Math 541 Fall 2010
Homework#7, November 30—Counting formula, class equation

Remark. Answers should be written in the following format:
i) Statement and/or Result.
ii) Main points that will appear in your explanation or proof or computation.
iii) The actual explanation or proof or computation.

1. Let G be a group acting on a set X.

   (a) Define the notion of an orbit $\mathcal{O}$ for the action of G on X.

   (b) Choose an element $x \in X$ and consider its orbit $\mathcal{O}_x$. Denote by $G_x = Stab_G(x) = \{g \in G;\ g \cdot x = x\}$ the stabilizer of x in G. The Counting formula tells us that

   $$\#G_x \cdot \#\mathcal{O}_x = ??$$

   (c) Assume that G is a p-group, i.e., G is of order $\#G = p^k$, for some prime p and $k \geq 1$. Suppose that G acts on a set X of cardinality $\#X$ which is not devisible by p. Show that there exists an element $x \in X$ which is a fixed point for G, i.e., such that $g \cdot x = x$ for every $g \in G$ (Hint: $\#X = \sum \#\mathcal{O}$ where the sum is over all the orbits for the action of G on X. If there is no fixed point then you should show using the counting formula that p devides each of the $\#\mathcal{O}$, hence $p|\#X$ contradiction!!).

2. Let G be a group.

   (a) Define the conjugation action of G on G. Define the notion of a conjugacy class C in G.

   (b) What is the class equation of G?

   (c) Compute the class equation for the groups S_3 and S_4.

Remarks
- You are very much encouraged to work with other students. However, submit your work alone.
- I will be happy to help you with the homeworks. Please visit me in my office hours.
- Start to think on your project.

Good Luck!