KENNETH KUNEN. ***The Foundations of Mathematics.*** Studies in Logic, Mathematical Logic and Foundations, vol. 19. College Publications, London, 2009, vii + 251 pp.

The book under review, written by my colleague Ken Kunen, is intended as a first logic book at the graduate level, not assuming any prior knowledge of logic but just the usual mathematical maturity of a beginning graduate student in mathematics (including some elementary facts from undergraduate algebra).

There are very few books of this kind on the market that I am aware of, Ebbinghaus/Flum/Thomas, Poizat and Hinman being the only three I can think of, but the first is more intended for German undergraduates (and so proceeds at a slightly slower pace), the second starts with general logic but then focuses mostly on model theory for a second semester, and the last is much more encyclopedic, covering at least two semesters of material all across logic. Kunen's book can be used for a fast-paced one-semester course (as I just did this semester) if one is willing to make some minor cuts.

The book is in three main chapters, with a brief introductory chapter on "Why logic?" and a brief intermezzo chapter on the philosophy of mathematics. The three main chapters address set theory, model and proof theory, and recursion theory, respectively. Most logic books start by defining first-order predicate logic and go all the way to the Completeness Theorem without talking about well-orderings or Choice, which makes the proof of the Completeness Theorem for uncountable languages awkward. Kunen chose the opposite approach, as a set theorist: He starts with the axioms of ZFC and gives an intuitive explanation of what "definable" means in the language of set theory so that he can talk about Comprehension and classes. He then spends about eighty pages on the fundamentals of set theory, like ordinals and cardinals and their arithmetic, induction and recursion, and variants of the Axiom of Choice. Some of the exercises address the independence of the axioms of ZFC.

The second chapter covers model and proof theory over about one hundred pages. It starts with a careful formal set-theoretic definition of Polish notation and unique readability and then proceeds to the definition of first-order syntax and semantics. A proof theorist may have a bit of a quibble with the proof calculus presented: All propositional tautologies are axioms, and free variables are not allowed in the proof calculus. But a few pages later, Kunen explains how one can introduce free variables in proofs through the back door, as constant symbols. The proof of the Completeness Theorem is probably the most careful writeup of this proof I have ever seen, with explanations on why the next step is necessary at every turn. The last forty pages of this chapter then address some further topics in model and proof theory, like elementary equivalence and elementary submodels, Horn theories and some weaker set and proof theories. (For time reasons, I skipped most of these forty pages.)

The last chapter covers recursion theory over fifty pages, but with a twist: As a true set theorist, Kunen uses HF as the universe for computation rather than ω, and Δ_1-definability as the definition of "computability", which may cause some recursion theorists to shake their heads. However, Kunen explains that it makes the coding significantly easier, and he is very careful in presenting in full the proof of the Incompleteness Theorems; he also explains that he chose this approach since it will be new to most students and so capture their interest better than hand-waving about Turing machines. As a recursion theorist myself, I have to admit I had qualms at first but in the end found Kunen consistent in his approach and chose to follow him pretty closely; he intersperses many comments about how this relates to computability as it is usually defined and even provides several justifications of the Church-Turing Thesis for his approach. (The only caveat is that one has to cover section II.17 on Δ_0-definability and absoluteness late in the model theory chapter to give the background necessary for that last chapter.)

Overall, I found Kunen's book masterfully written, with many carefully chosen examples, interesting philosophical comments, and a sufficient (albeit not abundant) number of exercises of varying difficulty, many with broad hints. He definitely espouses a more set-theoretical perspective in his presentation than most other general logic books, which leads to many arguments being more syntactic and proof-theoretic than model-theoretic, but he is being very consistent in this approach throughout the book. In summary, I enjoyed teaching from the book, my students also gave me plenty of positive feedback, so I wholeheartedly recommend it for a one-semester graduate logic course.

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