Math 340
Spring 2014
Final Exam Review
11 May 2014
Time Limit: 2 hours

Name (Print): ______________________

Teaching Assistant ______________________

This exam contains 9 pages (including this cover page) and 4 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

Turn off all cell phones and electronic devices.

Problem	Points	Score
1	25	
2	25	
3	25	
4	25	
Total:	100	

1. **Vector Spaces**

(a) (5 points) Let V be a vector space and suppose $v \in V$. What property must a vector v' satisfy in order to be the additive inverse of v?

(b) (10 points) For each of the following, circle whether or not the subset W is a subspace of the vector space V.

$V = \mathbb{R}^2$

$W = \{\text{circle of radius 1 centered at } (0,0)\}$

Subspace **Not Subspace**

$V = \mathbb{R}$

$W = \{\text{all points } x \in \mathbb{R} \text{ such that } x > 0\}$

Subspace **Not Subspace**

$V = M_3(\mathbb{R})$

$W = \{A \in M_3(\mathbb{R}) \mid A^t = A\}$

Subspace **Not Subspace**

$V = F(X), \text{where } X = \{1, 2, 3\}$

$W = \{f \in V \mid f(1) + f(3) = 0\}$

Subspace **Not Subspace**

(c) (10 points) Fix $A \in M_2(\mathbb{R})$, and let $W = \{B \in M_2(\mathbb{R}) \mid \mathrm{Tr}(AB) = 0\}$. Show that W is a subspace of $M_2(\mathbb{R})$.

2. **Basis & Dimension**

   (a) (5 points) Answer each question.

   1. Let V be a vector space. Define what it means for V to have dimension n.

   2. Let $\mathcal{B}$ and $\mathcal{C}$ be two bases for the vector space V, and let $v \in V$. What is the relationship between $[v]_{\mathcal{B}}$ and $[v]_{\mathcal{C}}$?

   (b) (10 points) Answer each question.

   1. **True** or **False**: Let $W = \text{Span}\left\{ \begin{pmatrix} 1 \\ -1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ 4 \\ 1 \\ 7 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ 0 \\ 5 \end{pmatrix} \right\} \subset \mathbb{R}^4$. The dimension of W is 4.

   2. **True** or **False**: Let $v \in \text{Span}\{v_1, \ldots, v_n\}$, where $v \neq v_i$ for all i. Then $\{v_1, \ldots, v_n, v\}$ is linearly independent.

   3. **Fill in the blank.** Suppose $\mathcal{B} = \{v_1, v_2\}$ and $\mathcal{C} = \{u_1, u_2\}$ are bases for V. Let $v \in V$, then

   $$[v]_{\mathcal{B}} = A[v]_{\mathcal{C}},$$

   where the matrix

   $$A = (\_\_\_\_ , \_\_\_\_).$$

   4. **Fill in the blank.** Let $V = M_2(\mathbb{R})$. The set $W = \{A \in M_2(\mathbb{R}) \mid A = A^t\}$ is a subspace of V. Then

   $$\dim W = \_\_\_\_\_\_\_\_.$$

(c) (10 points) Compute a **basis** for the space of solutions to the following system of equations. Compute the **dimension** of the space of solutions.

$$x - 4y + 2z - w = 0$$
$$2x + 4z + w = 0.$$

3. **Linear Transformations**

(a) (5 points) Let $T : V \to W$ be a linear transformation. Define what it means for T to be an isomorphism.

(b) (10 points) Answer each question.

1. **True** or **False**: Let $T : V \to W$ be a linear transformation. Then $0_W \in \operatorname{Im} T$.

2. **True** or **False**: Let $T : V \to W$ be a linear transformation. If $\ker T = \{0_V\}$ then T is one to one.

3. **Fill in the blank.**: Let $T : V \to V$ be a linear transformation, and suppose $\mathcal{B} = \{v_1, v_2\}$ is a basis for V. Let $v \in V$, and suppose that

$$[v]_{\mathcal{B}} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \qquad [T]_{\mathcal{B}} = \begin{pmatrix} 3 & 2 \\ -1 & 4 \end{pmatrix}.$$

Then

$$[T(v)]_{\mathcal{B}} = \begin{pmatrix} \_\_ \\ \_\_ \end{pmatrix}.$$

(c) (10 points) Let V be the vector space of 2×2 matrices with trace 0. Then

$$\mathcal{B} = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \right\}$$

is a basis for V. Consider the linear transformation $T : V \to V$ defined by $T(A) = A^t$. Compute $[T]_{\mathcal{B}}$.

4. **Eigenvalues & Diagonalization**

(a) (5 points) Let $T : V \to V$ be a linear transformation. Define what it means for T to be diagonalizable.

(b) (20 points) Consider

$$A = \begin{pmatrix} 4 & -2 \\ 3 & -1 \end{pmatrix}$$

and the associated linear transformation $T_A : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T_A(v) = Av$.

1. It turns out that the eigenvalues of T_A are 1 and 2. Show this by finding the characteristic polynomial $p_A(\lambda)$ of A and finding its roots.

2. For each eigenvalue $\lambda_1 = 1$ and $\lambda_2 = 2$ compute an associated eigenvector v_1 for λ_1 and v_2 for λ_2.

3. Write down the 2×2 matrix

$$C = (v_1, v_2)$$

and compute C^{-1}.

4. We know that $A = CDC^{-1}$ where $D = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$. Use this information to compute A^{17}.