Positive Realizability Morphisms and Tarski Models

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Definition 1A formula is said to be positive iff it is built from atomic formulas using only the connectives &, v and the quantifiers∀, ∃. □

Definition 2 A formula φ (x1,x2,...,xn) is preserved under homomorphisms iff for any homomorphisms f of a model A onto a model B and all a1,...,an in A if A |= [a1,...,an] B ⊨ [fa1,...,fan]. □

Theorem 1A consistent theory is preserved under homorphisms iff T has a set of positive axioms. □

Positive forcing (author 1981) had defined T* to be T augmented with induction schemas on the Generic diagram functions. That can effectively generates Tarskian models since Tarskian presentions can be created with Skolemization on arbitrary sentences on generic diagrams with the Skolem fucntions instantialting the generic diagram functions.

Proposition 1 Let ℜ and Ŋ be models for L. Then ℜ is isomorphically embedded in Ŋ iff Ŋ can be expanded to a model of the diagram of ℜ.□

Proposition 2 Let ℜ and Ŋ be models for L. Then ℜ is homomrphically embedded in Ŋ iff Ŋ can be expanded to a model of the positive diagram of ℜ.□

Let Σ be a set of formulas in the variables x1…xn. Let ℜ be a model for L. We say that ℜ realizes Σ iff some n-tuple of elements of A satisfies Σ in ℜ. ℜ omits Σ iff ℜ does not realize Σ. □

For our purposes we define a new realizability basis.

Definition 3 Let Σ (x1…xn) be a set of formulas of L. Say that a positive theory T in L positively locally realize Σ iff there is a formula φ (x1…xn) in L s.t.
(i) φ is consitent with T
(ii) for all σ ε Σ, T ⊨ φ or T ∪ σ is not consistent. □

Defintion 4 Given models A and B, with generic diagrams D_A and D_B we say that D_A homomrphially extends D_B iff there is a homomorhic emdedding f: A → B. □

Consdier a complete theory T in L. A formula φ (x1,.,xn) is said to be complete (in T) iff for every formula ψ(x1,.,xn), exactly one of T ⊨ φ → ψ or T ⊨ φ → ¬ ψ . A formula θ (x1,…,xn) is said to be completable (in T) iff there is a complete formula φ (x1,.,xn) with T models φ → θ. If that can't be done θ is said to be incompletable.

Theorem 2 Let L1, L2 be two positive languages. Let L = L1 ∩ L2. Suppose T is a complete theory in L and T1 ⊃ T, T2 ⊃ T are consistent in L1, L2, respectively. Suppose there is model M definable from a positive diagram in the language L1 ∪ L2 such that there are models M1 and M2 for T1 and T2 where M can be homorphically embedded in M1 and M2.
(i) T1 ∪ T2 is consistent.
(ii) There is model N for T1 ∪ T2 definable from a positive diagram that homomorphically extends that of M1 and M2. □

Theorem 3 (author ASL March 07) Let L1, L2 be two positive languages. Let L = L1 ∩ L2. Suppose T is a complete theory in L and T1 ⊃ T, T2 ⊃ T are consistent in L1, L2, respectively. Then

(i) $T1 \cup T2$ has a model M, that is a positive end extension on Models M1 and M2 for T1, and T2, respectively si

(ii) M is definable from a positive diagram in the language $L1 \cup L2$. □

Theorem 4 Considering a Tarskian presentation P for a theory T that has a positive local realization, with T* we can assert the following. Every formula on the presentation P is completable in T*. □

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