DENSE COMPUTABILITY, EFFECTIVELY DENSE COMPUTABILITY, AND RELATED REDUCIBILITIES

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I will discuss joint work with Eric Astor and Denis Hirschfeldt on algorithms which are correct with (asymptotic) density 1. A function $f : \omega \to \omega$ is called *densely computable* if there is a partial computable function φ such that f and φ agree on a set of density 1. This notion is weaker than coarse computability (where φ is required in addition to be total) and generic computability (where φ is required in addition to agree with f whenever φ converges). A function $f : \omega \to \omega$ is called *effectively densely computable* if f is witnessed to be densely computable by a partial function φ with a computable domain. This notion is stronger than both generic and coarse computability. We also study asymptotic approximations to these notions where we replace sets of density 1 by sets of lower density arbitrarily close to 1. We determine which Boolean combinations of these notions and their asymptotic approximations, as well as coarse and generic computability, are satisfiable, and show that all such satisfiable properties are satisfied by c.e. sets. We also study natural reducibilities which correspond to dense and effectively dense computability. We show that the Turing degrees can be embedded into the corresponding degree structures in a natural way. These embeddings are not surjective, and indeed we prove that sufficiently random sets have quasiminimal degree. We prove that, in the degrees corresponding to generic, dense, and effectively dense computability, all nontrivial upper cones have measure 0 and that there are minimal pairs in the dense degrees. This work has appeared in [1].

References

[1] Eric P. Astor, Denis R. Hirschfeldt, and Carl G. Jockusch, Jr., Dense computability, upper cones, and minimal pairs, *Computability*, vol. **8**, no. 2 (2019), 155–177.