Introduction to Representation Theory
Yale University - Fall 2016
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Tentative List of Topics

The combination of the lectures, projects, and homework assignments, of this course will enable us to cover part of the following topics:

1. **A motivation.**
   (a) Why some "nice" operators tend to diagonalize explicitly?
   (b) Example: The discrete Fourier transform (DFT).
2. **Groups.**
   (a) Category of groups.
   (b) Group action on sets. Category of G-sets.
   (c) Decomposition into irreducible actions. Counting principle.
3. **Representations of a finite group G.**
   (a) Basic definitions. Category of representations of a group G.
   (b) Irreducible representations. Schur's lemmas.
   (c) Natural constructions with representations.
   (d) Complete reducibility.
4. **Basic results about representations of finite groups**.
   (a) Intertwining numbers and their properties.
   (b) Decomposition of the regular representation.
   (c) Group algebra and its structure.
5. **Character theory.**
   (a) Definition of a character.
   (b) Schur's Orthogonality relations. Character rings.
6. **Frobenius reciprocity and Mackey theory.**
   (a) General notions from category theory. Restriction and induction functors.
   (b) Explicit construction of induction functor using equivariant sheaves.
   (c) Frobenius formula for the character of the induced representation.
   (d) Mackey's theory.

7. **Representations of abelian groups. Fourier transform.**
8. **Representations of semi-direct products.**
9. **Representations of symmetric groups.**
10. **Representations of the Heisenberg group.**
11. **Oscillator representation of the group** $Sp_{2n}(\mathbb{F}_q)$.
12. **Construction of irreps of the group** $SL_2(\mathbb{F}_q)$ **via quantization.**
13. **Construction of representations of** $SL_2(\mathbb{F}_q)$ **via** ℓ**-adic cohomology**
14. **Some results about representations of topological groups.**
    (a) Representations of commutative groups and Fourier transform.
    (b) Basic results about representations of the compact groups.
    (c) Representations of $G = SO(3)$ and spherical harmonics.
15. **Representation of the Lie Algebra** $sl_2(\mathbb{C})$.
    (a) Highest weight modules and their classification.
    (b) Explicit construction of Highest weight modules.
    (c) Complete reducibility.
    (d) sl_2-triples, Jacobson-Morozov theorem, Jordan canonical form.