

Introduction to Representations of Finite & Compact Groups (Course 843) Spring 2016 - Topics

1. Groups.

- (a) Basic definitions and properties.
- (b) Actions of groups on sets. Category of G -sets. Natural constructions with G -sets.
- (c) Counting principle and applications.

2. Representations of a finite group G .

- (a) Basic definitions. Category of representations of a group G .
- (b) Irreducible representations. Schur's lemmas.
- (c) Natural constructions with representations.
- (d) Complete reducibility.

3. Basic results about representations of finite groups.

- (a) Intertwining numbers and their properties.
- (b) Decomposition of the regular representation.
- (c) Group algebra and its structure.

4. Character theory.

- (a) Definition of a character.
- (b) Orthogonality relations. Character rings.

5. Frobenius reciprocity and Mackey theory.

- (a) General notions from category theory. Restriction and induction functors.
- (b) Explicit construction of induction functor using equivariant sheaves.
- (c) Frobenius formula for the character of the induced representation.
- (d) Mackey's theory.

6. Representations of abelian groups. Fourier transform.

7. Representations of semi-direct products.

8. Representations of symmetric groups.

9. Representations of the Heisenberg group.

10. **Weil representation of the group $G = SL(2, \mathbb{F}_q)$.**
11. **Representations of the group $G = SL(2, \mathbb{F}_q)$ & Θ -correspondence.**
12. **Some results about representations of topological groups.**
 - (a) Representations of commutative groups and Fourier transform.
 - (b) Basic results about representations of the compact groups.
 - (c) Representations of $G = SO(3)$ and Spherical harmonics.
13. **Representation of the Lie Algebra $sl_2(\mathbb{C})$.**