A remark on a characterization of non-forking in generic structures

Koichiro IKEDA

Let L be a countable relational language and $\mathbf{K}$ a class of finite L-structures with non-negative predimension. Then a countable L-structure M is said to be $\mathbf{K}$-generic, if (i) any finite $A \subset M$ belongs to $\mathbf{K}$; (ii) for any finite $A \leq B \in \mathbf{K}$ with $A \leq M$ there is a copy B' of B over A with $B' \leq M$; (iii) there are finite $A_0 \leq A_1 \leq A_2 \leq \cdots \leq M$ with $M = \bigcup_i A_i$. Let M be a saturated generic structure and $\mathcal{M}$ a big model of $\mathrm{Th}(M)$. In [1], [2] and [3], non-forking in $\mathcal{M}$ has been characterized as follows: For any $A \leq B, C \leq \mathcal{M}$ with $A = B \cap C$ *algebraically closed*, $\mathrm{tp}(B/C)$ does not fork over A if and only if B, C are free over A and $BC \leq \mathcal{M}$. It is seen that one cannot remove the condition that A is algebraically closed in the above statement. Our aim is to generalize the statement as follows: For any $A \leq B, C \leq \mathcal{M}$ with $A = B \cap C$, $\mathrm{tp}(B/C)$ does not fork over A if and only if B, C are free over A and $BC \cup \mathrm{acl}(A) \leq \mathcal{M}$. As a corollary, it can be proved that there is no saturated generic structure that is superstable but not ω-stable.

REFERENCES

[1] J.T. Baldwin and N. Shi, Stable generic structures, Annals of Pure and Applied Logic 79 (1996) 1–35

[2] V. Verbovskiy and I. Yoneda, CM-triviality and relational structures. Annals of Pure and Applied Logic 122 (2003), no. 1-3, 175–194

[3] F.O. Wagner, Relational Structures and Dimensions, Automorphisms of First-Order Structures, The Clarendon Press, Oxford University Press, New York, 1994, pp. 153–180

Faculty of Business Administration
Hosei University
2-17-1 Fujimi, Chiyoda
Tokyo 102-8160, Japan
ikeda@hosei.ac.jp