

HOMEWORK 5

MATH 571 FALL 2019

A register machine on the alphabet $A = \{a_0, a_1, \dots, a_k\}$ consists of a sequence of registers $R_0, R_1, \dots$, each of which may contain a word on A . Register R_0 is the input/output register. Initially all registers contain the empty word ε , except for maybe the register R_0 which contains the input for the machine.

An instruction for such a machine always starts with a label L , which is a natural number. We have the following 5 types of instructions:

- (1) $L : \text{Let } R_i = R_i + a_j$ (Appends the letter a_j at the end of the word in register R_i).
- (2) $L : \text{Let } R_i = R_i - a_j$ (Checks if the word in R_i ends with a_j then it deletes this a_j ; otherwise it leaves the word unchanged).
- (3) $L : \text{If } R_i = \varepsilon \text{ then } L' \text{ otherwise } L_0 \text{ or } L_1 \text{ or } \dots \text{ or } L_k$ (Checks if the word in register R_i is ε and if so goes to instruction labelled L' ; otherwise it looks at the last letter b of the word in R_i and goes to the corresponding instruction (if $b = a_i$ then it goes to L_i)).
- (4) $L : \text{Print}$ (Prints as output the word in register R_0).
- (5) $L : \text{Halt}$ (Stops the program).

We will abbreviate by “ L : Goto L' ” the instruction of the form

$L : \text{If } R_i = \varepsilon \text{ then } L' \text{ otherwise } L' \text{ or } L' \text{ or } \dots \text{ or } L'$

A program for such a machine is a finite sequence of instructions $I_0, I_1, I_2, \dots, I_n$, such that: I_i has label i ; every “If then” instruction refers to labels in the interval $[0, \dots, n]$; only the last instruction is Halt.

Exercise 1. Let $A = \{a, b, c\}$. Write a program for the register machine on alphabet A that halts precisely when its input is the word “ $abac$ ” and loops forever otherwise.

Exercise 2. Describe what the following program for the register machine on alphabet $\{a, b\}$ does when it is given as input a word in the alphabet $\{a, b\}$. What is the effect of the program specifically when the input is $abbababbbba$.

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| 0: If $R_0 = \varepsilon$ then 7 else 1 or 4 | 12: Let $R_2 = R_2 + b$ |
| 1: Let $R_0 = R_0 - a$ | 13: Goto 7 |
| 2: Let $R_1 = R_1 + a$ | 14: If $R_2 = \varepsilon$ then 21 else 15 or 18 |
| 3: Goto 0 | 15: Let $R_2 = R_2 - a$ |
| 4: Let $R_0 = R_0 - b$ | 16: Let $R_0 = R_0 + a$ |
| 5: Let $R_1 = R_1 + b$ | 17: Goto 14 |
| 6: Goto 0 | 18: Let $R_2 = R_2 - b$ |
| 7: If $R_1 = \varepsilon$ then 14 else 8 or 11 | 19: Let $R_0 = R_0 + b$ |
| 8: Let $R_1 = R_1 - a$ | 20: Goto 14 |
| 9: Let $R_2 = R_2 + a$ | 21: Print |
| 10: Goto 7 | 22: Halt |
| 11: Let $R_1 = R_1 - b$ | |

Exercise 3. Prove that the set of decidable sets over a fixed finite alphabet A is closed under union, intersection and complement.