Math 541 Fall 2010

Homework#5, November 2—Homomorphisms, Kernels and Normal subgroups

Remark. Answers should be written in the following format:
i) Statement and/or Result.
ii) Main points that will appear in your explanation or proof or computation.
iii) The actual explanation or proof or computation.

1. Denote by $S_n = Aut(\{1, ..., n\})$.
   (a) Define the natural action of S_n on the vector space of polynomials over $\mathbb{Q}$ in n variables $\mathcal{P} = \mathbb{Q}[x_1, ..., x_n]$.
   (b) Consider the discriminant polynomial $D(x_1, ..., x_n) = \prod_{1 \leq i < j \leq n} (x_j - x_i)$. Verify the equality $D^\sigma = sgn(\sigma) \cdot D$, where $D^\sigma = \sigma \cdot D$, as you defined in (a), and $sgn(\sigma) \in \{\pm 1\}$.
   (c) Show that the map $sgn : S_n \rightarrow \{\pm 1\}$ defined above is an homorphism.

2. Let H be a subgroup of G. Show that if index $[G : H] = 2$ then H is normal in G. Recall that the index is the number of left (or right) cosets of H in G.

Remarks

- You are very much encouraged to work with other students. However, submit your work alone.
- I will be happy to help you with the homeworks. Please visit me in my office hours.
- Start to think on your project.

Good Luck!