TOWARDS A NOTION OF p-ADIC CLOSURE FOR A RING

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In [1] L. Bélair generalizes results in [4] by giving an axiomatization, denoted by $ALpC_{e,f}$, of local p-adically closed rings of p-rank d, i.e. henselian local rings with residue field which are p-adically closed field of p-rank d ($d = ef$ with e and f fixed). The results extend the ones obtained by E. Robinson about the p-adic spectrum. Then he completes the theory $ALpC_{e,f}$ in a scheme of axioms $AIpC_{e,f}$ whose models are henselian valuation rings whose residue fields are p-adically closed fields of p-rank d and value groups are divisible.

Particular examples of models of $AIpC_{e,f}$ are provided and studied in Section 3 of [1]: if C is an affine curve on $\mathbb{Q}_p$ then L. Bélair shows that for any prime non maximal ideal $\mathfrak{p}$ of the ring of continuous definable functions $C \to \mathbb{Q}_p$, denoted $\mathcal{C}(C)$, we have $\mathcal{C}(C)/\mathfrak{p}$ is a model of $AIpC_{e,f}$.

To this effect, the powerful tool used in this paper is the existence of an homeomorphism between the p-adic spectrum of $\mathbb{Q}_p[V]$ (V is an affine variety over $\mathbb{Q}_p$) and the ring spectrum of $\mathcal{C}(V)$.

In [2], L. Bélair generalizes his previous approach to the ring of continuous definable functions on V over $\mathbb{Q}_p$, denoted by $\mathcal{C}(V)$, for any affine p-adic variety V. He studies, in particular, first-order properties of the quotient rings $\mathcal{C}(V)/\mathfrak{p}$ by a prime ideal $\mathfrak{p}$ of $\mathcal{C}(V)$.

This talk is devoted to give an account of the work in [3]. In this work, we introduce a new class of rings called p-adically closed rings. For this purpose, we build the p-adic closure of a given ring A in a similar way as in [5] for the real closure of a ring. Namely, the topological properties of the p-adic spectrum of A plays a central role in this construction.

Then it allows us to define the notion of a p-adically closed ring and to generalize the results from [1] and [2] to the p-adically closed rings.

References

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Date: May 14, 2007.
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