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Finite Reduction Trees in Modal Logic

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A standard proof-search procedure for a propositional formula A is an attempt to construct a model for $\neg A$ by using some tableau rules. In a logic like $S4$ the resulting tree may be infinite. By placing restrictions on the application of the rules, finite procedures can be obtained. These restrictions may be regarded as *metarules*.

In this talk I present the method of finite reduction trees that are derivations in a propositional reduction system. Such a system for a logic $\mathcal{L}$ consists of *one* rule:

$$(\mathsf{R}) \qquad \frac{F}{F_1, F_2, ..., F_n}$$

satisfying the following condition:

$F \in \mathcal{L}$ *if and only if* $F_1 \in \mathcal{L}$ *or ... or* $F_n \in \mathcal{L}$.

A reduction tree for a formula A is a finite tree Υ such that:

(1) The origin of Υ is A.

(2) If $F_1, ..., F_n$ are the immediate successors of a node F, then $F_1, ..., F_n$ are obtained from F by R.

(3) The end nodes of Υ are formulas whose validity is easy to decide.

Such reduction trees can be obtained from syntactic refutation procedures (see [1]) by *deleting* the applications of modus ponens and reverse modus ponens (B/A where $\vdash A \to B$). The completeness proof is a simple inductive argument. In each inductive step we have one application of one big rule R. As a result, we can have finite reduction procedures that are justified by *proofs* that are exact, simple, and constructive.

[1] T. Skura, Refutations, proofs, and models in the modal logic K4, *Studia Logica*, 70: 193-204, 2002.