Math 340 Spring 2014
Practice for Mid-Term Exam

This is a multiple choice exam. Circle the letter or number corresponding to the correct answer.

1. (25 pts) Let

$$A = \begin{pmatrix} 1 & 0 & 2 \\ -1 & 2 & 0 \\ 1 & 1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 1 & -1 \\ 2 & -1 & 1 \\ 0 & 2 & -3 \end{pmatrix}.$$

Mark the matrix below that can be filled in to form $B^{-1}AB$

(a) $\begin{pmatrix} * & -1/2 & -1 \\ * & 12 & -15 \\ * & 8 & -10 \end{pmatrix}$.

(b) $\begin{pmatrix} * & 1/2 & -1 \\ * & 12 & -15 \\ * & 8 & -10 \end{pmatrix}$.

(c) $\begin{pmatrix} * & -1/2 & -1 \\ * & 12 & -2 \\ * & 8 & -10 \end{pmatrix}$.

(d) $\begin{pmatrix} * & 1/2 & -1 \\ * & 12 & -2 \\ * & 8 & -10 \end{pmatrix}$.

2. There are two parts to this problem!

(a) (20 pts) Consider the following system of equations:

$$S: \quad \begin{cases} x + y + 2z = 1; \\ 2x + y + 5z = 3; \\ 4x + 3y + 9z = 5. \end{cases}$$

The set of solution $Sol(S)$ is equal to:

1. $\left\{ \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix} + t \cdot \begin{pmatrix} -3 \\ 4 \\ 1 \end{pmatrix} ; \ t \in \mathbb{R} \right\}$.
2. $\left\{ \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} + t \cdot \begin{pmatrix} -3 \\ 4 \\ 1 \end{pmatrix} ; \ t \in \mathbb{R} \right\}$.
3. $\left\{ \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix} + t \cdot \begin{pmatrix} -3 \\ 1 \\ 1 \end{pmatrix} ; \ t \in \mathbb{R} \right\}$.

4. $\left\{ \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} + t \cdot \begin{pmatrix} -3 \\ 1 \\ 1 \end{pmatrix} ;\ t \in \mathbb{R} \right\}$.

(b) (5 pts) Select the correct geometric interpretation of the solution of this system of equations.

1. Three parallel planes.
2. Three planes intersecting at a line.
3. A line through the origin.
4. A point.

3. (25 pts) Let A be an $n \times n$ matrix. Consider the linear transformation $T_A : \mathbb{R}^n \to \mathbb{R}^n$, defined of course by $T_A(\mathbf{v}) = A \cdot \mathbf{v}$. Let $\mathrm{Im}(T_A) = \{T_A(\mathbf{v});\ \mathbf{v} \in \mathbb{R}^n\}$. Which of the following is true:

(a) There exists a matrix A and vectors $\mathbf{u}, \mathbf{w} \in \mathrm{Im}(T_A)$, such that $\mathbf{u} + \mathbf{w} \notin \mathrm{Im}(T_A)$.

(b) There exists a matrix A such that $\mathrm{Im}(T_A)$ is empty.

(c) For all matrices A, vectors $\mathbf{u}, \mathbf{w} \in \mathrm{Im}(T_A)$, and $\alpha \in \mathbb{R}$, we have $\mathbf{u} + \mathbf{w} \in \mathrm{Im}(T_A)$, and $\alpha \cdot \mathbf{u} \in \mathrm{Im}(T_A)$.

(d) There exists a matrix A, a vector $\mathbf{u} \in \mathrm{Im}(T_A)$, and $\alpha \in \mathbb{R}$ such that $\alpha \cdot \mathbf{u} \notin \mathrm{Im}(T_A)$.

4. (25 pts) Let

$$A = \begin{pmatrix} 3 & 2 & 1 \\ 2 & 1 & 0 \\ 1 & 1 & -1 \end{pmatrix}.$$

For any invertible $C \in M_3(\mathbb{R})$ we have that $\det(CA^tC^{-1})$ is

(a) -2.

(b) 0.

(c) 1.

(d) 2.

Good Luck!