WHAT IF COMPUTERS COULD COUNT TO INFINITY?

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Abstract. This paper describes work in progress in weak systems of arithmetic and applications. We define a nonstandard version of the theory of polynomial time functions in which the usual fundamental principles of nonstandard mathematics, like saturation and transfer, are integrated. Our main result is that, over the finite numbers, the nonstandard poynomial time functions contain the recursive functions. Given the big difference in computational complexity between the standard polynomial time functions and the recursive ones, the result is surprising.

Hypercomputation with computers effectively executing infinitly many calculations in a finite time has already been considered in quantum mechanics and elsewhere. Our nonstandard theory of polynomial time functions is a very natural abstraction of this computational concept, and allows unexpected insights in the feasibility and power of such a hypercomputer.

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