Math 491 - Linear Algebra II, Fall 2015

Homework 8 - Jordan Form

Quiz on 4/30/15

Remark: Answers should be written in the following format:
A) Result.
B) If possible, the name of the method you used.
C) The computation or proof.

1. **Computing Jordan Forms.** For each of the following matrices, find a Jordan matrix J, to which it is similar.

$$A_1 = \begin{pmatrix} 4 & 6 & 0 \\ -3 & 5 & 0 \\ -3 & -6 & 1 \end{pmatrix} \; A_2 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix} \; A_3 = \begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 & 0 \\ -1 & 0 & 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 & 0 \\ 1 & 1 & 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{pmatrix}$$

2. **Classifying Matrices.** Find all matrices up to similarity in $M_{11}(\mathbb{Q})$ with minimal polynomial $m(x) = x(x-1)^2(x+4)^3$ and characteristic polynomial $p(x) = x^2(x-1)^5(x+4)^4$.

3. **Jordan Matrices and Similarity.**
   (a) How many Jordan matrices are there in $M_6(\mathbb{C})$ with minimal polynomial $m(x) = (x+2)^4(x-1)^2$.
   (b) How many matrices up to similarity are there in $M_6(\mathbb{C})$ with minimal polynomial $m(x) = (x+2)^4(x-1)^2$.

4. **A sufficient condition for similarity.** Let $A, B \in M_n(\mathbb{F})$ with $m_A(x) = m_B(x)$, and

$$p_A(x) = p_B(x) = (x-\lambda_1)^{d_1} \cdots (x-\lambda_k)^{d_k}.$$

   Suppose the $\lambda_i \in \mathbb{F}$ are distinct, and for all $1 \le i \le k$, $1 \le d_i \le 3$. Show that A and B are similar.