

**Math 748 - 001 Syllabus**  
**Algebraic Number Theory**  
**MWF 9:55 AM - 10:45 AM, Van Vleck B215**

3 credits by the traditional Carnegie definition;  $3 \times 50$ -minutes lectures/week

**A crash introduction to the course:** Number theorists study prime numbers as well as the properties of mathematical objects made out of integers (for example, rational numbers) or defined as generalizations of the integers (for example, algebraic integers). Being one of the oldest branch of math, Number Theory can be dated back to ancient Greece where people studied integer solutions of the Pythagorean equation and proved there are infinitely many prime numbers. Despite its long history, Number Theory is still going through huge progress in the 20th centuries with important applications to computer science, especially to cryptography discovered.

The class will be a beginner's guide to algebraic number theory, a rigorous introduction to the arithmetic of number fields: algebraic integers, geometry of numbers, Dirichlet's Unit Theorem, ideal class groups, first case of Fermat's Last Theorem; prime decompositions, Galois automorphisms. We will focus on number fields with a glimpse at function fields. If time permits, we will introduce the main results of Class Field Theory (no proof).

**Textbook:** *Algebraic Number Theory*, Jürgen Neukirch, Edition 1, Springer.

**Supplemental Textbook:** *Algebraic Number Theory*, Milne, freely available here. *Number Fields*, Marcus. freely available here.

**Pre-requisite:** MATH 541, 542 or Math 741.

**Instructor:** Yousheng Shi, Department of Mathematics, shi58@wisc.edu,  
Homepage:<https://people.math.wisc.edu/shi/>

**Office hour:** Monday 10:55am-11:55am, Wednesday 1:00-2:00pm or by appointments.  
The default way of office hour is via Zoom:<https://uwmadison.zoom.us/j/2398596908>. If you would like, we can also meet in my office which is Van Vleck 720.

**Canvas:** Canvas will be our main media of online communication. All course materials will be posted on Canvas.

**Grading policy:** Grade will be based on participating the class and weekly homework.

**Content of the course:** We will cover Chapter I, II of Neukirch which is roughly equivalent to the whole book of Milne. If time permits, we will go over Chapter IV of Neukirch.

### **Homework Policy**

There will be 12 homework problem sets, roughly speaking one set per week and no more than 5 problems per set. Each problem set will be announced each Friday and will be due next Friday. The problems will be graded and the lowest one will be dropped. You are welcomed to discuss the problems with your classmates, but are supposed to think about and write up the solutions independently. Any collaboration or used resources beside the textbook must be mentioned.