AXIOMATIZATIONS FOR INTERSECTIONS OF SUBSTRUCTURAL LOGICS.

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Substructural logics include classical, intuitionistic, linear, relevant and many-valued logics, and their study provides a framework for the comparative study of various logics. Substructural logics are defined as axiomatic extensions of the sequent calculus **FL**; the latter is essentially obtained from Gentzen's system **LJ** for intuitionistic propositional logic, by omitting the structural rules of *exchange*, *contraction* and *weakening*. *Residuated lattices* form algebraic semantics for substructural logics and algebraic methods have been applied successfully to the logical investigation.

Substructural logics ordered under inclusion, form a lattice. Given axiomatizations for two substructural logics, their join in the lattice (the least logic containing both of them) is axiomatized by the union of the axiomatizations. On the other hand, the meet of the two logics (the greatest logic containing both of them) is their intersection, but the intersection of their axiomatizations does not axiomatize the meet; for example, it can be empty.

Using algebraic methods, we provide axiomatizations for intersections of substructural logics that also work in case the language lacks the disjunction connective. In particular, we prove that the intersection of two finitely axiomatized subcommutative substructural logics has a finite axiomatization, as well. En route, we investigate the finitely subdirectly irreducible algebras of residuated (semi)lattices (if join is not included in the language) and show that if they form an elementary class, the finitely axiomatized extensions of the corresponding substructural logic are closed under intersections.

References

[1] N. Galatos, *Equational bases for joins of residuated-lattice varieties*, Studia Logica **76** (2004), no. 2, 227–240.

[2] N. Galatos, J. S. Olson, and J. G. Raftery, *Irreducible residuated semilattices and finitely based varieties*, manuscript.

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