Math 843
Representation Theory of Finite & Compact Groups, and Applications
FALL 2011

Instructor: Shamgar Gurevich, 317 VV.
Time and Location: Tue-Thu 9:30-10:45, Room VVB333.
Office Hours: Tuesday 11–12 & 1–2pm

Texts: The course notes. In addition the following books can be of some help: Algebra (M. Artin), Linear representations of finite groups (J.P. Serre), Representations of finite and compact groups (B. Simon), Character theory of finite groups (I.M. Isaacs), Group theory and physics (S. Sternberg), Categories for the working mathematician (S. Mac Lane).

Computer programs: It will be good for you to have the Matlab (you will need to buy it if you want) and Magma (I will distribute the free version) student's software programs.

Course assignments and grading: I will give several problems sets during the semester and you will need to present one of them with your team (two students) in our Monday seminar (5pm VV903). You will need to write notes of two lectures (and present them to me with your two students team). There will be a final project (two people per project) which you will need to present to class. We will have special days for projects presentations during the week of Dec. 5–9. They will count toward the grade as follows:

Notes	20%
Seminar	30%
Project	50%

Syllabus: We will study some parts of the following:

1. **Motivations.**
   (a) The general method of encoding a scientific problem in the language of a transform acting on a vector space, and solution to the problem using natural group representation "symmetries" of the transform.
   (b) Example: Computation of the sign of the Gauss sum, via the discrete Fourier transform, and its analysis using the Weil representation.
   (c) Example: Diagonalization of the discrete Fourier transform using the Weil representation.
   (d) Example: The fast Fourier transform using the Heisenberg representation.
   (e) Example: Tomography, Radon Transform and its inversion using the representation theory of SL_2.

2. **Category of G-sets.**
   (a) Motivations from the study of maps, and formulation of notion of symmetry of object in space.
   (b) Definition of the notion of category.
   (c) The categories of sets, groups, and G-sets. Notion of action of a group on a set. Basic decomposition theorem in the category G-sets.

3. **Category $Rep(G)$ of representations of a finite group G.**
   (a) Motivation from the study of diagonalizations of linear operators.
   (b) Definition of representations, and morphisms between them. The category $Rep(G)$.
   (c) Direct sum of representations, sub-representations, irreducible representations, complete reducibility (Maschke's theorem). Idea of averaging.
   (d) Basic problems of representation theory.
   (e) Schur's lemma.
   (f) Application for diagonalization of intertwiners.

(g) Examples:

   i. Irreducible representations of Abelian groups. Decomposition of the regular representation of Abelian group.
   ii. Irreducible representations of the irreducible representations of $T = Stab_{SO(3)}(P)$, $P =$Thetrahedron.
   iii. Irreducible representations of the finite Heisenberg group $H(\mathbb{F}_p)$.
   iv. Construction of the Weil representation of $SL_2(\mathbb{F}_p)$.

4. **Basic results about representations of finite groups.**

   (a) Intertwining numbers and their properties. Intertwining numbers between permutation representations.

   (b) Decomposition of the regular representation, and corollaries.

   (c) Application: Irreducible representations of the symmetry groups of the platonic solids, $Irr(A_4)$, $Irr(S_4)$, $Irr(A_5)$.

   (d) Group algebra and its structure. Geometric and spectral descriptions. The center of the group algebra. Corollaries.

   (e) Natural constructions of representations: direct sum, dual, tensor product, Hom.

5. **Character theory.**

   (a) Motivations: computation of intertwining numbers, natural basis for the center of the group algebra, explicit description of isotypic components of representations.

   (b) Definition of a character.

   (c) Answers to questions from motivations using Schur othogonality relations.

   (d) Schur orthogonality relations. Character of tensor product. Character rings.

   (e) Represenation of product.

6. **Induced representation.**

   (a) Motivations: $Irr(SL_2(\mathbb{F}_p)) =?$, Construction of irreducible representations of the Heisenberg group $H(\mathbb{F}_p)$.

   (b) General notions from category theory. Restriction and induction functors.

   (c) Frobenius formula for the character of induced representation.

Good Luck!