ 0.5 & 0 & 0 \\ 0 & 0.2 & 0 \end{bmatrix}$ be the matrix describing the growth of the population of this organism.

- (1) Describe the meaning of each value in this matrix.
- (2) Show that this population has a stable age distribution.
- (3) If there are 70 organisms in the first age group, how many must there be in the second and third in order for this to constitute a stable age distribution?

Problem 2. Suppose that at some initial point in time 100,000 people live in a certain city and 25,000 people live in its suburbs. Each year 5% of the city population moves to the suburbs and 3% of the suburban population moves to the city.

- (1) Assuming that the total population remains constant, what will the populations of the city and its suburbs be over the next five years? (round to the nearest integer).
- (2) Over the long term, how will the population be distributed between the city and its suburbs?

Problem 3. In a laboratory experiment, a mouse can choose one of two food types each day, type I or type II. Records show that if the mouse chooses type I on a given day, then there is a 30% chance that it will choose type I the next day, and if it chooses type II on one day, then there is a 60% chance that it will choose type II the next day.

- (1) Find a transition matrix for this phenomenon.
- (2) If the mouse chooses type I today, what is the probability that it will choose type I two days from now?
- (3) What is the steady state vector?

For problems 4 and 5 read “Application A: Recursion Relation; the Fibonacci Sequence” on page 50 from the textbook.

Problem 4. Let A be the matrix such that

$$\begin{bmatrix} u_n \\ u_{n-1} \end{bmatrix} = A \begin{bmatrix} u_{n-1} \\ u_{n-2} \end{bmatrix},$$

where u_n is the n -th Fibonacci number.

- (1) Show that A has two distinct eigenvalues and that the eigenvectors associated with each value are linearly independent.
- (2) Find D and S so that $A = S^{-1}DS$ and D is diagonal.

Problem 5. Use the presentation of A as $S^{-1}DS$, obtained in the previous problem to give a direct expression for the n -th Fibonacci number that does not depend on previous values of the sequence.