Notes for May 2nd.

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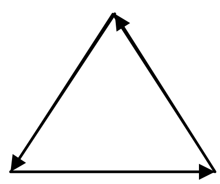
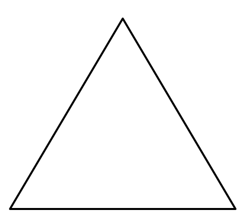
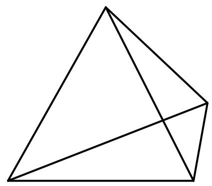
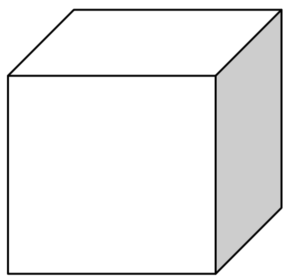
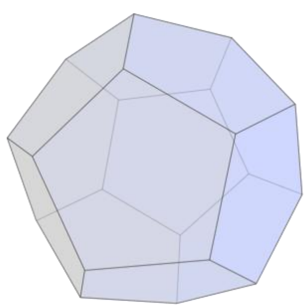
Symmetries in R^3

$X= R^3$

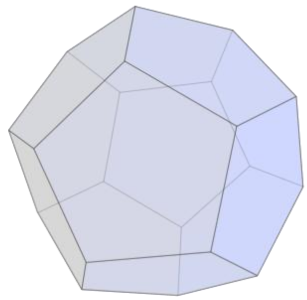
$G=SO(3)=\{A\in Mat(3, R^3)|\ \forall u,v\in R^3,$
$\langle Au,Av\rangle=\langle u,v\rangle,$
$\det(|A|)=1\}$

Correspondence:

Subset:	Subgroups
$y\subset R^3$, y →	$Stab_{SO(3)}(y)$
yz-plane  →	$C_3=\{\begin{pmatrix}1 & 0\\ 0 & r_\theta\end{pmatrix}$, $r_\theta=\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}$ $\theta\in(0,120,240)\}$
yz-plane P_n(**oriented** n-polygon) →	$C_n=\{\begin{pmatrix}1 & 0\\ 0 & r_\theta\end{pmatrix}$, $r_\theta=\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}$ $\theta=(360/n)*k$, $(k=0,1,2\ldots n-1)\ \}$
yz-plane  →	$D_3=\{\begin{pmatrix}1 & \\ & D_3\end{pmatrix}\}$
yz-plane, P_n	$D_n=\{\begin{pmatrix}1 & \\ & D_n\end{pmatrix}\}$

Pyramid, P 	T= $Stab_{SO(3)}(P) \cong A_4$
Cube, C 	O= $Stab_{SO(3)}(C) \cong S_4$
Dodcahedron, D 	I= $Stab_{SO(3)}(D) \cong A_5$

The Counting Formula:

D= 

Easy to find out: #I= $\#Stab_{SO(3)}(D)=12*5=60$

$\#Stab_{SO(3)}(Faces)=12$

Try to figure out: #Edges=#E(D)=30

#Vertices=#V(D)=20

Theory:

X--set

G:acting on X, finite

Definition: Let $x \in X$, a point.

(i) $G_x=Stab_G(x)=\{g\in G \mid g\cdot x=x\}$

(ii) $O_x=\{ g\cdot x \mid g\in \sigma \}$

Claim: $G_x<G$, subgroup.

Counting formula(orbit stabilizer formula):

$\#G=\# G_x \cdot \# O_x$, for $\forall x\in X$.

For problems above:
For #Vertices, G_v is stabilizer of a vertex v is the group of order 3 of rotation by multiples of $2\pi/3$ about that vertex. Thus 60=3*20, which means #Vertices=20.

Similar for #Edges, $|G_e|=2$, thus 60=2*30, which means #Edges=30.