Admissible rules for the multi modal logic of knowledge and time LTK_1

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Multi-modal propositional logics with operators representing both time and knowledge are particularly effective to describe the interaction of agents throughout the flow of time ([1, 2, 3, 4, 5, 6]). These systems are usually generated by adding to an existing propositional logic two sets of modalities: one to model the flow of time and one to describe agents' knowledge. The interaction of such modalities gives a precise account of the dynamic development of agents' knowledge.
However, despite the power of multi-modal propositional logics, multi-modal languages can only express formulae which are static in a way: the statements only fix a fact, and cannot handle a changing environment, although this is required to model human reasoning, computation and multi-agent environments. Sometimes it might be more useful to discover what follows given some premises, rather than knowing logical truths. For this purpose, inference rules, or logical consecutions, are a core instrument.

Our research aims at investigating a multi-modal propositional logic, LTK (Linear Time and Knowledge), which combines tense and knowledge modalities. This logic is semantically defined as the set of all $\mathcal{LTK}$-valid formulae, where $\mathcal{LTK}$-frames are multi-modal Kripke-frames combining a linear and discrete representation of the flow of time with special $S5$-like modalities, defined at each time cluster and representing agents' knowledge.

So far we have proved that: (i) LTK has the finite model property [1]; (ii) LTK has a finite axiomatisation [3]; (iii) LTK_1, a weaker version of LTK with only one agent operating in the system, is decidable with respect to its admissible inference rules [2]. Our latest results is to show that LTK_1 has a finite basis for admissible inference rules, i.e. all those rules under which the logic itself is closed.

References

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